Basic Electronics

Assessed Problems 3 - Solutions

APS3.1 (a) The total impedance is a series combination of the resistor and capacitor impedances

$$\tilde{Z}_R = R$$

$$\tilde{Z}_C = \frac{-j}{\omega C}$$

$$\tilde{Z} = \tilde{Z}_R + \tilde{Z}_C = R - \frac{j}{\omega C}$$

For a circuit current $\tilde{i}$ we can write

$$\tilde{v}_{in} = \tilde{i}\tilde{Z}$$

$$\tilde{v}_{out} = \tilde{i}\tilde{Z}_C = \frac{\tilde{v}_{in}}{R - j/\omega C} \times \frac{-j}{\omega C}$$

$$\tilde{g} = \frac{\tilde{v}_{out}}{\tilde{v}_{in}} = \frac{-j}{\omega CR - j}$$

$$|\tilde{g}| = \frac{\omega CR}{\sqrt{(\omega CR)^2 + 1}}$$

[1 mark]

(b) $|\tilde{g}| \to 0$ for $\omega \to 0$ and $|\tilde{g}| \to 1$ for $\omega \to \infty$ so this is a high-pass filter. [1 mark]

(c) By definition, at the cut-off frequency the filter output falls to -3 dB below the maximum output (equivalent to a factor $1/\sqrt{2}$).

$$\frac{1}{\sqrt{2}} = \frac{\omega_c CR}{\sqrt{(\omega_c CR)^2}}$$

The cut-off frequency for the filter is

$$\omega_c = \frac{1}{RC}$$

[1 mark]

(d) For $R = 120\,\Omega$, $C = 15.625\,\mu$F and $\omega = 400$ rad/s the circuit impedance is

$$\tilde{Z} = R - \frac{j}{\omega C} = 120 - j160$$

$|\tilde{Z}| = 200\,\Omega$ and $\arg(\tilde{Z}) = \phi = -\tan^{-1}(4/3)$ so

$$\tilde{Z} = 200e^{j\phi}\,\Omega$$

[1 mark]

(e) i. The input voltage has an amplitude of 10 V and has a maximum at time $t = 0$, which defines the phase angle to be zero so we can represent $\tilde{v}_{in}$ as

$$\tilde{v}_{in} = 10e^{j0} = 10\,\text{V}$$

The circuit current is

$$\tilde{i} = \frac{\tilde{v}_{in}}{\tilde{Z}} = 0.05e^{-j\phi}\,\text{A}$$

The amplitude of the current is $|\tilde{i}| = 50\,\text{mA}$.

ii.

$$\tilde{v}_{out} = \tilde{v}_R = \tilde{i}R = 6e^{-j\phi}\,\text{V}$$

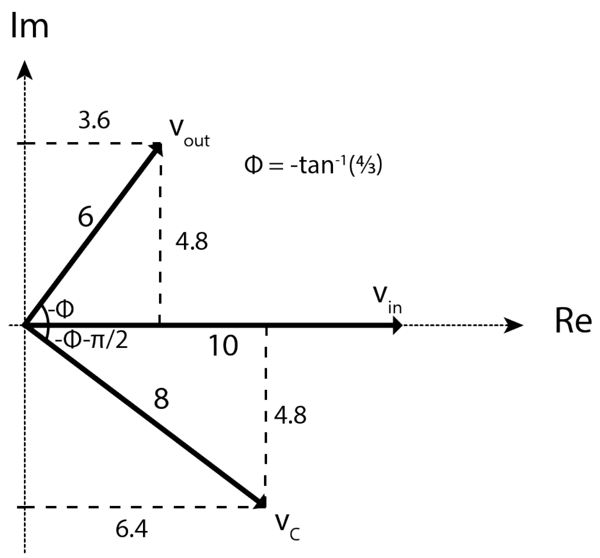
The amplitude of the resistor voltage is 6 V.

iii.

$$\tilde{v}_C = \tilde{i}\tilde{Z}_C = 0.05e^{-j\phi} \times (-160j) = 0.05e^{-j\phi} \times 160e^{-j\pi/2} = 8e^{-j(\phi+\pi/2)}$$

The amplitude of the capacitor voltage is 8 V. [3 marks]

(f) The phasor diagram shows the applied, resistor and capacitor voltages as phasors. The mark is awarded for the correct identification of the phasor lengths and relative angles.



[1 mark]

(g) The physical voltage on each component is the projection of the phasor onto the real axis. At time $t = 0$, as shown by the phasor diagram, the voltage across the capacitor is 6.4 V and the voltage across the resistor is 3.6 V which sums to the 10 V applied by the source, so KVL is obeyed. Equivalently, we can see that the vector sum of the capacitor and resistor phasors is the applied voltage. [1 mark]

(h) The phasor diagram shows that, since the phasors rotate anti-clockwise, the output voltage leads the input voltage. [1 mark]

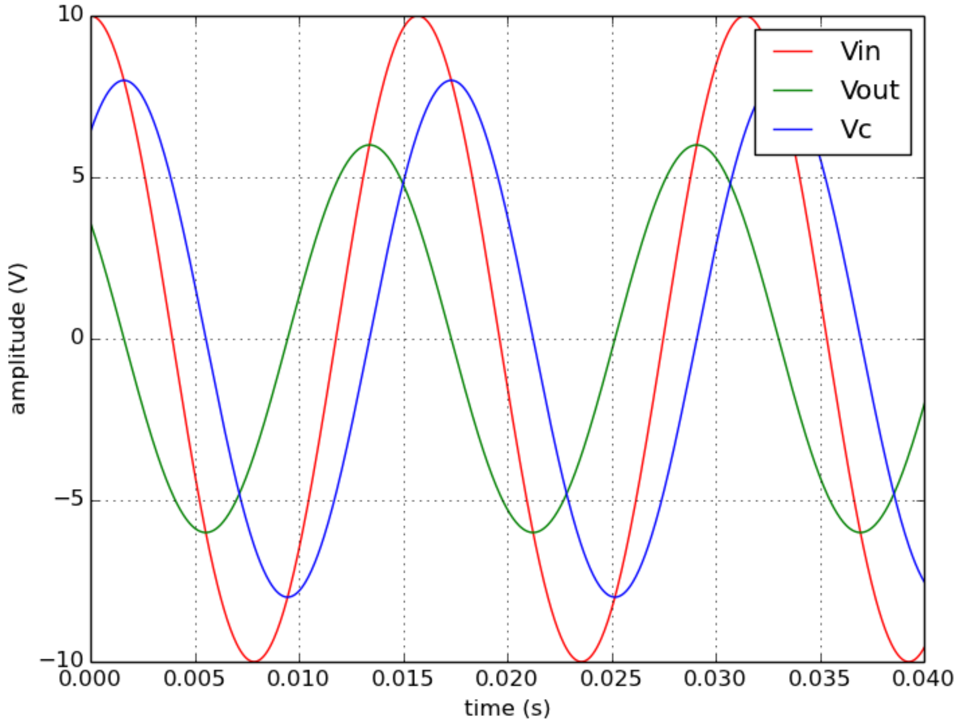
Notes:
To get the voltages as a function of time, multiply the phasor by $e^{j\omega t}$ and then take the real part.

$$v_{in} = 10\cos(\omega t)$$

$$v_{out} = 6\cos(\omega t - \phi)$$

$$v_C = 8\cos(\omega t - \phi - \frac{\pi}{2})$$

These are shown in the figure below



Here we can see that the output voltage leads the input and the capacitor voltage lags the input. The Python code can also be used to verify that, for all t, the sum of the resistor and capacitor voltages equals the input voltage.

Note: this is for information only and should match a LTSpice simulation of the circuit. This is ***not*** *required as part of the answer.*

CC
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