Mechanics 2017

Problem sheet 5

10 November 2017

Assessed problem

You need to hand in your answers to this question. Write clearly and make sure you show your working.

1. A ball is dropped from 3m onto a table; it bounces, with a coefficient of restitution $e = 0.7$. Take $g = 10ms^{-2}$.

   (a) How fast is the ball travelling when it first reaches the table?

   [2 marks]

   (b) What height does it reach after the first bounce?

   [2 marks]

   (c) How long does it take from being released to reach that height?

   [2 marks]

   (d) How long does it take to stop completely?

   [4 marks]

Hint: you will need to use the formula for a geometric series.

Homework problems

Attempt these in your own time; if you have problems, ask at your tutorial.

1. In 1D collisions, the final and initial speeds are related by

$$v_1 - v_2 = -\epsilon(u_1 - u_2)$$

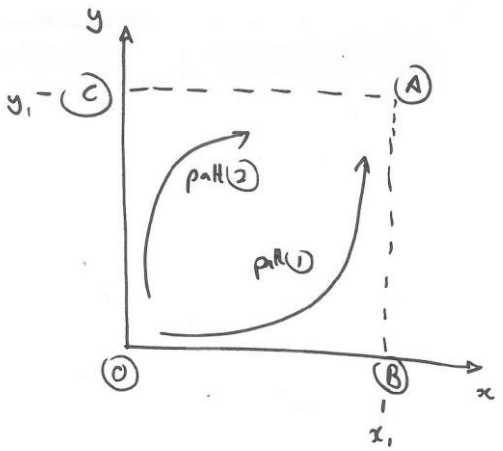
Where ϵ is the coefficient of restitution. Use this equation, plus momentum conservation, to derive the following expressions for the final speeds of both bodies, for the case when $u_2 = 0$, that is, the second body is initially at rest:

$$v_1 = u_1 \times \frac{m_1 - \epsilon m_2}{m_1 + m_2}$$

$$v_2 = u_1 \times \frac{m_1 + \epsilon m_1}{m_1 + m_2}$$

2. In the lectures, I stated that in 3D, unlike 1D, it was possible to have a force which is only a function of position, but which is not conservative. That is, we cannot define a potential associated with this force. For us to be able to define a potential, the change in kinetic energy on a body due to its motion under the force must be *independent* of the path taken between two points. Here, you will see from a simple example that it is straightforward to construct such a non-conservative force.

The force we consider is $F = (+ky, -kx, 0)$ where k is a constant. We calculate the work done by the force when a body moves from the origin, $O = (0,0,0)$ to point A at $(x_1, y_1, 0)$, over two different paths.



Path 1 is O-B-A, where B is located at $(x_1, 0,0)$.
Path 2 is O-C-A, where C is located at $(0, y_1, 0)$.

(a) Show that the work done on path 1 is $W_{OBA} = -kx_1y_1$
(b) Calculate W_{OCA}, the work done on path 2.

Since $W_{OBA} \neq W_{OCA}$, the work done is path dependent and this is not a conservative force.

Tutorial problem

This is a problem that your tutor might choose to cover during the tutorial. Since we've only had one lecture this week, I thought I would show you an old Comprehensive paper question (this is Q1 of Comp II, in 2000). This might be a good opportunity to discuss exam technique and how to answer questions: the end of term test is not too long away!

1. (i) On their round the world balloon flight, the balloonists throw an empty fuel tank overboard. Describe qualitatively how the vertical speed of the tank varies with time. You may assume that the density of the atmosphere does not vary with height.

(ii) The tank, of mass m is dropped from a height, h, when the balloon is travelling parallel to the Earth's surface with a speed, u. In addition to gravity, the falling tank experiences an air resistance force that is taken to be proportional to its speed with proportionality constant k. (Coriolis forces may be ignored.)

(a) Show that the equation of motion of the tank can be written as

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -k\mathbf{v} - mg\mathbf{j}, \qquad (1.1)$$

where the unit vector in the vertical, y-direction is $\mathbf{j}$ and the unit vector along the direction of travel of the balloon is $\mathbf{i}$.

(b) From equation (1.1), write down the components for the vertical and horizontal directions respectively and solve them for v_x and v_y as functions of time.

(c) Show that the velocity in the y-direction tends to the limit $-mg/k$.

(d) Show that the maximum horizontal distance travelled by the tank is mu/k.

(e) If $m = 20$ kg, $h = 10^4$ m, $u = 50$ m s^{-1} and the terminal speed of the tank is 50 m s^{-1}, estimate the time taken for the tank to reach 90% of its final speed. Hence or otherwise find an estimate for the horizontal distance between the balloon and the point at which the tank strikes the Earth.

(f) Explain, without detailed calculations, how a more realistic model of the atmosphere would alter the results obtained in parts (c) and (d).

(g) To what extent are we justified in assuming that air resistance is directly proportional to speed?