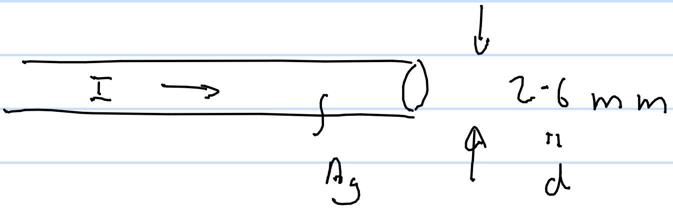
Section 6 : Current & Ohm's Law

Numerical Exercises

1.



$n = 5.8 \times 10^{28}\ m^{-3}$

$$I = \frac{dQ}{dt} = \frac{120}{0 \times 60} = 0.0875\ A \approx \underline{88\ mA}$$

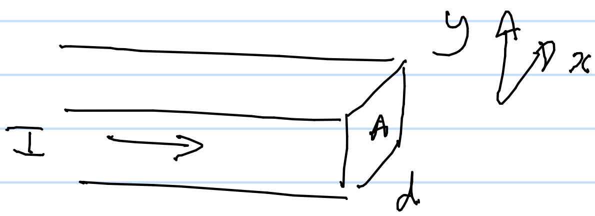
$$J = \frac{I}{A} = q\,v \Rightarrow v = \frac{I}{nqA} = \frac{I}{n\underset{=e}{q}\,\pi (d/2)^2}$$

$$\Rightarrow v = 1.8 \times 10^{-6}\ ms^{-1}$$

$$\underline{v = 1.8\ \mu m.s^{-1}}$$

Problems

1.



$d = \sqrt{A} = 0.25\ mm$

$$J = J_0 (1 - ax^2)(1 - by^2), \quad I = J_0 \int_{-d/2}^{d/2} \int_{-d/2}^{d/2} (1 - ax^2)(1 - by^2)\, dx\, dy$$

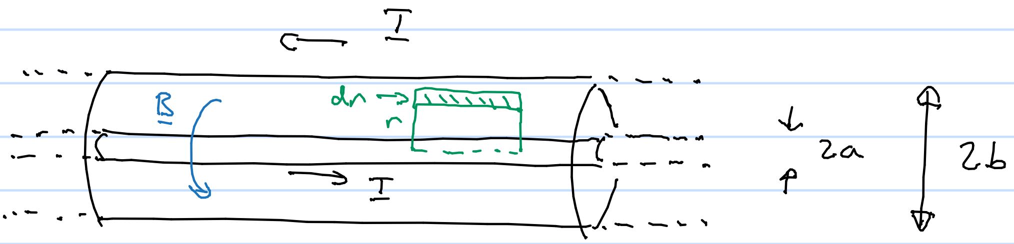
$$I = J_0 \left[x - \frac{ax^3}{3}\right]_{-d/2}^{d/2} \left[y - \frac{by^3}{3}\right]_{-d/2}^{d/2} = 4 J_0 \left[x - \frac{ax^3}{3}\right]_0^{d/2} \left[y - \frac{by^3}{3}\right]_0^{\frac{d}{2}}$$

$$I = J_0 d^2 \left(1 - \frac{ad^2}{12}\right)\left(1 - \frac{bd^2}{12}\right) = 2.47 \times 10^{-5}\ A$$

$$= \underline{25\ \mu A}$$

Section 8.5.3

(1)

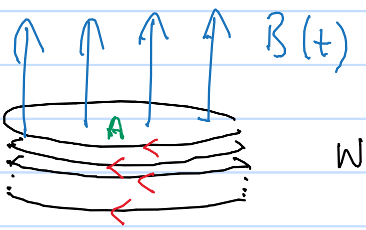


Flux through:

$$\mathcal{E} = -L\frac{dI}{dt} = -\frac{\partial \phi_B}{\partial t} \qquad B = \frac{\mu_0 I}{2\pi r}, \qquad d\phi_B = \frac{\mu_0 I}{2\pi r}\,\ell\, dr$$

$$\phi_B = \frac{\mu_0 \ell I}{2\pi}\ln\left(\frac{b}{a}\right) \Rightarrow \frac{\phi_B}{\ell} = \underbrace{\frac{\mu_0 \ln\left(\frac{b}{a}\right)}{2\pi}}_{= L/\ell} \cdot I$$

(2)

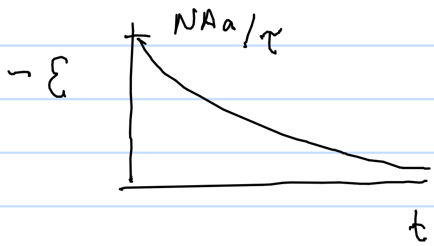


(a) $$\mathcal{E} = -\frac{\partial}{\partial t} B(t) . NA$$

$B(t) = a(1-e^{-t/\tau})$

$$= -\frac{NAa}{\tau}\, e^{-t/\tau}$$

(b) Induced current: $\leftarrow$



(c) $N = 50$, $A = 0.02\ \text{m}^2$, $\tau = 1/2.5$, $a = 0.012$ T

At $t = 0.5$ s $\quad |\mathcal{E}| = \frac{NAa}{\tau} e^{-t/\tau} = \underline{8.6\ \text{mV}}$

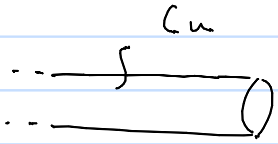
(d) $R = 15\ \Omega \Rightarrow I = |\mathcal{E}|/R = \underline{0.57\ \text{mA}}$

(3) $$q v B = q\frac{V}{l} \Rightarrow V = \underset{\sim 20\,\text{m (say)}}{l}\ \overset{300\,\text{ms}^{-1}}{v}\ \overset{18\,\mu\text{T}}{B} = \underline{0.11\ \text{V}}$$

7.2.6 Numerical Exercise

$n_{Cu} = 8.5 \times 10^{28} \text{ m}^{-3}$

$\rho_{Cu} = 1.7 \times 10^{-8} \ \Omega \text{ m}$

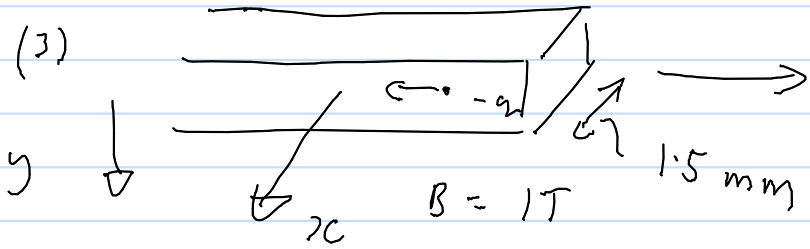
(1) 

$A \quad r = 0.5 \text{ mm} \qquad I = 1 A$

$$I = n e v (\pi r^2) \implies v = \frac{I}{\pi r^2 n e} = 94 \ \mu\text{m/s}$$

(2) $L = 100 \text{ m}$

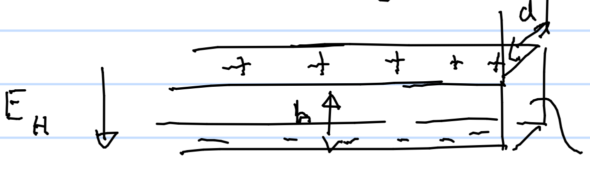
$$R = \frac{\rho L}{A} \ ; \quad P = I R^2 = I \left(\frac{\rho L}{\pi r^2}\right)^2 = \underline{4.7 \text{ W}}$$

(3) 

$I = 3 A$

$$-q \underline{v} \times \underline{B} = -q \begin{vmatrix} i & j & k \\ 0 & 0 & -v \\ B & 0 & 0 \end{vmatrix} = q v B \underline{j}$$

i.e. $-$ve charge accumulates on bottom surface



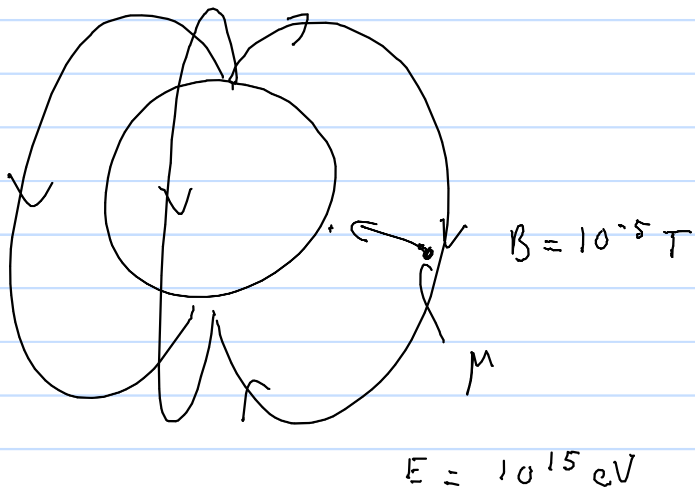
$$A n q \underbrace{E_H}_{\frac{V_H}{h}} = n q v B A = I B$$

$$V_H = I B / n q d$$

$$= \underline{0.15 \ \mu V}$$

7.2.7 Problems

(1) Please ignore this problem; it requires a formula not covered in the course. In case you're interested, here's the solution:



$v \simeq c = 3 \times 10^8 \ \mathrm{ms}^{-1}$

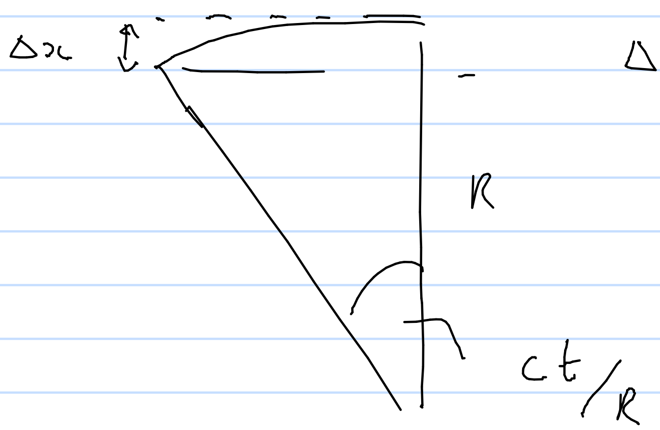
$m_\mu = 106 \ \mathrm{MeV}/c^2$

$= 1.89 \times 10^{-28} \ \mathrm{kg}$

$$\boxed{R = \frac{E}{cqB}}$$ ← Not covered

$$R = \frac{10^{15} \times q}{3 \times 10^8 \times q \times 10^{-5}} = 3.33 \times 10^{11} \ \mathrm{m}$$

$$\text{Time to reach ground} \simeq \frac{100 \times 10^3}{3 \times 10^8} = 3.33 \times 10^{-4} \ \mathrm{s}$$



$$\Delta x \simeq R\left(1 - \cos \frac{ct}{R}\right)$$

$$\simeq R \, \frac{1}{2}\left(\frac{ct}{R}\right)^2 = \frac{c^2 t^2}{2R}$$

$$= 1.5 \times 10^{-2} \ \mathrm{m} = \underline{15 \ \mathrm{mm}}$$

(2) Non relativistic, so $\frac{1}{2} m v^2 = q V$; $v = \left(\frac{2qV}{m}\right)^{\frac{1}{2}}$

(a) $V = 2\ kV \Rightarrow v = 2.65 \times 10^7$ m/s

Time to reach screen $= \frac{0.2}{2.65 \times 10^7} = 7.5 \times 10^{-9}$ s

Force $= q v B$

Sideways acc^n. $= \frac{q v B}{m}$

Deflection $= \frac{1}{2} \left(\frac{q v B}{m}\right)\left(\frac{d}{v}\right)^2 = \frac{1}{2} \frac{q B d^2}{m v} = 6.6 \times 10^{-3}$ m

$= \underline{6.6 \text{ mm}}$

(b) $V = 30\ kV$ Deflection $= 1.7 \times 10^{-3}$ m $= \underline{1.7 \text{ mm}}$

So yes, the effect does need to be taken into account.

(3) $q v B = \frac{m v^2}{R} \Rightarrow R = \frac{m v}{q B}$

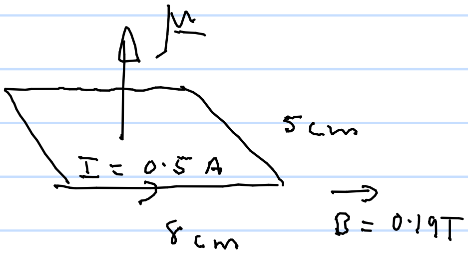
$m = 2 \times 10^{-13}$, $v = 2 \times 10^5$ m s^{-1}, $q = 1.0 \times 10^{-6}$ C

$B = 0.4$ T $\Rightarrow R = \underline{0.1 \text{ m}}$

Hence $R < h = 0.11$ m and particle will not emerge other side of field region.

7.3.7

(1)



$\underline{\tau} = \underline{\mu} \times \underline{B}$

$A = 5 \times 8 \times 10^{-4} = 4 \times 10^{-3} \text{ m}^2$

(a) $|\underline{\tau}| = \mu B = IAB = \underline{3.80 \times 10^{-4} \text{ Nm}}$

(b) $|\underline{\mu}| = IA = \underline{2.11 \times 10^{-3} \text{ Am}^2}$

(c) $A = d\left(\frac{L}{2} - d\right)$ $\quad \frac{dA}{dd} = \frac{L}{2} - 2d = 0 \Rightarrow d = \frac{L}{4}$

i.e. Area is max for a square $\Rightarrow d = 6.5$ cm

$A_{max} = \left(6.5 \times 10^{-2}\right)^2 = 4.23 \times 10^{-3} \text{ m}^2$

$\tau_{max} = 4.01 \times 10^{-4} \text{ Nm}.$