First Year Special Relativity - Lecture 2
The postulates of Relativity

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1 Introduction

The material in this lecture is covered in Young and Freedman, Secs. 37.1 and 37.3 and in McCall in Secs. 5.3, 5.6 and 5.7.

Having seen the example of rotational coordinate transforms and what classical physics assumed about relative motion, we will now look at why the Theory of Relativity arose and its postulates.

2 Maxwell's equations and the aether

One of the great successes of the discovery of what are now called the Maxwell equations was that they showed that light is waves of electric and magnetic fields. In particular, the equations in vacuum can be used to give a wave equation

$$\epsilon_0\mu_0\frac{\partial^2\vec{E}}{\partial t^2} - \nabla^2\vec{E} = 0$$

Here, the leading constant must be $1/c^2$, where c is the wave speed. This showed that the speed of light was directly related to two other quantities, the vacuum permittivity ϵ_0 and the vacuum permeability μ_0, as measured in electrostatic and magnetic experiments (respectively) in the laboratory, by $c^2 = 1/\epsilon_0\mu_0$, which fixes c. Hence, if Maxwell's equations are correct, c is a fundamental constant of nature and all light must travel at this speed in vacuum.

However, as opposed to all previously known physics laws, it turns out Maxwell's equations are not covariant under the classical Galilean transformation equations discussed in the previous lecture. The easiest way to see this is that the Galilean transformations change speed from u to $u' = u - v$, so this would imply that the speed of light should depend on the speed of the observer v. To reconcile these two apparently conflicting results, the idea of a medium called the 'aether' (sometimes spelt 'ether') was raised. This was based on an analogy with sound; this also has a speed in a medium (for sound, this is the air) calculable from the properties of the medium. However, sound does appear to have different speeds to moving observers. This is because the speed is relative to the medium and so sound only goes at the calculated speed in the frame where the medium is at rest. In other inertial frames, its speed will be dependent on its direction. Hence, the medium picks out a special inertial frame and breaks any inertial frame symmetry. Consequently physical laws will appear to be different in different frames and we would be able to tell if we were in the special frame where the medium is not moving (called 'at rest') as the speed of sound would be the same in all directions.

The aether was postulated to be the medium for light waves; it was the substance that was oscillating as the waves passed through the medium. As we can see light from distant stars, the aether was assumed to fill the whole Universe, even in the vacuum of outer space where there is nothing in terms of normal matter. Hence, as for sound, we would see light travels at a different speed than $1/\sqrt{\epsilon_0\mu_0}$ if we were moving relative to the frame where the aether is stationary and its speed would depend on direction. Effectively, the aether theory assumes the velocity transformation laws are correct but that the equation for the speed of light derived from Maxwell's equations only holds in one frame but is otherwise incorrect. This is of course testable; the Earth moves in orbit round the Sun with a speed of around 30 km/s. Measurements of the relative speed of light in two perpendicular directions taken at several different angles should show a difference in light speed when one direction aligns with the relative motion of the Earth and the aether. The difference is small ($\sim 10^{-4}$ of c) compared to the speed of light but measurable using a two-arm interferometer. The first such experiment sensitive enough to see a change in speed of this magnitude was done in 1887 by Michelson and Morley. They found no such effect and hence concluded that the aether theory must be wrong. This is the classic case of a 'failed' experiment which turned out to be extremely important, and really shows how critical is it to eliminate hypotheses as well as make discoveries.

With the aether theory rejected, Einstein realised the resolution of this dilemma was the other way round. He proposed that Maxwell's equations were correct, and hence $c^2 = 1/\epsilon_0\mu_0$ is always true for all observers, but it was the transformation equations which were wrong. Specifically, they needed to be changed so that light is observed as going at c in every inertial frame and the way Einstein found to do this was by allowing the *time* to be different in different frames. By changing the time in the right way, then the speed of light can appear the same for all frames.

3 Postulates of Relativity

Any theory must have some postulates (i.e. assumptions) from which the rest of the results arise. The theory then stands or falls on whether these assumptions are correct and this can only be verified experimentally. Relativity has only two such postulates:

1. The laws of physics are the same in all inertial frames, i.e. coordinate frames moving uniformly relative to each other.

2. The speed of light c in vacuum is independent of the speed of the light source and has the same value for all inertial observers.

Although Einstein published the Theory of Relativity over a hundred years ago, it is still the case that no violation of these two principles has ever been measured experimentally.

The first postulate is often itself called the 'Relativity Principle' as it concerns relative motion and says you can't tell how fast you are going in absolute terms. This is the reason we discussed rotations in the first lecture. In both cases, we consider how any system (such as a physics experiment) looks to different observers who are using different coordinate systems. Hence, different inertial frames are analogous to different rotated frames and hence this is a statement that all laws of physics are covariant under transformations between the coordinate systems of different inertial frames. It means there is a fundamental symmetry between all inertial frames. Also, as for rotations, the first postulate means it is impossible to build a physics experiment which can detect if it is moving in absolute terms; only relative motion is meaningful. It should be emphasised again that an inertial frame must have a constant velocity; acceleration itself can be easily detected, even directly by your body with no equipment needed.

The second postulate is basically a statement that Maxwell's equations are correct. As discussed above, the Michelson-Morley experiment found no effect due to an aether which would have indicated that Maxwell's equations were only approximate. We now believe the Maxwell's equations are exact (until quantum mechanics is included) and indeed they are covariant under Einstein's relativistic transformations. This was first noticed by Lorentz and this is why the actual transformations we will discuss in Lecture 4 are named after him. However, Lorentz treated the covariance of Maxwell's equations as being due to the specific properties of electromagnetic forces and still assumed that the Galilean transformations were correct. It was left to Einstein to fully understand their implications more generally.

We should be careful of the term 'observers' as used in the second postulate. For rotations of coordinate frames, there is no big conceptual problem here. We would assume the two people observing an experiment using two different coordinate systems would be able to see the whole experiment at all times. However, in relativity, there is often some initial confusion about the word 'observer' as it does imply visually seeing something, and relativity is all about the speed of light. However, this is incorrect; when we say 'observer' we actually mean the person has some system that allows them to record everything happening in all space at all times. It is as if they have video cameras everywhere thoughout space recording time-stamped images and the observer can look at these recordings at their leisure. Adding in the delay due to the finite speed of light to work out what they would actually visually see from one particular point in space in real time is a whole extra layer of complications. The visual appearance of relativistic objects to a single person is in fact quite an interesting study but not what we will consider in this course.

We will assume we can go to any inertial frame and there will be an (imaginary) observer who is at rest in the coordinate system of that frame. Hence, we can 'observe' any system from any speed we like, just as we could observe a system from any angle by using passive rotations. In the jargon, changing from one inertial frame to another is often refered to as doing a 'boost'. This implies an acceleration, which would be needed if the system was physically speeded up, i.e. by an active transformation, but in Relativity we will usually be considering passive transformations.

What happens to the origin in one frame relative to the other? By definition, if an inertial frame is moving at speed v relative to another frame, then the origin (or any other point) in the moving frame has speed v and so will move a distance vt in time t. Hence, an object at rest in that inertial frame will also have speed v. From the perspective of the observer in this frame, the original frame is moving in the opposite direction, but also with the same speed. The speeds could not possibly be different because, as we saw in the previous lecture, space is isotropic and there is nothing which could tell us which direction would be higher and which lower.

4 The light clock

We can get a very important result on the differences between time in different inertial frames very simply by considering a 'light clock'. This is an example of a 'thought experiment', where the consequences of a particular system are thought through, as performing a real experiment would be very difficult. A light clock is conceptually simple device which uses light bouncing off a mirror to measure time. A pulse of light is emitted, reflected by the mirror a distance d away, and then detected when it returns. The time taken in then the clock period T. Clearly, in the clock rest frame, the period is $T = 2d/c$.

Consider another frame where the clock moving at velocity v perpendicular to the separation of the light emitter and mirror. Figure 1 shows the situation in the rest frame and the moving frame.

The light pulse clearly has to go further in the moving frame. However, from the postulates of Relativity, it travels at speed c in all frames so the time taken must be longer; let the moving frame period be T'. By symmetry, the time taken for the light to get to the mirror is $T'/2$. In this time, the clock moves $vT'/2$ while the light goes $cT'/2$ and simply by Pythagoras

$$\left(\frac{cT'}{2}\right)^2 = \left(\frac{vT'}{2}\right)^2 + d^2$$

The solution is

$$T' = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{T}{\sqrt{1 - v^2/c^2}} = \frac{T}{\sqrt{1 - \beta^2}} = \gamma T$$

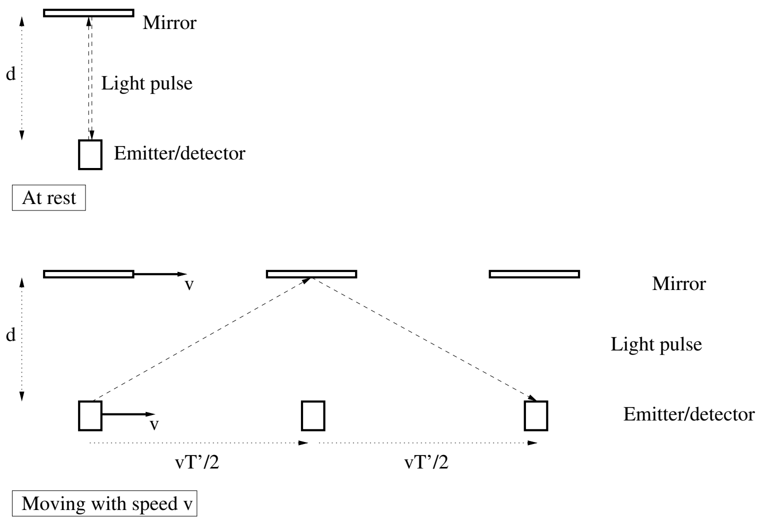

Figure 1: The lightclock at rest (top) and moving to the right with speed v (bottom).

where we have defined two new symbols which will be used throughout the course

$$\beta = \frac{v}{c}, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

These are both dimensionless; β is the velocity of the clock as a fraction of c (or in other words, in units of c), while γ can be considered to be a function of β. Another combination which often appears is $\gamma\beta$ and a useful relation to remember is

$$\gamma^2 - \gamma^2\beta^2 = \gamma^2(1-\beta^2) = 1$$

which is obvious from the definition of γ above. Figure 2 shows how γ and $\gamma\beta$ depend on β.

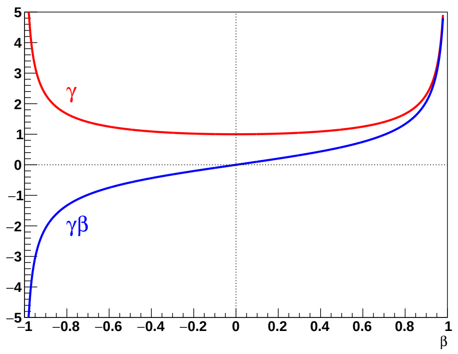

Figure 2: The functions γ and $\gamma\beta$ as a function of β.

Note $\gamma = 1$ for $\beta = 0$ and increases with $|\beta|$, going to $\gamma \to \infty$ as $\beta \to \pm 1$. If $|\beta| > 1$, then γ would become imaginary and hence so would T'. There is no physical meaning to this result so we will assume from now on that nothing can go faster than c. This applies both to the clock (if we were doing an active transformation) and to the observer (if we were doing a passive transformation). We will return to this point several times during the course.

5 Time dilation and proper time

Going back to the result on the clock periods, as $\gamma > 1$ for any moving frame, then $T' > T$, as stated earlier. This means that the clock has a longer period, i.e. is running slower, by a factor of γ in any moving frame compared with the rest frame of the clock. One issue is that as all inertial frames are equivalent, the same is true the other way around! Any observer in the rest frame of the clock will see time in the moving frame running slower than in the rest frame by the same factor. This seems completely contradictory; the solution arises from the fact that time does not only depend on the relative speed of the inertial frames but also on the object position in space, as we will see when we introduce the Lorentz transformations in Lecture 4.

This effect is called 'time dilation', as time has slowed down for the moving object. You might think this is an artifact of the light clock itself, as it uses light which is clearly has a special role in relativity. However, imagine a different type of clock, e.g. an old-fashioned alarm clock, also at rest in the light clock rest frame and right next to it. These will obviously remain in synchronisation and will tick at the same time in the rest frame. The pair of clocks as seen in a moving frame must give the same result, i.e. must tick synchronously, as it is the same system being viewed by different observers. This means the clockwork mechanism must also run slower by a factor of γ. Clearly, this must be true for every possible time-measuring device which anyone can ever dream up and this must include biological aging. Hence, we conclude the effect of time dilation is not related to the physical structure of the clock or any other object but is intrinsic to time itself. Time really does slow down for objects moving relative to an observer in another frame.

The minimum period of the light clock is clearly when $\gamma = 1$ for which $T' = T$. This is when $\beta = 0$ so the observer is in the rest frame of the clock. The time in the rest frame is therefore the shortest between two ticks. This time is called the 'proper time' (although there is no implication in the name that the time in other frames is in any sense 'improper', meaning not correct or not real) and the usual symbol for this is τ. Since any object, such as a human being, considers itself to be in its own rest frame, then the proper time measures the time as experienced by the object.