Mechanics 2017

Problem sheet 6

17 November 2017

Assessed problems

For these questions, you can use standard formulae we derived in the lectures.

A proton scatters elastically at an angle of 47° from a nucleus, which recoils at 65°.

1. What element was the nucleus?
   a. Hydrogen
   b. Helium
   c. Carbon
   d. Nitrogen
   e. Oxygen

[3 marks]

A lithium nucleus undergoes an elastic collision with an initially stationary proton.

2. What is the maximum angle through which the lithium nucleus can be deflected?
   a. The lithium nucleus is not deflected
   b. 180^0
   c. 90^0
   d. 14.5^0
   e. 8.21^o
   f. 0.37^0

[3 marks]

3. In this case, what is the angle between the paths of the outgoing lithium and the recoiling proton?
   a. 0^0
   b. 5.26^0
   c. 32.8^0
   d. 49.1^0
   e. 52.5^0

[4 marks]

Homework problem

Attempt this in your own time; if you have problems, ask at your tutorial.

This question is based on the SpaceX Falcon 9, a remarkable rocket. Like all launch vehicles, it uses multiple stages to increase mass to orbit, but uniquely it can soft-land its first stage back on Earth for re-use. Take $g = 10\ m/s^2$.

	Stage 1	Stage 2	Payload
Dry mass	18,000kg	5,000kg	13,000kg
Propellant mass	385,000kg	40,000kg	None
Specific impulse	311s	345s	N/A
Burn duration	180s	372s	N/A

Reminder: The *specific impulse* I_{sp} is a value, in seconds, describing how long a given rocket could hover against Earth's gravity with a kilogram of fuel. The effective exhaust speed is then $u = g \times I_{sp}$.

Rocket trajectories are actually very complicated with no margin for error. We consider a grossly simplified case, with no air resistance and we are just aiming to reach orbital velocity.

The key equation from lectures is $v = u \ln M_0/M_f - gt$ where M_0 and M_f are the initial and final masses, u the exhaust speed and t is the burn time.

(a) The main job of the first stage is to get the vehicle out of the atmosphere. For the first stage, calculate u and the total payload mass. Assuming it travels vertically, what is the speed of the rocket at the end of the first stage burn? The vehicle tips over as it rises, so we have this effective speed horizontally at the end of the burn. The first stage is released and falls away: we assume that we have reached orbital altitude of a few hundred km.

(b) Low Earth orbits take around 90 minutes: how fast must the rocket be going to achieve this? The second stage lights and accelerates horizontally (so you can ignore gravity) from its initial speed: can it reach orbital speed?

(c) Falcon rockets launch to the North-East from Cape Canaveral (at $28°N$): show that this makes it possible to reach orbit.

(d) SpaceX can now recover the first stage back to the launch site, which requires a burn to stop the stage's horizontal motion, a second burn at mid-altitudes and a final burn as it lands, on extensible legs. This must take at least 20 tons of the first stage's fuel. What is the reduction in final payload speed if this is the case?

If you are feeling geeky about Falcon 9 rockets – and why would you not be? – you might find this graphic interesting: https://i.redd.it/8mwqvbd2n5py.png

Tutorial problem

This is a problem that your tutor might choose to cover during the tutorial.

Nuclear fission reactors use neutrons emitted by the fission process to trigger subsequent fission events in other nuclei, leading to a chain reaction. The emitted neutrons are initially travelling too quickly, however, so a "moderator" material is used to slow them down. In this example we use deuterium as the moderator.

A neutron is released at a speed $v = 10^4 m/s$ and collides elastically with a stationary deuteron.

(a) What is the speed of the centre of mass in the reactor frame?

(b) Before the collision, what are the speeds of the neutron and deuteron in the centre of mass frame? What is the ratio of the total kinetic energy in the reactor and centre of mass frames?

(c) What is the maximum fraction of the neutron's kinetic energy that can be transferred to the deuteron? If half of the maximum energy is transferred on average in each collision, estimate how many collisions are required to bring the neutron's speed below $10^3 m/s$.

(d) If half of the maximum energy is transferred, what is the angle in the reactor frame through which the neutron is scattered? Why is it desirable for this to be a large angle?

(e) If carbon was used as a moderator instead of deuterium, how many collisions would be required to slow the neutron?

You will find useful the following formulae that we proved in the lectures:

Fractional kinetic energy transferred to a stationary target:

$$\frac{E'_{K,2}}{E_{K,1}} = 4\frac{m_1 m_2}{(m_1 + m_2)^2}\sin^2\left(\frac{\theta^*}{2}\right)$$

Relation between lab frame and centre of mass frame scattering angles:

$$\tan\theta = \frac{\sin\theta^*}{\frac{m_1}{m_2} + \cos\theta^*}$$