EM Assessed Problem Sheet - McCall **ANSWERS** **March 22, 2018**

1. (i) **ANSWER:** The force due to the electric field $qE\hat{\mathbf{y}}$ and the (Lorentz) force due to the magnetic field $qB(\mathbf{v} \times \hat{\mathbf{z}})$ combine to give the equation of motion

$$m\frac{d\mathbf{v}}{dt} = q\left(E\hat{\mathbf{y}} + B\mathbf{v} \times \hat{\mathbf{z}}\right) ,$$

for which the components are

$$\frac{dv_x}{dt} = \Omega v_y , \ \frac{dv_y}{dt} = \frac{qE}{m} - \Omega v_x , \ \frac{dv_z}{dt} = 0 ,$$

where $\Omega = qB/m$.

[1 mark for each equation of motion. Deduct a mark if expression for Ω not given or incorrect.] [3 marks]

(ii) **ANSWER:** Following the hint, setting $\tilde{v} = v_x + iv_y$ allows us to write

$$\frac{d\tilde{v}}{dt} = \frac{d}{dt}(v_x + iv_y) = \Omega v_y + i\frac{qE}{m} - i\Omega v_x = i\frac{qE}{m} - i\Omega(v_x + iv_y) = i\frac{qE}{m} - i\Omega\tilde{v} .$$

i.e.

$$\frac{d\tilde{v}}{dt} + i\Omega\tilde{v} = i\frac{qE}{m} .$$

Solving this as a linear ODE[1]:

$$\tilde{v} = e^{-i\Omega t}\left[\int e^{i\Omega t}\left(i\frac{qE}{m}\right)dt + C\right] ,$$

where C is a complex constant. Effecting the integration yields

$$\tilde{v} = \frac{qE}{\Omega m} + Ce^{-i\Omega t} ,$$

The initial condition that the particle starts from rest fixes $C = -qE/\Omega m = -E/B$, so that

$$v_x + iv_y = \frac{E}{B}\left[1 - \cos\Omega t + i\sin\Omega t\right]$$

Taking real and imaginary parts and adding (trivially) $v_z = 0$ yields

$$v_x = \frac{E}{B}(1 - \cos\Omega t) , \ v_y = \frac{E}{B}\sin\Omega t , \ v_z = 0 .$$

[Full marks for any other correctly executed method.] [3 marks]

[1] The formula $y = e^{-\int P dx}\left[\int e^{\int P dx} Q dx + C\right]$ solving $dy/dx + P(x)y = Q(x)$ was covered in the complex analysis course in term 1.

[This question continues on the next page . . .]

(iii) **ANSWER:**

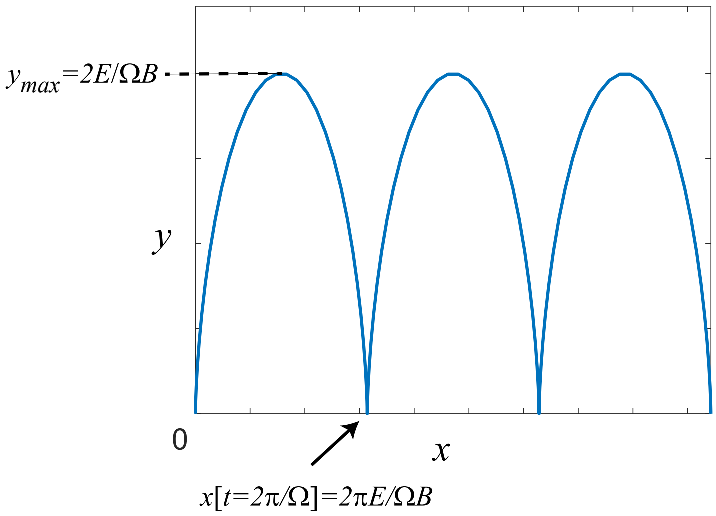
$$x(t) = \int_0^t v_x dt = \frac{E}{B}\int_0^t (1 - \cos\Omega t)dt = \frac{E}{B}t - \frac{E}{\Omega B}\sin\Omega t\ ,$$

$$y(t) = \int_0^t v_y dt = \frac{E}{B}\int_0^t \sin\Omega t dt = \frac{E}{\Omega B}[1 - \cos\Omega t]\ ,$$

$$z(t) = \int_0^t v_z dt = 0\ .$$

[No marks if there is any slip.] [1 mark]

(iv) **ANSWER:** Were the $(E/B)t$ term missing from $x(t)$ the trajectory would be a circle of radius $(E/\Omega B)$ centred at $(x, y) = (0, E/\Omega B)$. The particle would start at the origin and rotate clockwise. Consider the particle confined to the rim of a stationary circular wheel of radius $(E/\Omega B)$, centred at $(x, y) = (0, E/\Omega B)$. The trajectory just described is represented by the particle running round the rim with angular frequency Ω. The presence of the $(E/B)t$ term can be accounted for by allowing the wheel to roll along the x-axis to the right with speed E/B. Note also that the charge comes to rest on the x-axis at the end of each cycle. The resulting trajectory (called a cycloid) is therefore



[1 mark for general shape, 1 mark for noting the time for at least one return to the x-axis, 1 mark for maximum y-excursion. Give a bonus mark (but total not to exceed 10) if there is erudite supporting discussion] [3 marks]

[Total 10 marks]