Vibrations and Waves – Tutorial Questions VnW4

Note: Please also bring the tutorial problem sheets from previous weeks to this week's tutorial.

1. Which of the following waves (written in vector form, where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are unit vectors in the x, y and z directions) are transverse waves and which are longitudinal, which are travelling waves and which are standing waves and in which direction are the travelling waves propagating?

   i) $\boldsymbol{\psi}(z,t) = A\hat{\mathbf{x}}\cos(\omega t - kz)$

   ii) $\boldsymbol{\psi}(z,t) = A\hat{\mathbf{z}}\cos(\omega t)\sin(kz)$

   iii) $\boldsymbol{\psi}(x,t) = A\hat{\mathbf{x}}\cos(\omega t + kx)$

   iv) $\boldsymbol{\psi}(z,t) = A_1\hat{\mathbf{x}}\cos(\omega t - kz) + A_2\hat{\mathbf{z}}\sin(\omega t - kz)$

   v) $\boldsymbol{\psi}(z,t) = A_1\hat{\mathbf{x}}\cos(\omega t - kz) + A_2\hat{\mathbf{z}}\cos(\omega t + kz)$

   vi) $\boldsymbol{\psi}(z,t) = A\hat{\mathbf{x}}\cos(\omega t - kz) + A\hat{\mathbf{x}}\cos(\omega t + kz)$

2. The following waves describe disturbances on stretched strings, where z is distance in metres and t is time in seconds. For each wave, determine the wave amplitude, angular frequency, frequency in Hz, period, wavenumber, wavelength and phase velocity. Ensure you give the units for each quantity. Now calculate the *transverse* velocity and *transverse* acceleration of the string for each wave at $(z,t) = (0.0, 1.0)$.

   i) $\psi(z,t) = 0.2\cos(4\pi t - z/5)\text{m}$

   ii) $\psi(z,t) = 80\sin[\pi(t - 10z)]\text{mm}$

3. Two stretched strings are joined together at $x = 0$. String 1 is has 4 times the mass per unit length as string 2. The tensions in both strings are the same.

   i) The wave speed for string 1 is measured to be $5\,\text{m}\,\text{s}^{-1}$. What is the wave speed for string 2?

   A travelling transverse harmonic wave on string 1 is incident on the boundary between the strings. In complex form the wave is $\psi_\text{i} = A_i\,\text{e}^{\text{i}(\omega t - k_1 x)}$.

   ii) Write down similar expressions for the transmitted and reflected waves.

   iii) What are the wavenumbers of the reflected and the transmitted waves?

   iv) Both the displacement and its gradient must be continuous at the join (at $x = 0$). Use these boundary conditions to write down two equations relating the amplitudes of the three waves.

   v) Hence derive expressions for the amplitude transmission and reflection coefficients in terms of the wavenumbers of the waves. Then re-express these coefficients in terms of the wave speeds $v_1 = \omega/k_1$ and $v_2 = \omega/k_2$.

   vi) If the amplitude of the incident wave is 0.1m, calculate the amplitudes of the transmitted and reflected waves. Is the reflected wave inverted? What fraction of the power of the incident wave is reflected?