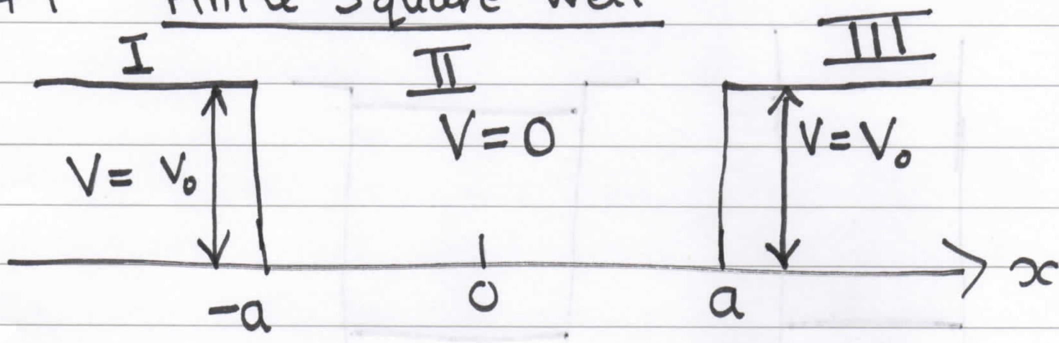
MPH marksphysicshelp

14. Wells and Steps

14·1 Finite Square Well



In region II (as ∞ well eqn. 13·4)

$$\psi_{II}(x) = A\cos kx + B\sin kx \qquad [14\cdot1]$$

$$k = \left(\frac{2mE}{\hbar^2}\right)^{1/2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V_0\psi(x) = E\psi(x)$$

In region I + III

$$\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) = (V_0 - E)\,\psi(x)$$

or

$$\frac{d^2}{dx^2}\psi(x) = \frac{2m(V_0-E)}{\hbar^2}\psi(x) \qquad [14\cdot2]$$

~~define~~ solutions:

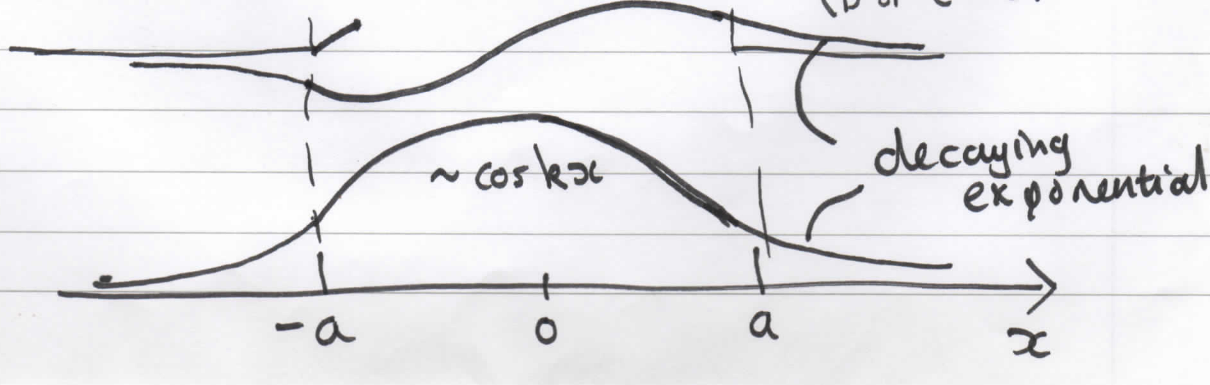
$$\psi_{I,III}(x) = C\exp(Kx) + D\exp(-Kx) \qquad [14\cdot3]$$

where $$K = \left(\frac{2m(V_0-E)}{\hbar^2}\right)^{1/2} \qquad [14\cdot4]$$

K imaginary if $E > V_0$ (free)
K real if $E < V_0$ (bound)

$$\psi_{I,III}(x) \sim e^{-Kx} \qquad [14\cdot5]$$

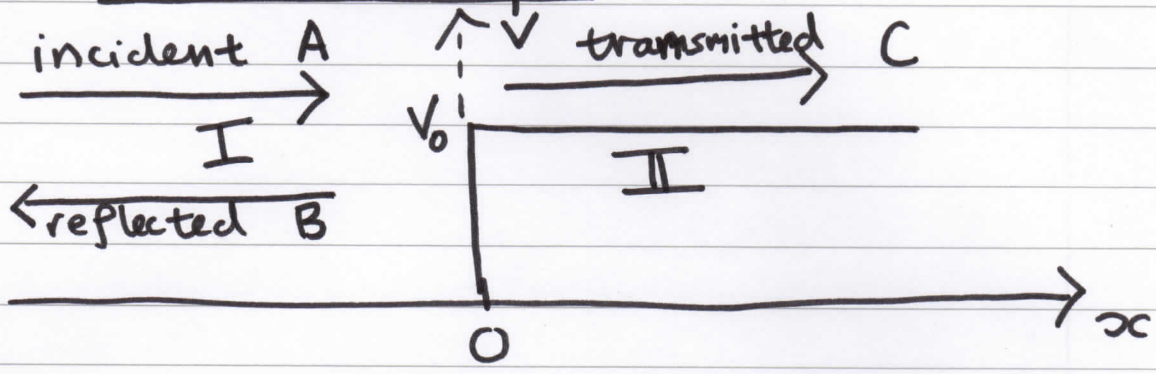
(D or C = 0)



<u>energy eigenvalues</u> can only be found graphically or numerically

- always at least 1 bound state
- $E_n(V_0) < E_n(\infty)$
- For V_0 'large' $E_1(V_0) \simeq E_1(\infty)$

14·2 Potential Step



Plane wave solutions in both regions for $E > V_0$.

Task to work out <u>transmission</u> & <u>reflection</u> probability

Define <u>flux</u>: no of particles crossing unit area per s

$$\text{flux} = \underset{(m^{-3})}{\text{particle density}} \times \underset{(m\,s^{-1})}{\text{velocity}} \quad (m^{-2}s^{-1}) \quad [14{\cdot}6]$$

[For an ∞ plane wave, to avoid an ∞ number of particles we implicitly understand the plane wave is of finite spatial extent - but much longer than scale of step]

In region I:

$$\psi_I(x) = A e^{ik_I x} + B e^{-ik_I x} \quad (a) \quad [14{\cdot}7]$$

where $k_I^2 = \frac{2m}{\hbar^2} E$ (b)

In region II $\quad \psi_{II} = C e^{ik_{II} x}$ (a) $\quad$ [14.8]

$k_{II}^2 = \frac{2m}{\hbar^2}(E - V_0)$ $\quad$ (b)

flux $= |C|^2 |e^{ik_{II}x}|^2 \times \hbar k_{II}/m = \frac{\hbar k_{II} |C|^2}{m}$

14.3 Reflection and Transmission Probability

In case $E > V_0$: $\psi(x=0)$ continuous from (14.7 + .8)

$$A + B = C \quad [14.9]$$

$\frac{d\psi(x)}{dx}$ at $x=0$ also continuous (4.7 + .8)

$$ik_I A - ik_I B = ik_{II} C \quad [14.10]$$

rearranging :

$$\left.\begin{aligned} A &= \left(1 + \frac{k_{II}}{k_I}\right)\frac{C}{2} \\ B &= \left(1 - \frac{k_{II}}{k_I}\right)\frac{C}{2} \end{aligned}\right\} \quad [14.11]$$

Reflection coefficient R :

$$R = \frac{\text{reflected flux}}{\text{incident flux}} = \frac{|B|^2 \hbar k_I/m}{|A|^2 \hbar k_I/m} = \frac{|B|^2}{|A|^2}$$

$$R = \left(\frac{k_I - k_{II}}{k_I + k_{II}}\right)^2 \quad [14.12]$$

$$T = \frac{|C e^{ik_{II}x}|^2 \times \hbar k_{II}/m}{|A e^{ik_I x}|^2 \times \hbar k_I/m} = \frac{4 k_I k_{II}}{(k_I + k_{II})^2} \quad [14.13]$$

$$R + T = 1 \quad [14.14]$$