Mechanics 2017

Problem sheet 2

20 October 2017

Assessed problem

You need to hand in your answers to this problem. Make sure you write carefully, show your working and use units in your answers where appropriate.

Gliders (aircraft without engines) are subject to two main sources of drag, through which they lose energy. "Form drag" is due to friction and turbulence caused by the motion of the glider through the air and is proportional to the square of the speed. "Induced drag" is a side effect of the wing creating lift, is larger at slower speeds and is proportional to the inverse square of the speed:

$$D_{Form} = Av^2$$

$$D_{Induced} = Bv^{-2}$$

Total drag is the sum of these two and is the rate of energy loss, in watts, of the aircraft in steady flight.

(a) When the glider is flying at a constant forward speed in still air it descends at a constant rate. From where does the energy come to balance that lost through drag?

[1 mark]

(b) Show that the rate of sink (that is, the vertical component of the velocity) of a glider of mass m is given by

$$v_s = \frac{Av^2 + Bv^{-2}}{mg}.$$

You can assume that, since the glider descends at a very shallow angle, the drag acts horizontally.

[2 marks]

(c) Hence show that the forward speed which minimises the sink rate is

$$v_{ms} = \sqrt[4]{B/A}.$$

[1 mark]

(d) Sketch the curve of sink rate versus speed, labelling it carefully. You can ignore very slow and very fast speeds and your sketch should be qualitative, with no numbers.

[2 marks]

Imperial College Gliding Club owns three gliders, including "296," a high performance single seat aircraft. For this glider, $A = 2.83kg/s$, $B = 6.64\times 10^5 kg\ m^4 s^{-5}$ and with a pilot, $m = 550kg$. Take $g = 9.8m/s^2$.

(e) Sophie flies in "296" to 1000m over the Hampshire countryside. If there are no vertical air motions, how long can she now stay in the air? If she flies through still air at the minimum sink speed, how far can she reach? If, however, she is flying into a $10\ m/s$ headwind at the minimum sink speed, how far can she fly?

[4 marks]

Homework problems

Attempt these in your own time; if you have problems, ask at your tutorial.

1. The Tesla Model S is a fully electric car which I would like to own. With a full charge of its 85kWh battery pack (that is, a pack containing enough energy to dissipate 85kW for one hour), Tesla claims a range of 283 miles when driving at 60 mph. 1 mile is 1609m.

   (a) What is its power consumption, in watts, at 60mph?

   (b) Assuming that all of this power is used to overcome friction, what is the frictional retarding force?

   (c) The car weighs 2000kg; take $\mathrm{g} = 10\mathrm{m/s^2}$. How much power is required to drive the car up a 10% incline – a steep hill – at the same speed? What is the range in this case?

   (d) What is the gradient of a hill if the car can coast down it, using no power, at the same speed?

2. A particle of mass 2kg follows the trajectory

$$\mathbf{r} = \mathbf{i} + 15\mathrm{t}\mathbf{j} - 5\mathrm{t}^2\mathbf{k}\ \mathrm{m}$$

   where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors. Calculate the particle velocity, and the force exerted upon it, at $\mathrm{t} = 2\mathrm{s}$. Can you think of an example of when a particle would undergo such a motion?

3. A particle drops vertically under the force of gravity and a retarding viscous force $\mathrm{F} = -\mathrm{Rv}$ proportional to its speed v. Take $\mathrm{g} = 10\mathrm{m/s^2}$.

   (a) Show that, when released from rest, its speed at time t is given by

$$v = \frac{mg}{R}\left(1 - e^{-\frac{Rt}{\mathrm{m}}}\right).$$

   (b) If the particle has a mass of $100\mu\mathrm{g}$ and reaches a limiting speed of 10 m/s, how long did it take to reach 90% of this speed?

Tutorial problem

This is a problem that your tutor might choose to cover during the tutorial.

On the 14th of November 2013, Thunder Law threw a basketball into a hoop 109 feet 9 inches (33.45m) away; you can watch it (not in the tutorial, please!) on YouTube. Take $g = 9.8ms^{-2}$. We ignore air resistance and for simplicity we're going to assume that Mr Law released the ball at the same height as the hoop (10 feet): not a terrible approximation given that he is 6'3" tall.

(a) The player releases the ball at a speed v and inclination (angle to the horizontal) α. Show that it travels a distance

$$x = \frac{v^2}{g} \sin 2\alpha$$

How long does the ball take to travel this distance?
The identity $\sin 2\theta = 2 \sin\theta \cos\theta$ will prove useful.

(b) A classic ballistics problem: by differentiating the distance with respect to α, show that the maximum range occurs when $\alpha = 45°$. If Mr Law did release the ball at $45°$, how fast did he have to throw it to get it into the hoop?

(c) The hoop has a 46cm diameter. Assuming the ball was originally perfectly on target, at how different an angle would Mr Law have had to throw the ball to make the bottom of it hit the front rim?

(d) Calculate how long it should have taken for the ball to drop into the hoop. From the video, it seemed to take about 2.5s.