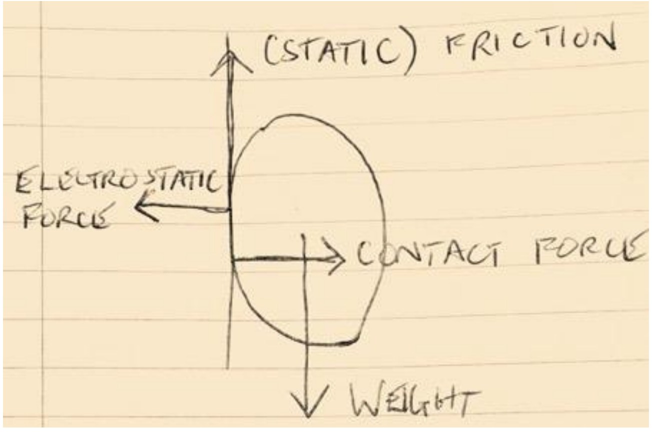
Study handout 1 hints, answers and solutions for non-assessed questions

1. You rub a balloon on your head and stick the balloon on the wall. Has the balloon gained or lost electrons or is it impossible to say? Given that the wall is electrically neutral how is it possible to stick the charged balloon to the wall? Sketch a free body diagram for the stuck balloon.

In the absence of any further information one cannot say in which direction the electrons are transferred and more information or a basic experiment with a known charge would have to be carried out to see. It happens to be that the electrons go from the hair to the balloon though.

The free body diagram is of this form:

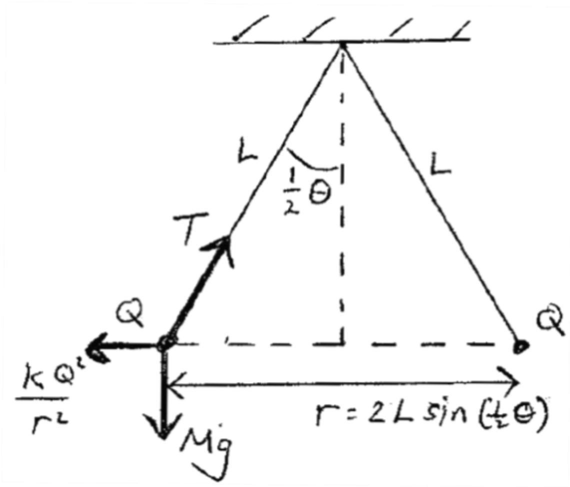


Clearly the friction must equal the weight and the electrostatic force must equal the contact for for linear equilibrium but notice the relative positions of the forces needs to be in the sense noted for rotational equilibrium as well.

As to how a charged object sticks to a neutral think about what the charges in the neutral object will want to do when a charged object approaches.

2. Two charged spheres both of mass M and are attached to light strings of length L and attached to the same point in a uniform gravitational field. It the charges are both charged to the same value $+Q$ find an expression for Q if the equilibrium position is with the strings at an angle θ apart. What is the tension in the strings? How could this arrangement be used to test Coulomb's law? Would it be better or worse than a torsion balance?

A sketch of the arrangement showing all the forces on the left hand charge is shown below. The forces on the right hand charge are the mirror image but aren't shown.



Vertically the weight of the sphere is exactly balanced by the vertical component of the tension in the string i.e. $mg = T\cos\frac{\theta}{2}$ so the value for the tension in the string is $T = \frac{mg}{\cos\frac{\theta}{2}}$. This makes sense as when the angle is zero the tension is equal to the weight and as the angle approaches $180°$ it becomes progressively difficult for the vertical component to match the weight hence it tends to infinity.

Horizontally the electrostatic force given by Coulomb's law is exactly balanced by the horizontal component of the tension i.e. $k\frac{Q^2}{r^2} = T\sin\frac{\theta}{2}$. Incorporating the value of the tension given above and the distance r from the figure gives $\frac{Q^2}{4L^2\sin^2\frac{\theta}{2}} = \frac{mg\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$. This can be rearranged to give a value for the charge of $Q = \sqrt{\frac{mg}{k}}4L\sin\frac{\theta}{2}\sqrt{\tan\frac{\theta}{2}}$.

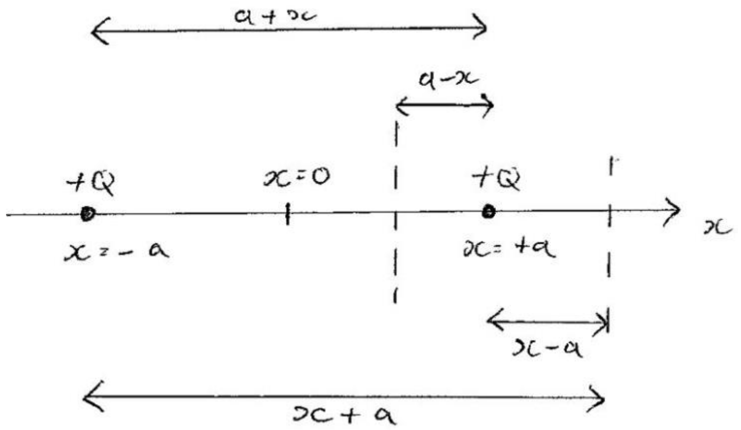
This final result isn't particularly edifying but does makes sense with extremes of angle again (zero charge at zero angle and infinite charge for $180°$). It also shows that for small angles and charges the charge is directly proportional to $\theta^{3/2}$.

As to whether this would be a good way to test Coulomb's law is all relative. It wouldn't really be easy to design an accurate experiment. A torsion balance would probably be better as the complication of the weight of the strings wouldn't be a factor.

3. Two point charges of value $+Q$ are fixed in position at $x = -a$ and $x = +a$. Find the value of the force on another charge $+q$ as a function of position along the x-axis. Sketch a graph of force vs. position and comment on any interesting features. For $x \gg a$ find an approximate expression for the force. Would the charge experience any force at all at $x = 0$?

This question requires Coulomb's law and the principle of superposition.

First of all consider the setup and let's guess the nature of some of the results:



Before making any calculations we can surmise the following:

a. **The force as a function of x, $F(x)$, is going to be symmetric about the origin so the final expression should be an even function**

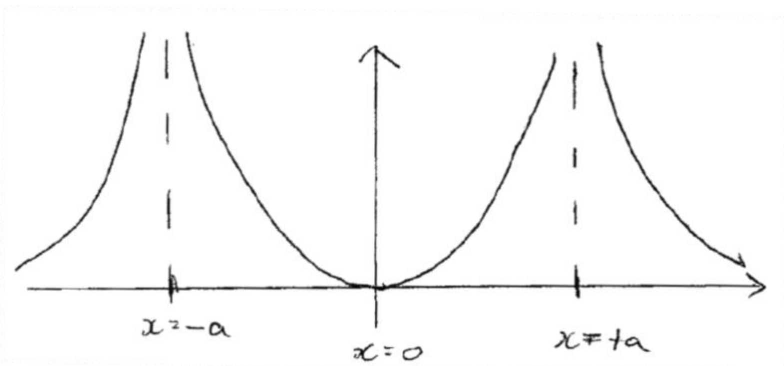
b. **The force should tend to zero as $x \to \pm\infty$; in fact for $|x| \gg a$ the force should approximately that of the force between two charges of values $+q$ and $+2Q$**

c. **The force should be exactly zero at $x = 0$**

d. **There should be positive singularities at $x = \pm a$ which tend to $+\infty$ when approaching from either side**

e. **The function should be continuous everywhere else**

Without making any calculations it is thus a fair bet that the plot should look something like this:



Let's see what happens when we do the maths:

If we consider the positive direction of forces as left to right, for $x > a$ the force is given by $k\frac{Qq}{(x-a)^2} + k\frac{Qq}{(x+a)^2} = \frac{kQq}{x^2}\left(\left(1+\frac{a}{x}\right)^{-2} + \left(1-\frac{a}{x}\right)^{-2}\right)$.

This has a singularity at $x = a$ (agreeing with (d) and diminishes with x (agreeing with (b)).

When $x \gg a$ using the binomial theorem the expression becomes $\frac{kQq}{x^2}\left(1 - 2\frac{a}{x} + 1 + 2\frac{a}{x} + O(\left(\frac{a}{x}\right)^2)\right) \approx 2\frac{kQq}{x^2}$ which is just the force due to a point charge of values $2Q$ and a (agreeing with (b)).

For $x < -a$ the expressions will essentially be the same as this with the forces acting left to right i.e. with left to right as positive the force expression is the same as for $x > a$ but with a minus sign on the front. As all x terms are squared this gives an even function for $|x| > a$ agreeing with (a).

Between the charges the force is given by $-k\frac{Qq}{(a-x)^2} + k\frac{Qq}{(a+x)^2} = \frac{kQq}{x^2}\left(\left(1 + \frac{x}{a}\right)^{-2} - \left(1 - \frac{x}{a}\right)^{-2}\right)$.

This has singularities at $x = \pm a$ in agreeing with (d) and is equal to zero at the midpoint agreeing with (c) – the test charge experiences no force here and if it is stationery and constrained to the x-axis is in stable equilibrium as the slightest deviation will cause it to be pushed back to the origin. If not constrained to the x-axis it is in unstable equilibrium as is will get pushed other dimensions on deviation.

When $a \gg x$ using the binomial theorem the expression becomes $\frac{kQq}{a^2}\left(1 - 2\frac{x}{a} - 1 - 2\frac{x}{a} + O(\left(\frac{x}{a}\right)^2)\right) \approx -4\frac{kQq}{a^3}x$ so the force is proportional to the displacement at small displacements from the origin and acts in the opposite direction to the displacement.

If constrained to the x-axis for small displacements the charge would thus oscillate with simple harmonic motion of angular frequency given by $m\omega^2 = 4\frac{kQq}{a^3}$ where m is the mass of the charge q and thus a frequency of $T = \pi\sqrt{\frac{ma^3}{kQq}} \equiv 2\sqrt{\frac{\epsilon_0 m(\pi a)^3}{qQ}}$.

All parts of the detailed solution agree with the initial sketch of the plot thus no further modification is needed.

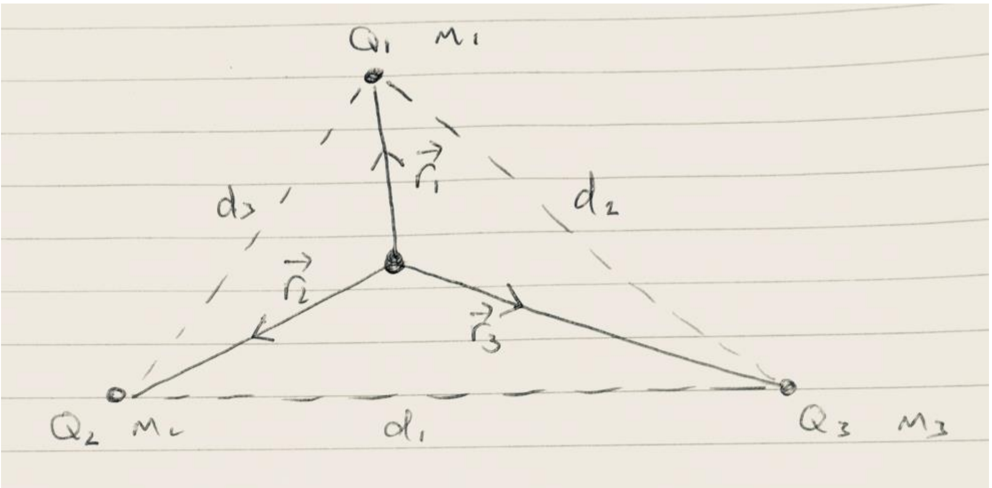
4. On page 9 of the lectures we saw a calculation to see the force between a point charge and radial line of charge. Repeat the calculation but for a line of charge perpendicular to the one in the example.

This is incorporated into the full solution for question 17.

5. **The full solution for this former classwork is included as a separate PDF.**

6. Three positively charged spheres form the vertices of a triangle. When the spheres are released from rest they all move in a straight line. Show that for this to happen the triangle they form remains similar to the original triangle at all times. The motion takes place in a vacuum and although the spheres are not massless gravitational forces can be ignored.

This is a type of three body problem and without having done anything of this nature before and in the absence of any hints it is certainly difficult. The first thing to do is sketch a diagram to show the geometry of the situation. The $\vec{r_i}$ vectors are the displacements from the centre of mass arrangement to the sphere; the d_i scalars are the distances between the spheres.



There are three arbitrary fixed charges with three arbitrary fixed masses at the vertices of a triangle of arbitrary shape. The charges can have any sign or magnitude and their masses can be any magnitude. They will all move and in the general case will certainly not move in a straight line. In this problem we are told that they all move in straight lines and given that, essentially to show the shape of the triangle stays the same.

There are several ways in which this might be tackled; here we present just one. To start there a few things to bear in mind which could constitute hints. If you had problems getting going have a try using the diagram and the hints before going for the full solution:

(1) The centre of mass remains at a fixed position as there are no external forces and thus setting this as the origin of the coordinate system is a sensible starting point.

(2) If an object moves in a straight line the direction of its acceleration in in the same direction as its position vector.

(3) If the ratio of the distances from the centre of mass remains the same in time then although the size of the triangle may get bigger or smaller, plus rotate about the centre of mass, the angles must stay the same and thus the triangle stays similar.

Onwards, considering sphere 1: Using Newton's second law, Coulomb's law and the principle of superposition gives

$$\overrightarrow{F_1} = m_1\overrightarrow{a_1} = k\frac{Q_1Q_2}{d_3^3}(\overrightarrow{r_1} - \overrightarrow{r_2}) + k\frac{Q_1Q_2}{d_2^3}(\overrightarrow{r_1} - \overrightarrow{r_3})$$

The mathematical expression for the centre of mass being at the origin is

$$m_1\overrightarrow{a_1} + m_2\overrightarrow{a_2} + m_3\overrightarrow{a_3} = m_1\overrightarrow{v_1} + m_2\overrightarrow{v_2} + m_3\overrightarrow{v_3} = m_1\overrightarrow{r_1} + m_2\overrightarrow{r_2} + m_3\overrightarrow{r_3} = 0$$

Substituting $\overrightarrow{r_3}$ into the dynamics equation gives, after a little algebra

$$m_1\overrightarrow{a_1} = kQ_1\left[\frac{Q_2}{d_3^3} + \frac{Q_3}{d_2^3}\left(\frac{m_1}{m_3} + 1\right)\right]\overrightarrow{r_1} + kQ_1\left[\frac{Q_3}{d_2^3}\frac{m_2}{m_3} - \frac{Q_2}{d_3^3}\right]\overrightarrow{r_2}$$

Now consider hint (2). For this to be correct it means that, using the above equation,

$$m_1\overrightarrow{a_1} = kQ_1\left[\frac{Q_2}{d_3^3} + \frac{Q_3}{d_2^3}\left(\frac{m_1}{m_3} + 1\right)\right]\overrightarrow{r_1}$$

i.e. the acceleration of sphere 1 is proportional to its own position vector alone, and,

$$kQ_1\left[\frac{Q_3}{d_2^3}\frac{m_2}{m_3} - \frac{Q_2}{d_3^3}\right]\overrightarrow{r_2} = 0$$

which leads to

$$\frac{Q_3}{d_2^3}\frac{m_2}{m_3} = \frac{Q_2}{d_3^3}$$

which in turn implies that

$$\frac{Q_1}{m_1}d_1^3 = \frac{Q_2}{m_2}d_2^3 = \frac{Q_3}{m_3}d_3^3 = \lambda$$

The first term in included by symmetry. We are getting somewhere now as we have found that a constant term at any given time for each sphere. Of course charge and mass are invariant, but the distances do change. So λ is a constant in sense that it is always identical at any given time for all three spheres though it does vary in absolute value from one moment to the next.

It takes some more algebra but now using the above expression in the equation for the acceleration gives

$$\overrightarrow{a_1} = \frac{k}{\lambda}\frac{Q_1Q_2Q_3}{m_1m_2m_3}(m_1 + m_2 + m_3)\overrightarrow{r_1}$$

and by symmetry similar equations apply to the other spheres i.e. $\overrightarrow{a_2} = \alpha\overrightarrow{r_2}$ and $\overrightarrow{a_3} = \alpha\overrightarrow{r_3}$ where $\alpha = \frac{k}{\lambda}\frac{Q_1Q_2Q_3}{m_1m_2m_3}(m_1 + m_2 + m_3)$.

To stress once again, the "constant" varies is time but is always identical for all three spheres at any given time.

So given hint (3) as the ratio of the spheres' accelerations is always the same, so is the ratio of their velocities and their distances from the centre of mass.

One can go a little further to find expressions for the vertex angles as a function of the quantities labelled above and show that these remain constant as well (and thus give some rigour to hint 3) but the problem is now essentially solved.

7. In what ways are heat conduction and electrical conduction similar and different?

A question which could take entire textbooks to answer but one of the key things to think about it what the cause is, what the effect is and what the property of the material that resists the effect occurring is.

8. We say that scalar quantities have a magnitude only and whereas vectors have a magnitude and direction. But it is common to talk about the direction a current travels in. Is it therefore proper to refer to current as a vector?

No. Refer to lectures 1 and 2; this is revisited in section 6.

9. The Bohr model of a hydrogen atom consists of an electron orbiting a single positively charged proton in a circle according to Newton's second law, akin to a planet orbiting the Sun but with the mutual force of attraction being electrostatic rather than gravitational. Compute the approximate value of the electrical current produced by the orbiting electron according to this model.

Using $F = ma$ with Coulomb's law and the equation for acceleration in uniform circular motion of $a = \frac{v^2}{r}$ the equation of motion for the electron in this model can be written as $m\frac{v^2}{r} = k\frac{e^2}{r^2}$ where m is the electrons's mass (about 9.1×10^{-31} kg), v its speed, e the charge on the electron and r the atom's radius (about 53 pm).

This rearranges to give a electron speed of $v = e\sqrt{\frac{k}{rm}}$. The question doesn't require this to be calculated but plugging in values gives a value of approximately $2,200\ \text{kms}^{-1}$.

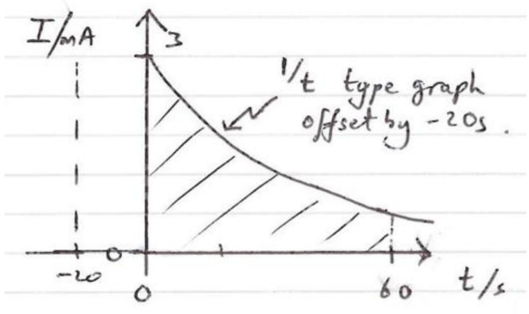
This means the electron takes $\frac{2\pi r}{v}$ seconds to make one complete circuit, or that in one second $\frac{v}{2\pi r} = \frac{e}{2\pi}\sqrt{\frac{k}{mr^3}}$ circuits are completed. Again, it isn't necessary to put in numbers yet but plugging in values gives a whopping 6.6×10^{15} circuits per second.

So to find the current generated by the electron, we simply invoke the definition of average current – charge flowing per unit time – and note that this must have a value of the proton's charge multiplied by the number of times it passes a point in one second which is thus $\frac{e^2}{2\pi}\sqrt{\frac{k}{mr^3}}$.

Plugging in values gives a current of approximately 1.1 mA. This may seem a little high but do remember this question is "for fun" only! Electrons do not behave according to classical mechanics and the Bohr model is not useful for describing this kind of physics. The motion is governed by quantum mechanics as will be explained in several course over the course of the degree.

This problem is a real one and is taken from actual lab data on the year 2 Solid State experiment.

Plotting current vs. time:



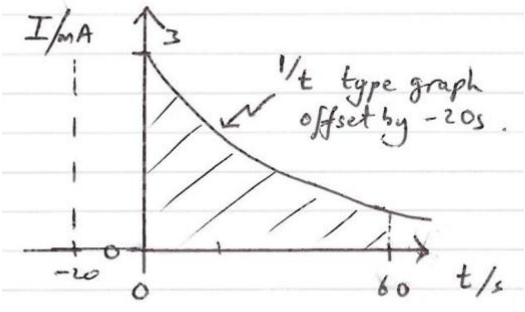
As current is defined by $I = \frac{dq}{dt}$ the charge that has passed is given by the area under the curve.

So $q_{total} = \int_0^{60} \frac{0.06}{20+t} dt = 0.06[\ln(20+t)]_0^{60} = 0.06 \ln\frac{80}{20} = 0.06 \ln 4\ C \approx 83\ \text{mC}$

10. In a second year lab experiment a student measures the current through a light emitting diode with time, and approximates the current flow to an empirical equation of $I = \frac{0.06}{20+t}$ Amperes where t is the time in seconds. Estimate the charge that has passed through the ammeter in 1 minute.

experiment.

Plotting current vs. time:



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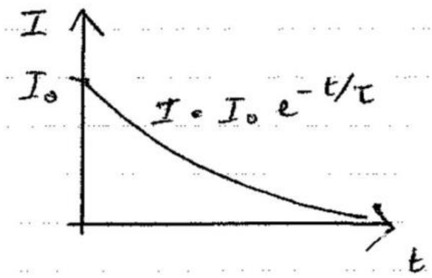
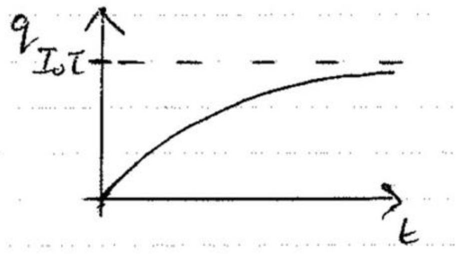
11. A current passes through a resistor with a time dependence given by $I = I_0 e^{-t/\tau}$ where I_0, the initial current, is a constant, and τ is also a constant, known as the time constant. (This is a feature of circuits where a capacitor discharges through a resistor.) Sketch I vs. t and find a value for the charge that has passed through the resistor as a function of time. What value does this tend to when $t \gg \tau$?

Plot of current with time is an exponential "decay" (completely accurate and widely accepted terminology though I'm personally not fond of it) curve:



The rate of flow of charge can be written $\frac{dq}{dt} = I_0 e^{-t/\tau}$ so the total charge that has passed during time T can be derived by $q = I_0 \int_0^t e^{-t'/\tau} dt' = I_0\tau\left[e^{-t'/\tau}\right]_t^0 = I_0\tau\left(1 - e^{-t/\tau}\right)$.

For large times the charge passed tends to $I_0\tau$; the plot of charge passed with time is shown below:



12. What distance from an electron would be required for its own individual field strength to be similar to that of the surrounding field if it were (a) in the Earth's atmosphere on calm day, (b) between the ground and a thundercloud the moment before a lighting strike, (c) between the plates generating the field in Millikan's oil drop experiment?

Using $E = \frac{kQ}{r^2}$ the distances can be found from $r = \sqrt{\frac{kQ}{E}}$. So:

(a) for the Earth's field strength of approximately $100\ \mathrm{NC^{-1}}$ so the electron's field strength matches the magnitude at a distance of about $3.8\ \mu\mathrm{m}$.

(b) before a lightning strike the field is about $3\ \mathrm{MNC^{-1}}$ and the corresponding distance is about $22\ \mathrm{nm}$.

for Millikan's experiment the field is typically of $20\ \mathrm{kNC^{-1}}$ and the corresponding distance is about $270\ \mathrm{nm}$.

13. If a free electron moves under the influence of the Earth's electric field alone how far would it travel (in terms of distance and time) before reaching relativistic speeds?

Using $F = Eq$ with $F = ma$ gives a constant acceleration of $a = \frac{Eq}{m}$.

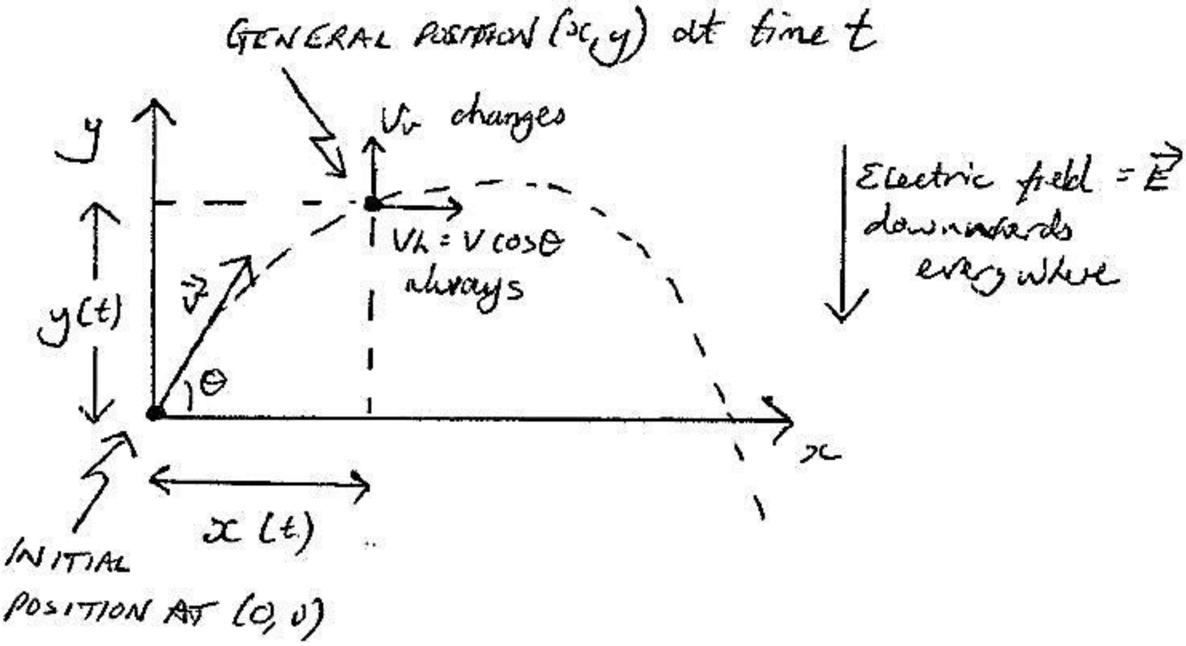
As the acceleration is constant the equations $v^2 = u^2 + 2as$ and $v = u + at$ can be used, so setting $u = 0$ so the electron is stationary to begin with gives $s = \frac{v^2}{2a} = \frac{v^2 m}{2Eq}$ and $t = \frac{v}{a} = \frac{vm}{Eq}$.

Using values of $v = 3 \times 10^8\ \mathrm{ms^{-1}}$, $m = 9.1 \times 10^{-31}\ \mathrm{kg}$, $E = 100\ \mathrm{NC^{-1}}$ and $q = 1.6 \times 10^{-19}\ \mathrm{C}$ gives a distance of $\sim 2.6\ \mathrm{km}$ and a time of $\sim 17\ \mu\mathrm{s}$.

14. Show that a charged particle moving in a uniform electric field always moves in a parabolic path.

Consider a particle released from point considered as the origin in an xy coordinate system. Let the particle have charge $+q$, mass m, initial speed v and the angle of the initial velocity to the x-axis is θ. The electric field in the system is uniform and aligned along $-y$ at all points.

A schematic diagram of the situation is shown below:



The question asks us to show that the path of the particle is parabolic. In mathematical terms, this means that we need to show that the path in the xy- coordinate system is governed by an equation of the form $y = ax^2 + bx + c$ where a, b and c are constants.

This is an identical problem to showing that a mass in a uniform gravitational field moves in a parabola. Many of you will have done this during A levels (in maths mechanics as opposed to physics normally) and may well know exactly how to proceed. If you haven't then it may be difficult to know where to start.

The key is to split the motion of the particle into horizontal and vertical components:

Horizontally (i.e. along x):

There is no component of the field, and thus no acceleration of the particle along this direction. The intial horizontal component of the velocity along this axis is $v\cos\theta$ and this stays constant at all times and positions.

So after any time t the particle will have moved a horizontal distance given by $x(t) = v\cos\theta\, t$.

Vertically (i.e. along y):

The field has value E so the force on the charge is $-Eq$ and is constant. Using $F = ma$ this gives a constant downward acceleration of the charge of $a = -\frac{Eq}{m}$. The negative sign means the

acceleration is in the direction of negative y. This means the particle, which initially has an upward component of velocity, will have this component linearly reduced to zero and then start becoming more and more negative; it will hence follow a path given by the dashed line in the figure above.

As the acceleration is constant to find the vertical distance moved after time t the equation for constant acceleration $s = ut + \frac{1}{2}at^2$ can be used with:

$$s \equiv y(t)$$

$u \equiv v \sin\theta$ **(a constant)**

$$a \equiv -\frac{Eq}{m}$$

Hence $y(t) = v \sin\theta\, t - \frac{Eq}{2m}t^2$

We now have two equations as a function of time - $x(t) = v\cos\theta\, t$ **and** $y(t) = v\sin\theta\, t - \frac{Eq}{2m}t^2$ **so the t can be eliminated to get y as a function of x. Writing** $t = \frac{x}{v\cos\theta}$ **leads to** $y = v\sin\theta \times \frac{x}{v\cos\theta} - \frac{Eq}{2m}\left(\frac{x}{v\cos\theta}\right)^2$ **which simplifies and rearranges to give**

$$y = -\frac{Eq}{2mv^2\cos^2\theta}x^2 + \tan\theta\, x$$

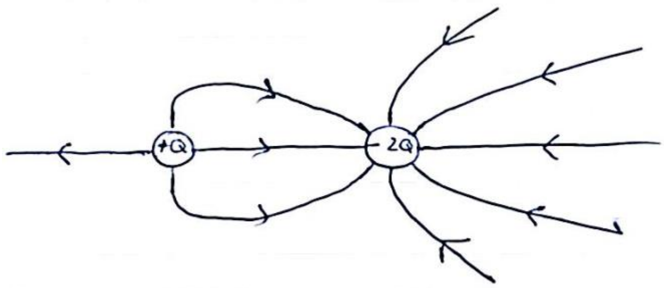
and the proof is complete as this is the equation of a parabola as described above with $a = -\frac{Eq}{2mv^2\cos^2\theta}$**,** $b = \tan\theta$ **and** $c = 0$**.**

15. In lectures we said that test charges do not necessarily move along field lines. Under what circumstances would a test charge move along a field line?

There is a similar argument involved with question 6 here. Can it ever happen is a field line is not straight?

16. Sketch electric field lines for the following arrangements of charges: (a) two separated charges of magnitudes $+Q$ and $-2Q$, (b) two separated charges of magnitudes $+Q$ and $+2Q$, (c) three charges each of magnitude $+Q$ at the corners of an equilateral triangle.

(a) First of all the wrong answer:



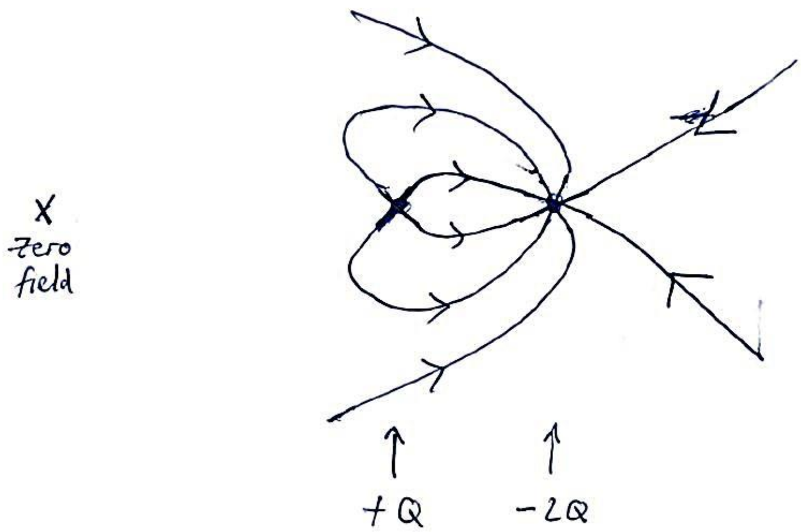
Now why is it wrong? It is what you might quite easily draw and is an easy mistake it is to make. The problem is the line shooting out left from the positive charge: close to the charge this will indeed be left to right but a long way away the field should look more like that of a point charge of value $-Q$ and act right to left. Indeed there must be a point of zero field strength somewhere to the left of the positive charge.

To find the location arrange the coordinate system as having the positive charge at the origin, the negative charge at $x = +D$ and the zero spot to the left of the origin at $x = a$ where a is a negative number to be determined.

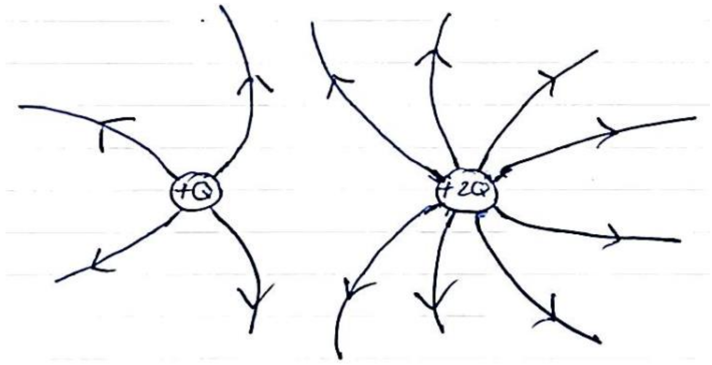
The charges are D apart and the zero field spot is at $x = -a$ with the positive charge at the origin. The field strength at this spot is thus $-k\frac{Q}{a^2}$ due to the positive charge and $+k\frac{2Q}{(D+a)^2}$ due to the negative charge where positive is left to right.

If the field is zero at this point then a is given by $-k\frac{Q}{a^2} + k\frac{2Q}{(D+a)^2} = 0$ which has a solution of $|a| = \frac{1}{\sqrt{2}-1}D \approx 2.4D$ so the zero spot is that much to the left of the positive relative to the separation.

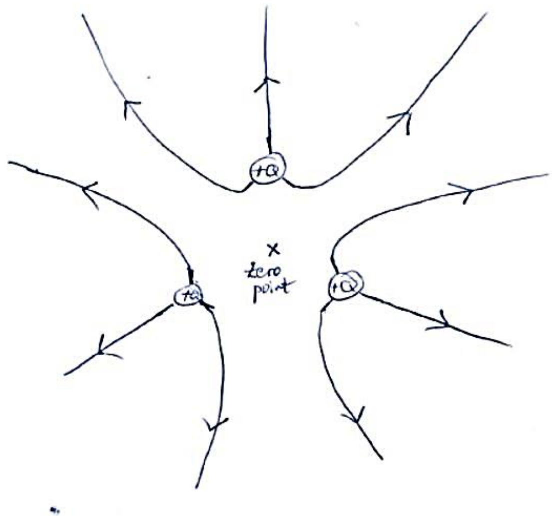
A more representative set of field lines are thus:



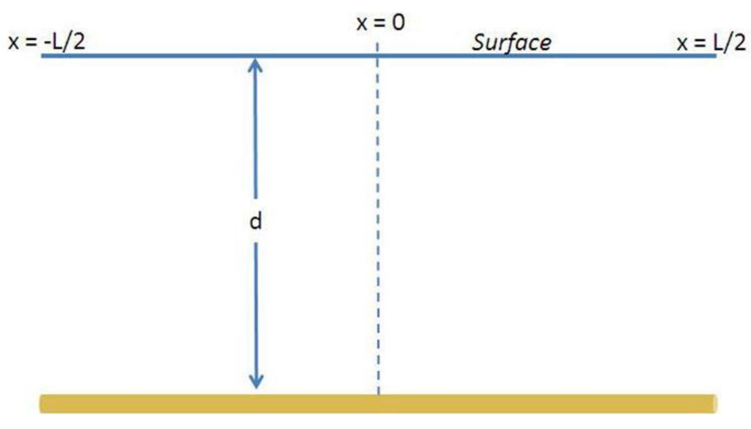
(b)



(c)



17. Calculate the field strength as a function of distance from (a) an infinitely long line of charge of uniform charge density λ Cm^{-1} (b) an infinite sheet of charge of charge density σ Cm^{-2}.



Consider the magnitude of the electric field strength due to the line along $x = 0$ at a distance d away as indicated on the figure.

The magnitude of the field strength due to a small element of length δx will be given by $\delta E = \frac{k\lambda\delta x}{r^2}$ where r is the distance to the element which can be written according to $r^2 = d^2 + x^2$ thus $\delta E = \frac{k\lambda\delta x}{d^2+x^2}$.

This field strength has a direction along the line connecting it with the $x = 0$ point at the surface.

We now must integrate the field strength across the whole length of the line to get the total.

Recognising that the horizontal components of the field strength will cancel either side of the mid-point, the vertical components of the field strength only - given by $\frac{k\lambda d\delta x}{(d^2+x^2)^{3/2}}$ - are required.

So the integral required is $E = \int_{-L/2}^{L/2} \frac{k\lambda d}{(d^2+x^2)^{3/2}}.dx$.

This is not an integral that you would be expected to recall and you would definitely be given it in an exam. Looking it up gives $= k\lambda d\left[\frac{x}{d^2\sqrt{d^2+x^2}}\right]_{-L/2}^{L/2} = \frac{k\lambda}{d}\left[\frac{1}{\sqrt{1+d^2/x^2}}\right]_{-L/2}^{L/2} = \frac{2k\lambda}{d}\left(1+\frac{4d^2}{L^2}\right)^{-1/2}$.

In the limit that $L \to \infty$ the expression becomes $E = \frac{2k\lambda}{d} = \frac{\lambda}{2\pi\epsilon_0 d}$.

This simplifies to the field due to a point mass for $d \gg L$ as one might expect.

(b) For a sheet of charge this is a similar problem but considering the sheet as several aligned strips of width δx and infinite length each contributing a small proportion of vertical field component given by $\delta E = \frac{2k\sigma}{\sqrt{R^2+x^2}} \cdot \frac{R}{\sqrt{R^2+x^2}}$ where R is an arbitrary height above the sheet.

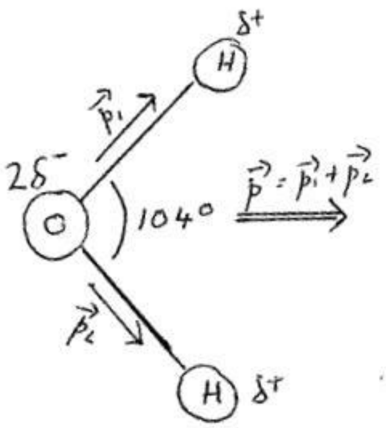
The total field is thus $E = \int_{-\infty}^{\infty} \frac{2k\sigma R.dx}{R^2+x^2} = 2k\sigma R.\frac{1}{R}\left[\tan^{-1}\frac{x}{R}\right]_{-\infty}^{\infty}$ which solves to give $E = 2k\sigma\pi = \frac{\sigma}{2\epsilon_0}$.

18. We discussed the behaviour of an electric dipole in a uniform electric field. How would a dipole behave in a non-uniform electric field?

In general there would be translation plus rotation.

19. In a water molecule, the separation of the effective charge centres is approximately 940 pm. Given the value of the dipole moment of water, and the bond angle in the notes, estimate the magnitude of the effective charges on the three atoms in the molecule.

Herewith a reproduction of figure 2.8 in the lecture notes showing a schematic representation of a water molecule befitting the analysis of this problem:



In this representation the oxygen atom is thought of as two overlapping point charges, each of value $-\delta$, and each hydrogen atom is thought of as a point charge of value $+\delta$.

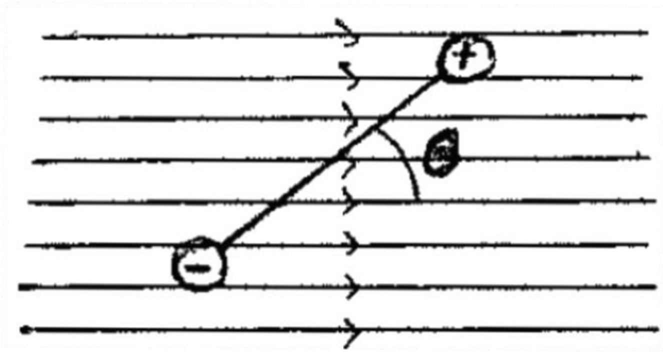
Using the definition of dipole moment, if the hydrogen - oxygen separation is d, the magnitude of each dipole moment is δd and the angle between the two is $104°$.

As the dipole moments add as vectors this gives a "vertical" cancellation of the effects and a horizontal summation of the two components of $\delta d \cos 52°$ each i.e. the total dipole moment is $2\delta d \cos 52°$ horizontally.

As the notes state the total magnitude of this dipole moment is approximately 6.2×10^{-30} Cm this gives a value for the charges of $\delta = \frac{6.2\times10^{-30}\ \text{Cm}}{2d \cos 52°}$. Plugging in the separation stated in the question gives charges of $\sim 1.3 \times 10^{-18}$ C $\cong 8.1e$.

20. Show that a dipole placed at a small angle $\theta \ll \pi\, rad$ does indeed undergo an angular oscillation that is simple harmonic in nature with period $2\pi\sqrt{\frac{I}{pE}}$.

Considering figure 2.9 again:



Using the rotational form of Newton's second law, i.e. $\boldsymbol{\Gamma} = \boldsymbol{I}\frac{d\omega}{dt}$ and the dynamics of the situation showing the magnitude of the torque on the dipole to be $\boldsymbol{qEd}\sin\boldsymbol{\theta}$ and always acting to restore the dipole to equilibrium, it can be written $\boldsymbol{I}\frac{d^2\theta}{dt^2} = -\boldsymbol{qEd}\sin\boldsymbol{\theta}$.

This differential equation has no exact analytical solution but for small angles the approximation $\boldsymbol{I}\frac{d^2\theta}{dt^2} \approx -\boldsymbol{qEd\theta}$ holds. Recall from mechanics (see Young and Freedman or my own textbook ;-) that this means the angular acceleration must be simple harmonic in nature with angular frequency given by $\boldsymbol{\omega}^2 = \frac{qE}{d}$. As $\boldsymbol{\omega} = \frac{2\pi}{T}$ this gives an expression for the period of oscillation of $\boldsymbol{T} = \boldsymbol{2\pi}\sqrt{\frac{I}{qEd}}$ as required.