Friday 9th February 2018
(Answers available on Monday 19th February)

Quantum Physics Problem Sheet 2

1. The peak in the radiation spectrum from the Sun is at $\lambda = 470nm$. Treating the Sun as a Black Body, what is its surface temperature? For what range of temperatures does the peak of the BB spectrum move through the visible $(400nm \leq \lambda \leq 800nm)$?

   What is the fractional difference in the radiated power between the temperatures that correspond to peak emission intensity at these two wavelength limits?

2. In a photoelectric effect experiment, the cathode was illuminated with EM radiation of three different frequencies and the stopping potential (potential required to stop all the electrons reaching the anode) V_0 was measured for each.

Frequency ν (10^{15} Hz)	Stopping Potential V_0 (V)
0.75	1.0
1.00	2.0
1.25	3.0

   (i) Make a sketch of stopping potential against frequency and hence estimate the work function of the metal in eV.

   (ii) From the slope of your line, estimate the value of Planck's constant h. How close is this to the accepted value?

3. In a photoelectric effect experiment, a current was observed when the metallic cathode was illuminated with light of wavelength 310 nm, but none was observed with longer wavelength light.

   (i) Estimate the work function of the cathode in eV.

   (ii) What is the energy (in eV) of a photon of wavelength 200 nm?

   (iii) What is the maximum kinetic energy (in eV) of the electrons emitted if the cathode is illuminated by light of wavelength 200 nm?.

4. The energy flux of starlight reaching us from a sixth-magnitude star (approximately the faintest that can be seen by the naked eye) is 1.4×10^{-10} Wm^{-2}. If you are looking at such a star, how many photons enter your eye per second? On average, how many photons are inside your eye at any one time? Do these estimates provide any evidence that the human eye can detect single photons? (Assume that the diameter of your dark-adapted pupil is 0.7 cm, the length of your eye is 4 cm, and the wavelength of the light is 500 nm.)

5. Electrons of energy 100 eV pass through a narrow slit of width $1\,\mu$m.

   (a) What is the distance between the zeros of intensity on either side of the central peak of the electron diffraction pattern 1 m away from the slit?

   (b) If the electron energy were increased to 200 eV where would the zeros be located?

   (c) If 100 eV electrons were replaced by 10 eV muons where would the zeros be located?

6. Neutron diffraction is often used to study the atomic positions and atomic-scale magnetic fields in solids where a spatial resolution of $\approx 10^{-10}m$ is usually required. Suggest a reasonable value for the de Broglie wavelength of the neutrons used in such experiments. Find the kinetic energy (in eV) of neutrons of this wavelength. Use the equation $E = 3k_BT/2$ to translate the kinetic energy into an equivalent temperature.

Assessed Problem

7. X-rays of energy 37 keV are Compton scattered by a thin metal foil through an angle of 75^o.

   (i) What is the wavelength of these scattered photons (in m) ?

   (ii) Would the kinetic energy of the recoiling electrons be sufficient to allow some of them to escape from the metal (Yes or No)?

   (iii) What value of the scattering angle θ would give the largest change in wavelength (in degrees) ?

   (iv) What is the maximum possible wavelength of the scattered photon (in m)?

Tutorial Problem

8. (Q40-63 from Young and Freedman) Nuclear fusion reactions at the centre of the Sun produce gamma-ray photons with energies of order 1 MeV. By contrast, what we see emanating from the Sun's surface are visible photons with wavelengths of order 500 nm. Models of the solar interior explain this wavelength difference by suggesting that every photon is Compton scattered about 10^{26} times during its journey from the centre of the Sun to the surface.

   (i) Estimate the average increase in wavelength of a solar photon per Compton-scattering event.

   (ii) Find the angle in degrees through which the photon is scattered in the scattering event described in part (i). (Hint: a useful approximation is $\cos\phi \approx 1 - \phi^2/2$, which is valid when ϕ is $\ll 1$ radian.)

   (iii) It is estimated that a photon takes about 10^6 years to travel from the core to the surface of the Sun. Find the average distance that light can travel within the interior of the sun without being scattered.

As you can see, the Sun is *very* opaque, and the radiation has plenty of opportunity to reach equilibrium with the matter before emerging. This explains why the Sun emits black-body radiation.

(Discuss with your tutor if you think this question presents a realistic model of radiation transport in the Sun)

Physical Constants

$$\begin{aligned}
h &\approx 6.63 \times 10^{-34}\,\mathrm{Js} \\
\hbar &\approx 1.05 \times 10^{-34}\,\mathrm{Js} \\
c &\approx 3.00 \times 10^{8}\,\mathrm{ms^{-1}} \\
e &\approx 1.60 \times 10^{-19}\,\mathrm{C} \\
m_e &\approx 9.11 \times 10^{-31}\,\mathrm{kg}
\end{aligned}$$