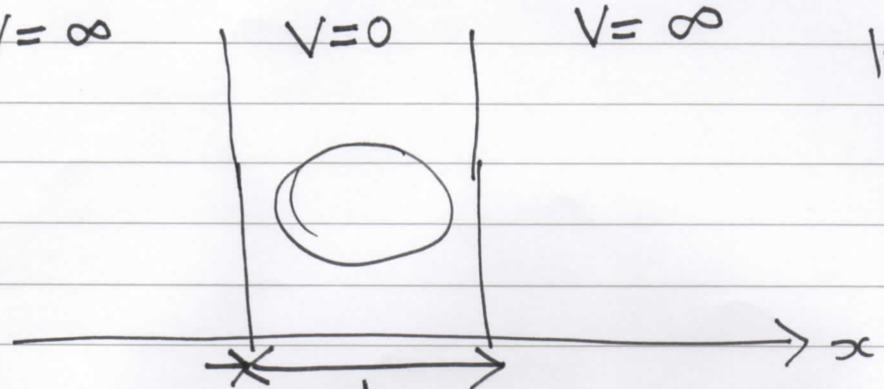
MPH

9.0 Confined Particles

9.1 Particle in a box

Let us consider a potential ∞ everywhere except in a region of width d

$V=\infty$ $V=0$ $V=\infty$



Infinite potential will exclude particles of finite ~~potential~~ energy

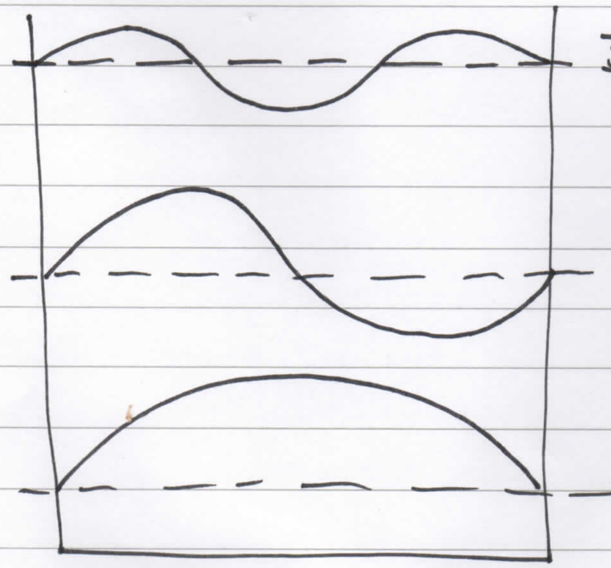
Particle will be confined inside this (1-d) box

In the box (ignoring time dependence)

$$\psi(x) = a \exp\left(\frac{i}{\hbar} p x\right) \quad \text{free patricle}$$

$$\text{or} \quad a' \cos\left(\frac{p}{\hbar} x\right) \quad \text{or} \quad a' \cos\left(\frac{2\pi}{\lambda} x\right) \quad 9.1$$

At boundary (walls) $\psi(x) \to 0$ (similar to optical cavity of L4)



$\frac{3\lambda}{2} = d$

$\lambda = d$

$\frac{\lambda}{2} = d$

Only discrete set of de Broglie wavelengths exist inside the box.

$$\boxed{d = \frac{n\lambda}{2}} \qquad 9.2$$

n integer that characterizes the state

$$\boxed{\lambda_{deB} = \frac{2d}{n}} \qquad 9.3$$

but ~~pe~~ since $p = \frac{h}{\lambda_{deB}}$

$$\Rightarrow \quad \boxed{p = \frac{nh}{2d}} \qquad 9.4$$

only discrete values in box

(actually $p = \pm nh/2d$) Momentum quantised

If $d = 10\,m$ ~~[illegible]~~

$$\cancel{\Delta} p_1 = \frac{h}{2d} = 3.3 \times 10^{-35}\ kg\,m\,s^{-1}$$

~~grains~~ momentum ~~conservation~~ quantisation won't be important (or observable)

but if $d = 10^{-10}\,m$

$$\cancel{\Delta} p_1 = \frac{6.6 \times 10^{-34}}{2 \times 10^{-10}} = 3.3 \times 10^{-24}\ kg\,m\,s^{-1}$$

of the scale of momentum of a 'typical' electron in matter.

9.2 Energy Quantisation

Energy of particle $E = \underset{\text{potential}}{V} + \underset{\text{kinetic}}{T}$

$V = 0$ (inside box) $\quad T = \frac{p^2}{2m}$ (Correspondence principle)

$$\therefore \quad \boxed{E_n = \frac{p_n^2}{2m} = \left(\frac{nh}{2d}\right)^2 \frac{1}{2m} = \frac{h^2 n^2}{8md^2}} \qquad 9.5$$

– Energy quantisation of confined particle

— $E_n \propto n^2$ n is a 'quantum number'

— same ~~analysis sec~~ outcome (qualitatively) for a 3d box.

Scales (treat as ∞ square well)

electron in 'atom sized' box (e.g. Coulomb potential)

$d \sim 10^{-10}\,m$

$$E_1 = \frac{h^2}{8m_e(10^{-20})} = 6.01 \times 10^{-18}\,J$$

$$= 37.6\,eV$$

(round about atomic scale)

'nuclear sized' box for a proton

$d \sim 8.7 \times 10^{-16}\,m$

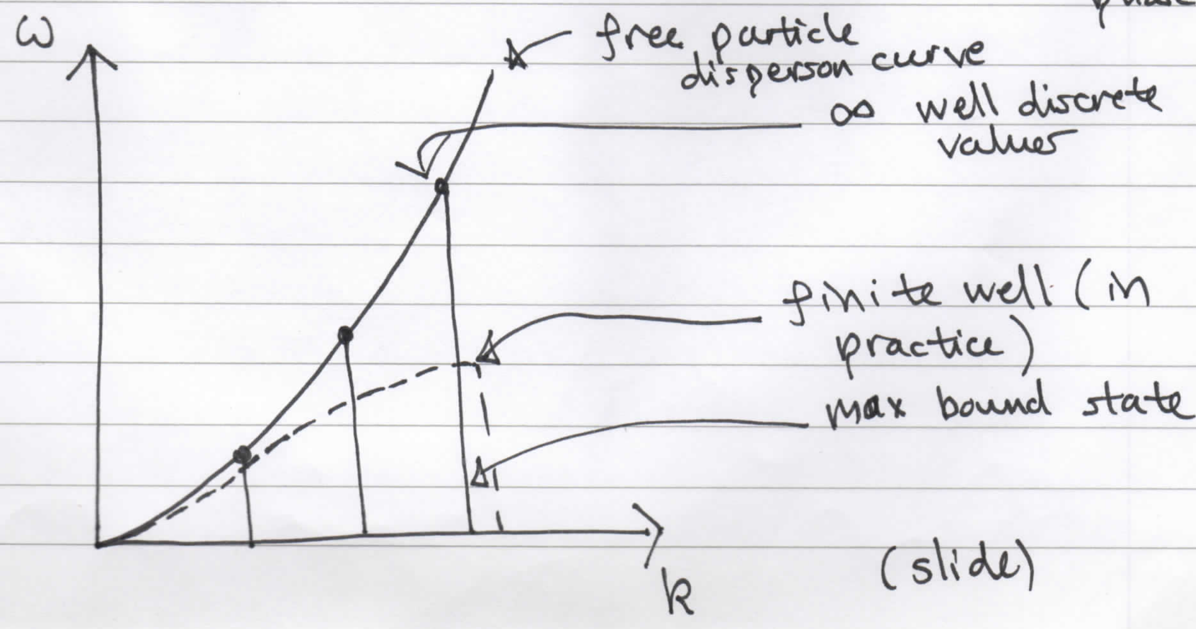
$$E_1 = \frac{h^2}{8m_p(8.7 \times 10^{-16})^2} = 528\,MeV$$

(nuclear scale)

electron in 'quantum dot'

$d = 10^{-9}\,m$ $m_e = 9.11 \times 10^{-31}\,kg$ (in practice electron has a different 'dressed' mass in condensed phase)

$E_1 = 0.375\,eV$



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