Vibrations and Waves - Assessed Problem Sheet 1 (Dr P. Tangney)

A wrecking ball of mass $m = 1000$ kg and radius $R = 0.5$ m hangs from a crane by a cable of length ℓ and is at equilibrium when the cable is vertical. In Cartesian coordinates, the position of the center of the ball relative to its position at equilibrium is (x, y, z), where z is its height above its equilibrium position.

1. Prove that

$$z = \frac{x^2 + y^2}{2\ell} \tag{1}$$

is a good approximation if $\ell \gg \sqrt{x^2 + y^2}$. [2 marks]

2. Now suppose that the ball is set in motion and that nothing impedes its motion apart from a damping force $F_d = -m\gamma\dot{x}$, where γ is a constant. If the ball's initial position and velocity are suitably chosen, the lateral motion (in the $xy-$plane) can be accurately described by the equations

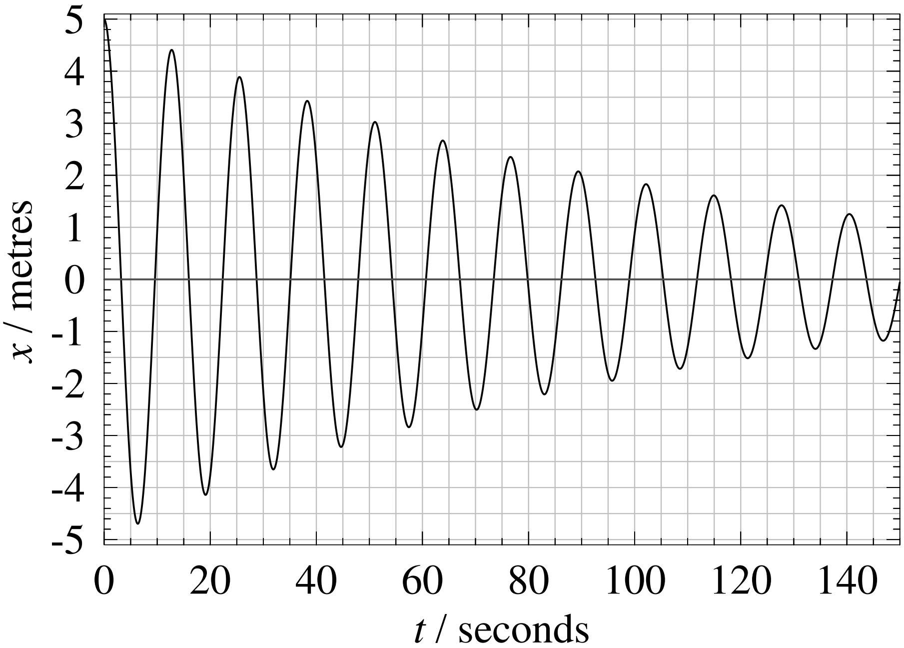
$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \tag{2}$$

$$\ddot{y} + \gamma\dot{y} + \omega_0^2 y = 0 \tag{3}$$

where $\omega_0 \equiv \sqrt{g/\ell}$ is a constant. Apart from the linear form of the damping force, what other simplifying physical assumptions have been made to deduce these equations? When $\gamma = 0$, what is the easiest way of calculating, from the set of initial conditions $\{x(0), y(0), \dot{x}(0), \dot{y}(0)\}$, whether or not it is possible that these physical assumptions will be violated in the ensuing motion?

[2 marks]

For the remaining questions you can assume that Eqs. 1, 2 and 3 are accurate.



The plot above is of x versus time, t, when the ball is swinging freely.

3. Use the plot to estimate the $Q-$factor and ω_0. Hence calculate the damping constant, γ. [1 marks]

4. Given that the coefficient $b = m\gamma$ for linear fluid resistance on a spherical particle can be approximated by the formula $b = 6\pi\eta R$, where η is the *dynamic viscosity* of the fluid and R is the radius of the particle, calculate the dynamic viscosity that would be required to produce the observed damping. Is it likely that the damping is caused by air resistance, given that $\eta_{\text{air}} \approx 1.8 \times 10^{-5}$ Pa s? Suggest some other likely causes of the damping. [1 mark]

A circus stunt is performed in which the wrecking ball is released from rest from the position $(x(0), y(0)) = (9.0, 0.0)$ metres, while a performer stands facing it with the tip of his nose at position $(-9.0, 0.0)$ metres.

5. While waiting for the ball to arrive, the performer realises that he has neglected to account for either energy dissipation or the finite radius of the ball when calculating where to stand. Is he safe? Justify your answer with calculations. [1 marks]

6. If everything else remained unchanged, would it be possible to change the outcome (i.e. whether or not the performer is hit by the ball) by releasing the ball from the same point but with an initial velocity in the $xy-$plane of

   (a) $(\dot{x}, \dot{y}) = (0, v)$?

   (b) $(\dot{x}, \dot{y}) = (v\cos\alpha, v\sin\alpha)$?

   v and α can take any values for which Eqs. 1, 2, and 3 remain accurate approximations. If the answer to (b) is "yes", is it "yes" for any value of α? Explain and justify your answers. [3 marks]