Basic Electronics

Further Problems 3 - Solutions

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1. For an input $V_i(t) = 3\sin(12566t)$ we will have an output $V_o(t) = |V_o|\sin(12566t + \phi)$ since the frequency remains unchanged while the magnitude and phase of the output are modified according to

$$\frac{|V_o|}{|V_i|} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

and

$$\tan\phi = -\omega CR$$

Putting in the values given we have $\omega CR = 1.88$ and hence the magnitude of the output voltage is 1.41 V with a phase (lag) of -1.08 rad.

$$V_o(t) = 1.41\sin(12566t - 1.08)$$

2. For the low-pass filter we apply the input voltage across the series combination of resistor and capacitor and take the output voltage across the capacitor only so it forms a frequency-dependent voltage divider with

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

and phase

$$\tan\phi = -\omega CR$$

as derived in the course notes. The time-constant CR is

$$CR = 10^{-5}\,\mathrm{s}$$

Since the output lags the input we have $\phi = -\pi/4$ from which we find ωCR=1 hence

$$\omega = 10^5 \ \mathrm{rad/s}$$

and

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\sqrt{2}}$$

from which

$$V_{in} = 2\sqrt{2}\,\mathrm{V}_{RMS} = 4\,\mathrm{V}\ \text{amplitude}$$

3. (a) $\tilde{Z}_C = -j/\omega C$ and using impedance divider

$$\frac{\tilde{v}_{out}}{\tilde{v}_{in}} = \frac{\tilde{Z}_C}{R + \tilde{Z}_C} = \frac{1}{j\omega CR + 1} = \frac{1-j}{2}$$

$$\frac{|\tilde{v}_{out}|}{|\tilde{v}_{in}|} = \frac{1}{\sqrt{2}}$$

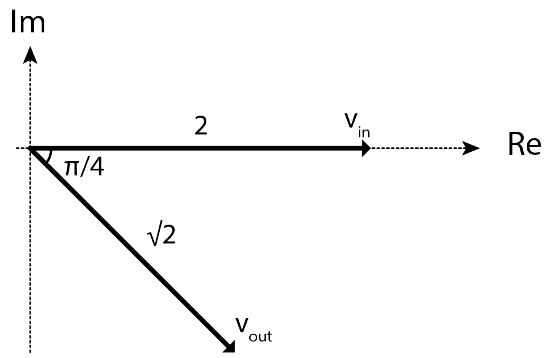
$$|\tilde{v}_{out}| = \sqrt{2}\,\mathrm{V}$$

(b)

$$\phi = \tan^{-1}(-1) = -\frac{\pi}{4}\,\text{rad}$$

Negative phase so output lags the input.

(c) Phasor diagram



(d) Inductor impedance same magnitude as capacitor so

$$\omega L = \frac{1}{\omega C}$$

$$L = \frac{1}{\omega^2 C} = R^2 C$$

4. (a) For the reactive components the total impedance is given by

$$\frac{1}{\tilde{Z}} = j\left(\omega C - \frac{1}{\omega L}\right)$$

The gain is given by the impedance divider formula

$$\tilde{g} = \frac{\tilde{v}_{out}}{\tilde{v}_{in}} = \frac{\tilde{Z}}{R + \tilde{Z}} =$$

$$|\tilde{g}| = \frac{1}{\sqrt{1 + R^2\left(\omega c - \frac{1}{\omega L}\right)^2}}$$

(b) For $\omega = \omega_0 = 1/\sqrt{LC}$, $\tilde{Z} \to \infty$ so $|\tilde{g}| = 1$. For high and low frequencies, $\tilde{Z} \to 0$ so $|\tilde{g}| \to 0$

(c) For high frequencies the impedance of the capacitor $\to 0$ and for low frequencies the impedance of the inductor $\to 0$.

(d) This is a band-pass filter.

(e) Let the frequency at which $|\tilde{g}| = 1/\sqrt{2}$ be ω_c. Substitute into the expression for $|\tilde{g}|$ giving

$$R\left(\omega_c C - \frac{1}{\omega_c L}\right) = \pm 1$$

This results in a pair of quadratic equations with four possible solutions

$$\omega_c = \frac{1 \pm \beta}{2RC},\ \omega_c = \frac{-1 \pm \beta}{2RC}$$

where for convenience

$$\beta = \sqrt{1 + \frac{4R^2C}{L}}$$

Since $R,\ C,\ L$ are real and non-zero, $\beta > 1$ and there are only two positive values of ω_c

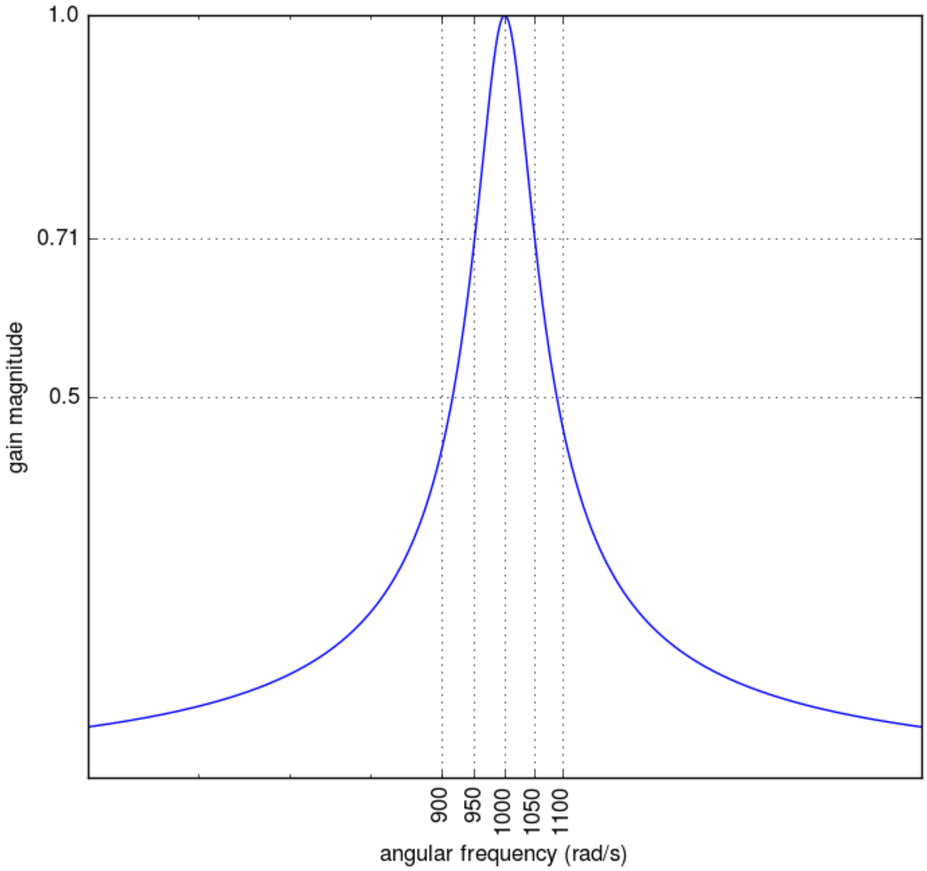
$$\omega_c = \frac{1+\beta}{2RC},\ \omega_c = \frac{-1+\beta}{2RC}$$

The difference is the bandwidth

$$\Delta\omega = \frac{1}{RC}$$

(f) $\omega_0 = 1000\,\text{rad/s}$, $\Delta\omega = 100\,\text{rad/s}$. The filter will pass frequencies in the range 950 to 1050 rad/s, but the filter is not 'flat' within this pass-band (see gain plot below).
*Note:*This type of filter could be used to select a particular frequency from a noisy signal, e.g. a radio-tuner, where the tuning frequency could be selected with a variable capacitor.

(g) Halving the resistance will double the bandwidth.



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