Quantum Physics Answer Sheet 2

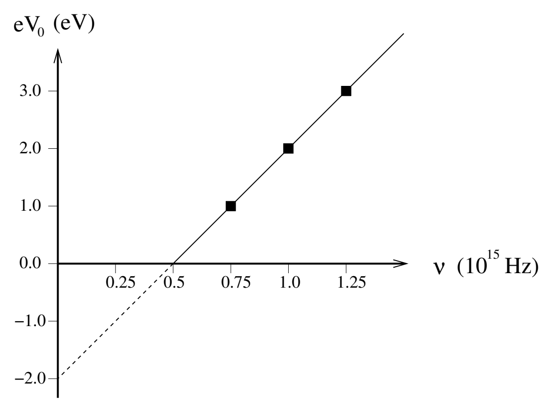
Photons

1. We need to use Wein's law that relates the wavelength at which the black-body spectrum peaks λ_{max} to the temperature T, this states $\lambda_{max} = b/T$ where the constant $b = 2.9 \times 10^{-3} mK$. We therefore find $T = 2.9 \times 10^{-3}/4.7 \times 10^{-7} = 6170K$ for the temperature of the surface of the Sun.

   Using the same formula we find $T_{800nm} = 3625K$ and $T_{400nm} = 7250K$.

   The power radiated per unit area by a black-body is given by Stefan's law $P = \sigma T^4$ where $\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$ Stefan's constant. The fractional change in radiated power will be simply $R = (T_{400nm}/T_{800nm})^4 = 16$.

2. (i) The data look like this:



   Since the y intercept is $-W$, the work function W is 2.0 eV.

   (ii) The slope of the line is

$$\frac{(3.0-1.0)\,\mathrm{eV}}{(1.25-0.75)\times 10^{15}\,\mathrm{Hz}} = \frac{2\times 1.60\times 10^{-19}\,\mathrm{J}}{0.5\times 10^{15}\,\mathrm{s}^{-1}} = 6.4\times 10^{-34}\,\mathrm{Js}\;.$$

   Assuming that the experimental errors were reflected in the precision with which the measured values were quoted, this is consistent with the accepted value of 6.63×10^{-34} Js. (The fact that the three data points lie on a perfect straight line is suspicious, though.)

3. (i) Light of wavelength greater than $\lambda_{\max} = 310\,\mathrm{nm}$ is incapable of producing a current. Hence the work function W is given by:

$$W = \frac{hc}{\lambda_{\max}} = \frac{6.63\times 10^{-34}\times 3.00\times 10^{8}}{310\times 10^{-9}} \approx 6.42\times 10^{-19}\,\mathrm{J}\;.$$

$$W = \frac{6.42 \times 10^{-19}}{1.60 \times 10^{-19}} \approx 4.00\,\mathrm{eV} .$$

(ii) The energy of a photon of wavelength 200 nm is

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{200 \times 10^{-9}} \approx 9.95 \times 10^{-19}\,\mathrm{J} ,$$

or

$$E = \frac{9.95 \times 10^{-19}}{1.60 \times 10^{-19}} \approx 6.22\,\mathrm{eV} .$$

(iii) The stopping potential V_0 at 200 nm is given by Einstein's equation, $W + eV_0 = E$. Hence

$$V_0 = 6.22 - 4.00 = 2.22\,\mathrm{V} .$$

The maximum KE of the emitted electrons is 2.22 eV.

4. The total energy entering each eye per second is the energy striking a unit area per second times the area of the pupil:

$$\begin{aligned} \text{energy entering eye per second} &= 1.4 \times 10^{-10} \times \pi(0.0035)^2 \\ &\approx 5.39 \times 10^{-15}\,\mathrm{J} . \end{aligned}$$

Average number of photons entering eye per second is

$$\begin{aligned} \frac{\text{energy entering eye per second}}{\text{energy per photon}} &= \frac{5.39 \times 10^{-15}}{hc/\lambda} \\ &= \frac{5.39 \times 10^{-15} \times 500 \times 10^{-9}}{6.63 \times 10^{-34} \times 3.00 \times 10^{8}} \\ &\approx 13,500 . \end{aligned}$$

The average number of photons inside eye at any one time is

$$\begin{aligned} &\frac{\text{number entering per second} \times \text{length of eye}}{\text{distance a photon travels per second}} \\ &= \frac{13500 \times 0.04}{3.00 \times 10^{8}} \approx 1.8 \times 10^{-6} . \end{aligned}$$

The actual number of photons in the eye is almost always zero.

Since light always arrives as individual photons, all light detectors must be capable of detecting individual photons. A more interesting question is whether the arrival of a single photon is sufficient to trigger one of the detectors (rods and cones) in the retina, or whether it is necessary to bombard that detector with many photons in close succession. Given that the eye takes much less than 1 s to process a new image, it is reasonable to assume that the "memory" of the detectors is less than, say, 0.2 s. Any effects caused by photons that arrived more than 0.2 s ago can therefore be ignored. Within 0.2 s, only 2,700 photons enter the eye, all of which are focused onto the small area of the retina where the image is formed. Are there more than 2,700 detectors in this area? I have no idea, but I doubt it. In other words, the ability of the eye to see the star provides no convincing evidence that the detectors in the retina are triggered by single photons — it may be necessary to hit the same detector with several photons in quick succession.

5. A 100 eV electron can be accurately described by a non-relativistic treatment so the electrons momentum $p = \sqrt{2m_e E} = \sqrt{2 \times 9.11 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}} = 5.40 \times 10^{-24} kgms^{-1}$. We can then compute the de Broglie wavelength of the electron as $\lambda = h/p = 6.63 \times 10^{-34}/5.40 \times 10^{-24} = 1.228 \times 10^{-10} m$.

   For the diffraction pattern of an electron passing through a rectangular slit, just as in optics, the angle between the centre of the diffraction pattern and one of the first zeros on either side is given by $\Delta\theta = \lambda/w$ where w is the width of the slit. Therefore at a distance of 1 m from the slit the distance between the intensity zeros for the diffraction of 100 eV electrons is $\Delta y = 2\Delta\theta = 2\lambda x/w = 2 \times 1.228 \times 10^{-10} \times 1/1 \times 10^{-6} = 2.456 \times 10^{-4} m$.

   If the energy of the electron is doubled (from 100 eV to 200 eV) then the momemtum increases by $\sqrt{2}$ and the wavelength and diffraction are reduced by a factor of $1/\sqrt{2}$. So now the separation of the zeros will be $\Delta y = 2.456 \times 10^{-4}/\sqrt{2} = 1.737 \times 10^{-4}$.

   A muon has a mass $m_\mu = 207 m_e$, so in evaluating the momentum the mass is increased by this factor and the energy decreased by a factor 10 compared to the electron case considered. Therefore the momentum is changed by a factor $\sqrt{207/10} = 4.55$, so we can find the width of the diffraction pattern due to the muons directly from our first result to be $\Delta y = 2.456 \times 10^{-4}/4.55 = 5.40 \times 10^{-5} m$.

6. In order to use neutron diffraction to study atomic positions and atomic-scale magnetic fields, the neutron de Broglie wavelength must be comparable to the size of an atom:

$$\lambda \;\approx\; 10^{-10}\,\mathrm{m}\ .$$

   The kinetic energy of the neutron is thus:

$$\begin{aligned}\frac{1}{2}mv^2 \;&=\; \frac{p^2}{2m} \;=\; \frac{h^2}{2m\lambda^2} \;\approx\; \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times 10^{-20}} \\ &\approx\; 1.32 \times 10^{-20}\,\mathrm{J} \;\approx\; 0.083\,\mathrm{eV}\ .\end{aligned}$$

   This is the same as the average energy $3k_BT/2$ of a neutron in thermal equilibrium at temperature

$$T \;\approx\; \frac{2 \times 1.32 \times 10^{-20}}{3 \times 1.38 \times 10^{-23}} \;\approx\; 640\,\mathrm{K}\ .$$

7. (i) (i) The wavelength of the incident photons is

$$\lambda \;=\; \frac{hc}{E} \approx \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{37 \times 10^{3} \times 1.60 \times 10^{-19}} \;\approx\; 3.36 \times 10^{-11}\,\mathrm{m}\ .$$

The change in wavelength is given by the Compton formula with $\theta = 75^o$:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \approx \frac{6.63 \times 10^{-34} \times 0.5}{9.11 \times 10^{-31} \times 3.00 \times 10^8} \approx 1.80 \times 10^{-12}\,\mathrm{m}\ .$$

Combining the values of λ and $\lambda' - \lambda$ gives the wavelength of the scattered photons:

$$\lambda' \approx 3.54 \times 10^{-11}\,\mathrm{m}\ .$$

[4 marks]

(ii) (ii) The energy lost by a photon as it scatters is:

$$\begin{aligned} E - E' &= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \\ &= 6.63 \times 10^{-34} \times 3.00 \times 10^8 \left(\frac{1}{3.36 \times 10^{-11}} - \frac{1}{3.54 \times 10^{-11}}\right) \\ &\approx 3.01 \times 10^{-16}\,\mathrm{J} \\ &\approx 1881\,\mathrm{eV}\ . \end{aligned}$$

Comment: This answer is the difference of two much larger numbers (the incoming and outgoing photon energies) and is subject to considerable rounding error. For example, if you store intermediate values such as λ and λ' to full calculator precision, the final result changes by several eV.

All this energy is transferred to the electron as recoil energy. The work function of a typical solid is only 5 or 10 eV, so some of the recoiling electrons will certainly escape from the metal. So the answer is YES.

[2 marks]

(iii) The largest change in wavelength would be obtained when $\theta = 180^0$, (then $(1 - \cos\theta = 2$ in which case $\lambda' - \lambda = 2h/mc \approx 4.85 \times 10^{-12}$ m.)

[2 marks]

(iv) The maximum possible wavelength of the scattered photon (assuming only one scattering) is $3.36 \times 10^{-11} + 4.85 \times 10^{-12} \approx 3.85 \times 10^{-11}$ m.

[2 marks]

8. (i) The initial photon wavelength is

$$\lambda_{\text{init}} = \frac{hc}{E} \approx \frac{6.63 \times 10^{-34} \times 3.00 \times 10^8}{10^6 \times 1.60 \times 10^{-19}} \approx 1.24 \times 10^{-12}\,\mathrm{m}\ .$$

The final photon wavelength after 10^{26} Compton scattering events is 500 nm. If we assume that each scattering event increases the photon wavelength by the same amount $\Delta\lambda$, we obtain

$$10^{26}\Delta\lambda \approx (500 \times 10^{-9} - 1.24 \times 10^{-12})\,\mathrm{m}\ ,$$

and hence

$$\Delta\lambda \approx 5 \times 10^{-33}\,\mathrm{m}\ .$$

(ii) The Compton formula says that

$$\Delta\lambda \;=\; \frac{h}{mc}(1-\cos\theta)\ .$$

Since $\Delta\lambda$ ($\approx 5\times10^{-33}\,\mathrm{m}$) $\ll h/mc$ ($\approx 2.43\times10^{-12}\,\mathrm{m}$), the average scattering angle θ must be very small (so that $\cos\theta$ is very close to 1). We can therefore make the approximation $\cos\theta\approx 1-\theta^2/2$ to obtain $\Delta\lambda\approx h\theta^2/2mc$, and hence

$$\begin{aligned}\theta &\approx \sqrt{\frac{2mc\Delta\lambda}{h}}\\ &\approx \sqrt{\frac{2\times9.11\times10^{-31}\times3.00\times10^{8}\times5\times10^{-33}}{6.63\times10^{-34}}}\\ &\approx 6.42\times10^{-11}\,\text{radians}\\ &\approx 3.68\times10^{-9}\,\text{degrees}\ .\end{aligned}$$

(iii) In 10^6 years, a photon travels a distance:

$$\begin{aligned}d \;=\; ct &\;=\; 3.00\times10^{8}\times60\times60\times24\times365\times10^{6}\\ &\approx 9.46\times10^{21}\,\mathrm{m}\ .\end{aligned}$$

During this time, it scatters 10^{26} times. Hence, the average distance travelled by a photon between scattering events is $9.46\times10^{21}/10^{26}\approx 9.46\times10^{-5}\,\mathrm{m}$ or roughly 0.1 mm.