MECHANICS PROBLEM SHEET 2

HOMEWORK PROBLEMS ANSWERS

Q1.

60mph. 85kWhr total power

(a) Total range at 60mph is 283 miles.

This takes $\frac{283}{60} \times 3600 = 16{,}980\,s$.

We dissipate $85\,kWhr = 85 \times 10^3 \times 3600$
$= 306 \times 10^6\,J$ in this time.

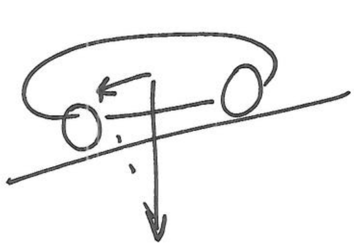
So, power $= \frac{306 \times 10^6}{16{,}980} = \underline{18\,kW}$.

(b). Power = Force × velocity.

$V = \frac{60 \times 1609}{3600} = 26.8\,m/s$

So, force $F = \frac{P}{V} = \frac{18 \times 10^3}{26.8} = \underline{672\,N}$.

(c).



mg

Component of weight retarding car is $M \times g \times 10\%$
$= 2000 \times 10 \times 0.1 = 2000\,N$.

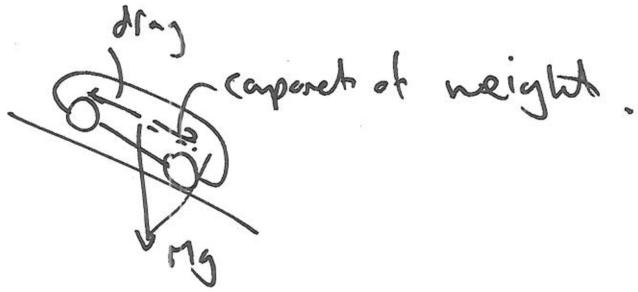
So total force $= 672 + 2000 = 2672\,N$.

Power $= F \times v = 2.6 \times 10^3 \times 26.8 = \underline{71.6\,kW}$

Range is $\dfrac{85 \times 10^3}{71.6 \times 10^3} \times 60 = \underline{71.2\text{ miles}}$

(Electric cars do not like hills.)

(d).



component of weight equals drag

$\Rightarrow \quad Mg \times A = 672$

$A = \dfrac{672}{20{,}000} = \underline{\underline{32\text{ gradient}}}$

Q2.

$$\underline{r} = \underline{i} + 15t\,\underline{j} - 5t^2\,\underline{k} \text{ m.}$$

Velocity,

$$\underline{v} = \frac{d\underline{r}}{dt} = \dot{\underline{r}} = 0\underline{i} + 15\underline{j} - 10t\,\underline{k} \text{ m/s.}$$

At $t = 2$s,

$$\underline{v} = 15\underline{j} - 20\underline{k} \text{ m/s.}$$

Force,

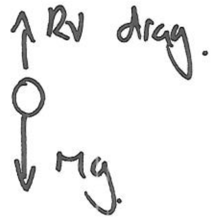
$$\underline{F} = m\underline{a}$$

$$\underline{a} = \frac{d^2\underline{r}}{dt^2} = \frac{d}{dt}(\underline{v}) = -10\,\underline{k} \text{ m/s}^2.$$

So, force $\underline{F} = 2 \times \underline{a} = -20\,\underline{k}$ N.

Motion is a constant acceleration of 10 m/s² downwards – freefall in Earth's gravitational field.

Q3.



(a). Take v positive upwards.

$$F = M\frac{dv}{dt}.$$

$$-Mg - Rv = M\frac{dv}{dt}.$$

$$\Rightarrow \quad dt = -M\frac{dv}{Mg+Rv}.$$

Trick: define $w = Mg + Rv$.

then, $\frac{dw}{dv} = R \quad \Rightarrow \quad dv = \frac{dw}{R}.$

So, $$dt = -\frac{M}{R}\frac{dw}{w}.$$

Integrate:

$$\int_0^t dt = -\frac{M}{R}\int_{w_0}^{w}\frac{dw}{w}$$

$$t = -\frac{M}{R}\ln\left(\frac{w}{w_0}\right).$$

$$w = w_0 e^{-\frac{Rt}{M}}.$$

Substitute back w,

$$Mg + Rv = w_0 e^{-Rt/M}.$$

What is w_0?

At $t=0$, $v=0$.

$$\Rightarrow Mg + 0 = w_0 \underbrace{e^{-0}}_{=1}.$$

$$\Rightarrow w_0 = Mg.$$

So, $Rv = Mg\left(e^{-Rt/M} - 1\right)$

$$v = \frac{Mg}{R}\left(e^{-Rt/M} - 1\right).$$

This is true upwards; we know v is always downwards, so magnitude,

$$|v| = \frac{Mg}{R}\left(1 - e^{-Rt/M}\right).$$

(b) Limiting speed is when $t = \infty$.

$$v_\infty = \frac{Mg}{R} = 10. \quad \text{So, } \frac{M}{R} = 1.$$

At 90% speed,

$$\frac{v}{v_\infty} = 0.9 = \left(1 - e^{-Rt/M}\right) = 1 - e^{-t}.$$

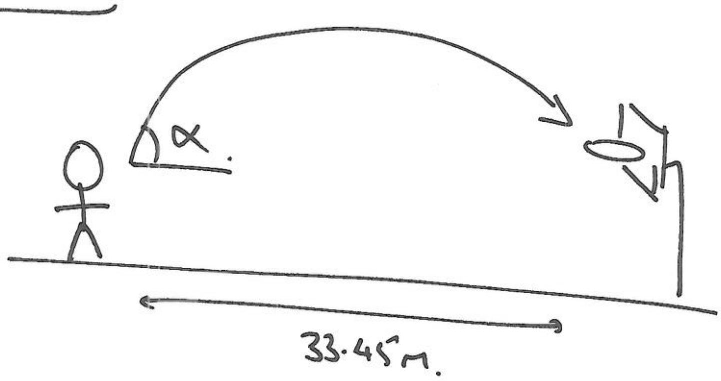
$$e^{-t} = 0.1.$$

$$\Rightarrow e^{+t} = 10$$

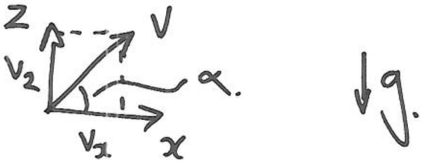
$$t = \ln(10) = \underline{\underline{2.3\text{ s}}}$$

MECHANICS PROBLEM SHEET 2

TUTORIAL PROBLEM.



(a). At release:



Choose z positive upward.

Initial horizontal speed $v_x = v\cos\alpha$

" vertical " $v_z = v\sin\alpha$.

<u>Vertical</u> motion:

g positive, points downwards.

$$z(t) = \underbrace{v_z}_{\text{initial}} t - \frac{1}{2}gt^2$$

when it goes in the hoop, $z = 0$.

$$\Rightarrow v_z t = \frac{1}{2}gt^2$$

$\div t$.

$$\Rightarrow t = \frac{2v_z}{g} = \frac{2v\sin\alpha}{g}.$$ Time taken.

<u>Horizontal</u> motion:

$$x(t) = \underbrace{v_x}_{\text{initial}} t.$$

When balls goes in hoop, $t = \frac{2v\sin\alpha}{g}$

$$\Rightarrow \quad x = v_x t = v\cos\alpha \times \frac{2v\sin\alpha}{g}$$

$$= \frac{v^2}{g} \times \underbrace{2\cos\alpha\sin\alpha}_{\text{identity } = \sin 2\alpha}$$

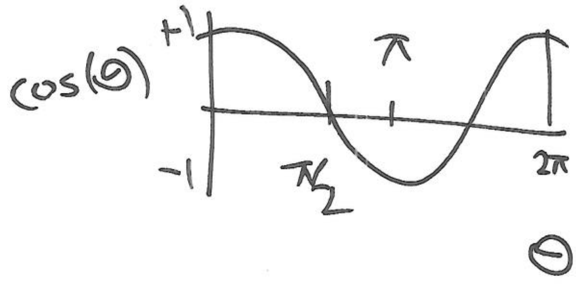
$\Rightarrow$ distance travelled,

$$\underline{x = \frac{v^2 \sin 2\alpha}{g}}$$

(b). Differentiate distance with respect to α,

$$\frac{dx}{d\alpha} = \underbrace{\frac{v^2}{g}}_{\text{all constant}} \times \frac{d}{d\alpha}(\sin 2\alpha) = \frac{v^2}{g} \times 2\cos 2\alpha.$$

This is zero when $\cos 2\alpha = 0$.

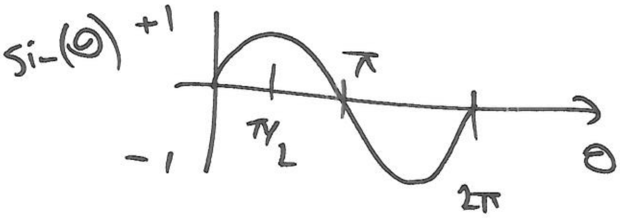


$\Rightarrow 2\alpha = \frac{\pi}{2} \quad \Rightarrow \alpha = \frac{\pi}{4}$, $\underline{45°}$

Is this a maximum or minimum?

$$\frac{d^2x}{d\alpha^2} = \frac{2v^2}{g} \times \frac{d}{d\alpha}(\cos 2\alpha) = \frac{2v^2}{g} \times -2\sin(2\alpha).$$

$$= -\frac{4v^2}{g}\sin(2\alpha).$$



when $\alpha = \pi/4$, $\sin(2\alpha) = +1$.

v^2 and g are positive (so is 4),

so $-\frac{4v^2}{g}\sin(2\alpha) < 0.$

$\Rightarrow$ Maximum

If $\alpha = 45°$, for $x = 33.45\,m$,

$$x = \frac{v^2 \sin 2\alpha}{g}$$

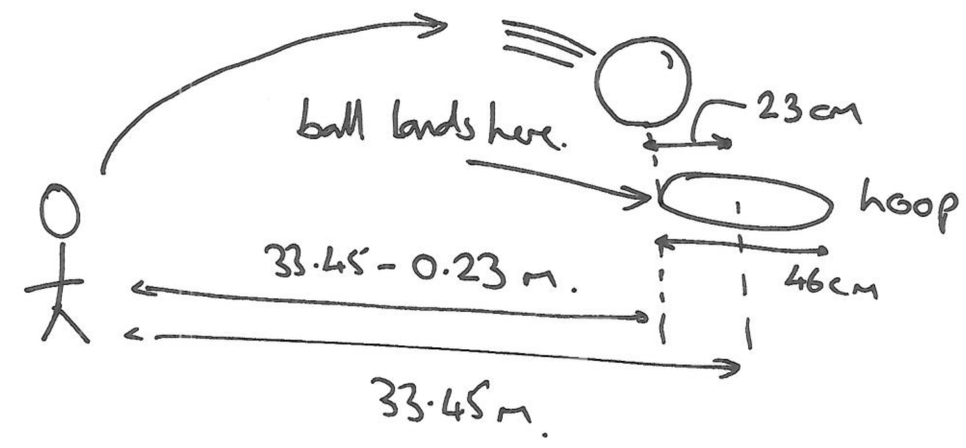
$$\Rightarrow v^2 = xg$$

$$v = \sqrt{xg}$$

Numbers,

$v = 18.1\,m/s$ - fast!!!

(c).



Now,

$$x = 33.45 - \frac{0.46}{2} = 33.22\text{m}.$$

Same v as before,

$$x = \frac{v^2 \sin 2\alpha}{g}$$

$$\Rightarrow \quad \sin 2\alpha = \frac{xg}{v^2}$$

Numbers, $\sin 2\alpha = 0.991$

$$\Rightarrow \quad 2\alpha = 83.27^\circ$$

$$\underline{\alpha = 41.63^\circ}$$

Less than 4° from a perfect hit.

That's why they're paid so much.

(d) $t = \dfrac{2v \sin\alpha}{g}$. For 45°, $\cancel{t = \dfrac{2v}{g} = 3.7\text{s}}$ $\quad t = \dfrac{2v}{g} \times \underbrace{\sin 45^\circ}_{0.707} = \underline{2.6\text{s}}$

Drag is probably the biggest difference. Makes trajectory more like this:

than this: