January 2019

Mark Scheme

Mock Paper (set1)

Pearson Edexcel GCE A Level Mathematics

Pure Mathematics 2 (9MA0/02)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100

2. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **o.e.** – or equivalent (and appropriate)
   - **d** or **dep** – dependent
   - **indep** – independent
   - **dp** decimal places
   - **sf** significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

   A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Question	Scheme	Marks	AOs
1	$\cos\theta - \sin\left(\frac{1}{2}\theta\right) + 2\tan\theta = \frac{11}{10}$		
(a)	$1 - \frac{1}{2}\theta^2 - \frac{1}{2}\theta + 2\theta \approx \frac{11}{10}$	M1	1.2
		A1	1.1b
	$\Rightarrow \frac{1}{2}\theta^2 - \frac{3}{2}\theta + \frac{1}{10} \approx 0 \Rightarrow 5\theta^2 - 15\theta + 1 \approx 0$ *	A1*	2.1
		(3)	
(b)	$\theta = 0.068$ is valid because θ is small $\theta = 2.932$ is not valid because θ is large	B1	2.3
		(1)	
			(4 marks)

Question 1 Notes:

(a)	
M1:	At least two of either $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\sin\left(\frac{1}{2}\theta\right) \approx \frac{1}{2}\theta$ or $\tan\theta \approx \theta$ substituted into the given equation
A1:	Substitutes $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\sin\left(\frac{1}{2}\theta\right) \approx \frac{1}{2}\theta$ and $\tan\theta \approx \theta$ into the given equation to obtain a correct (un-simplified) approximation or equation. E.g. $1 - \frac{1}{2}\theta^2 - \frac{1}{2}\theta + 2\theta \approx \frac{11}{10}$ or $= \frac{11}{10}$
A1*:	Obtains $5\theta^2 - 15\theta + 1 \approx 0$ (condone $5\theta^2 - 15\theta + 1 = 0$) with no errors seen in their working
(b)	
B1:	States $\theta = 0.068$ is valid because θ is small; and $\theta = 2.932$ is not valid because θ is large
(b)	
Alt 1	$\text{LHS} = \cos\theta - \sin\left(\frac{1}{2}\theta\right) + 2\tan\theta$
B1:	States $\theta = 0.068$ is valid and $\theta = 2.932$ is not valid based on testing these two values in the original equation **Note:** $\theta = 0.068 \Rightarrow \text{LHS} = 1.0999...$ & $\theta = 2.932 \Rightarrow \text{LHS} = -2.3980...$
	Note: $\theta = 0.068218... \Rightarrow \text{LHS} = 1.1002...$ & $\theta = 2.931782... \Rightarrow \text{LHS} = -2.3984...$

Question	Scheme	Marks	AOs
2	$x = 6t+1,\ \ y = 5 - \dfrac{4}{3t},\ \ t \neq 0$		
	$\left\{t = \dfrac{x-1}{6} \Rightarrow\right\}\ \ y = 5 - \dfrac{4}{3\left(\dfrac{x-1}{6}\right)}$	M1	1.1b
		A1	1.1b
	$\Rightarrow\ y = 5 - \dfrac{4}{\left(\dfrac{x-1}{2}\right)}\ \Rightarrow\ y = 5 - \dfrac{8}{x-1}\ \Rightarrow\ y = \dfrac{5(x-1)-8}{x-1}$ $\Rightarrow\ y = \dfrac{5x-13}{x-1},\ \ x \neq 1 \qquad \{a = 5, b = -13, k = 1\}$	A1	2.1
		(3)	
Alt 1	$\left\{t = \dfrac{4}{3(5-y)} \Rightarrow\right\}\ \ x = 6\left(\dfrac{4}{3(5-y)}\right)+1$	M1	1.1b
		A1	1.1b
	$\Rightarrow\ x = \dfrac{8}{(5-y)}+1\ \Rightarrow\ (x-1)(5-y) = 8\ \Rightarrow\ 5x - xy - 5 + y = 8$ $\Rightarrow\ 5x - 5 - 8 = xy - y\ \Rightarrow\ 5x - 13 = y(x-1)\ \Rightarrow\ y = \dfrac{5x-13}{x-1},\ \ x \neq 1$	A1	2.1
		(3)	
		(3 marks)	

Question 2 Notes:

M1:	An attempt to eliminate t
A1:	Achieves a correct equation in x and y only which can be un-simplified or simplified
A1:	Uses correct algebra to show $y = \dfrac{5x-13}{x-1},\ x \neq 1$

Question	Scheme	Marks	AOs
3 (a)	$\left\{y = x^2 + kx + 14 - 8(x-5)^{-1} \Rightarrow\right\} \dfrac{dy}{dx} = 2x + k + 8(x-5)^{-2}$	M1 A1	1.1b 1.1b
	$\left\{\text{At } x = 3, \dfrac{dy}{dx} = 0 \Rightarrow\right\} \dfrac{dy}{dx} = 2(3) + k + \dfrac{8}{(3-5)^2} = 0$	dM1	1.1b
	$\Rightarrow 6 + k + \dfrac{8}{4} = 0 \Rightarrow 6 + k + 2 = 0 \Rightarrow k = -8$ *	A1*	2.1
		(4)	
(b)	$\dfrac{d^2y}{dx^2} = 2 - 16(x-5)^{-2} = 2 - \dfrac{16}{(x-5)^3}$		
	When $x = 3$, $\dfrac{d^2y}{dx^2} = 2 - \dfrac{16}{(3-5)^3}$	M1	1.1b
	$\dfrac{d^2y}{dx^2} = 4 > 0 \Rightarrow$ {local} minimum {stationary point at P}	A1	2.1
		(2)	
(b) Alt 1	E.g. $x = 2.9$, $\dfrac{dy}{dx} = 2(2.9) - 8 + 8(2.9-5)^{-2} = -0.38594... < 0$ $x = 3.1$, $\dfrac{dy}{dx} = 2(3.1) - 8 + 8(3.1-5)^{-2} = 0.41606... > 0$ $\Rightarrow$ {local} minimum {stationary point at P}	M1 A1	1.1b 2.1
		(2)	
(c)	**<u>Criteria 1</u>** (Accept any one of the two following points) • At $x = 7$, $\dfrac{d^2y}{dx^2} = 2 - \dfrac{16}{(7-5)^3} = 0$ • $\dfrac{d^2y}{dx^2} = 2 - \dfrac{16}{(x-5)^3} = 0 \Rightarrow (x-5)^3 = 8 \Rightarrow x = 2 + 5 \Rightarrow x = 7$		
	<u>Criteria 2</u> (Accept any one of the two following points) • At $x = 6.9$, $\dfrac{d^2y}{dx^2} = -0.33... < 0$ and at $x = 7.1$, $\dfrac{d^2y}{dx^2} = 0.27... > 0$ • $\dfrac{d^3y}{dx^3} = \dfrac{48}{(x-5)^3}$ and at $x = 7$, $\dfrac{d^3y}{dx^3} \left\{= \dfrac{48}{(7-5)^3} = 6\right\} \neq 0$		
	At least one of Criteria 1 or Criteria 2	M1	2.1
	Both Criteria 1 and Criteria 2 (with correct calculations) and concludes the curve has a {non-stationary} point of inflection at $x = 7$	A1	2.4
		(2)	
(d)	Sign change method is not valid because either • the curve is not defined at $x = 5$ • the curve is not continuous over the interval (4.5, 5.5)	B1	2.4
		(1)	
		(9 marks)	

Question 3 Notes:	
(a)	
M1:	At least one of either $x^2 \to \pm Ax$ or $kx \to k$ or $-\dfrac{8}{(x-5)} \to \pm B(x-5)^{-2}$; $A, B \neq 0$
A1:	$\dfrac{dy}{dx} = 2x + k + 8(x-5)^{-2}$, which may be un-simplified or simplified
dM1	**dependent on the previous M mark**
	Complete strategy of substituting $x=3$ into their equation for $\dfrac{dy}{dx}$ and setting $\dfrac{dy}{dx}$ equal to 0
A1*:	Correctly shows $k = -8$ with no errors in their working
(b)	
M1:	Evidence of substituting $x = 3$ into an expression for $\dfrac{d^2y}{dx^2}$ which is in the form $\pm\alpha \pm \beta(x-5)^{-3}$; $\alpha, \beta \neq 0$
A1:	For a correct calculation, a valid reason and a correct conclusion
(b) Alt 1	
M1:	Uses $\dfrac{dy}{dx}$ which is in the form $\pm\alpha x - 8 \pm \beta(x-5)^{-2}$; $\alpha, \beta \neq 0$ and finds values for $\dfrac{dy}{dx}$ either side of $x=3$
A1:	For correct calculations, a valid reason and a correct conclusion
(c)	
M1:	See scheme
A1:	See scheme
(d)	
B1:	States that the sign change method is not valid together with an acceptable reason as indicated in the scheme

Question	Scheme	Marks	AOs
4	$\mathrm{f}(x) = x^3 - 6x^2 + 7x + 2, \quad x \in \mathbb{R}$		
(a)	$\mathrm{f}(x) = (x-2)(x^2 - 4x - 1)$	M1	2.2a
		A1	1.1b
		(2)	
(b) **(i), (ii)**	{Note: $(x-2) = 0 \Rightarrow x_Q = 2$ is known and at P, R, $(x^2 - 4x - 1) = 0$ }		
	$(x-2)^2 - 4 - 1 = 0$ **or** $x = \dfrac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$	M1	1.1b
	$\Rightarrow x_P = 2 - \sqrt{5}$ and $x_R = 2 + \sqrt{5}$	A1	1.1b
		(2)	
(c)	$\sin^3\theta - 6\sin^2\theta + 7\sin\theta + 2 = 0, \quad -\pi \leqslant \theta \leqslant 12\pi,$		
	Deduces that there are 14 real solutions for $-\pi \leqslant \theta \leqslant 12\pi$	B1	2.2a
	Correct justification. E.g. Both • $\sin\theta = 2$ and $\sin\theta = 2 + \sqrt{5} = 4.236...$ have no real solutions and either • $\sin\theta = 2 - \sqrt{5} = -0.236...$ has 2 real solutions for each interval of 2π. So there are 12 real solutions in the interval $[0, 12\pi]$ and 2 real solutions in the interval $[-\pi, 0]$ • $\sin\theta = 2 - \sqrt{5} = -0.236...$ has 2 real solutions for each interval of 2π. So there are 12 real solutions in the interval $[-\pi, 11\pi]$ and 2 real solutions in the interval $[11\pi, 12\pi]$ • $\sin\theta = 2 - \sqrt{5} = -0.236...$ has 2 real solutions for each interval of 2π. So there are 14 real solutions in the interval $[-2\pi, 12\pi]$ and no real solutions in the interval $[-2\pi, -\pi]$ • $\sin\theta = 2 - \sqrt{5} = -0.236...$ has two real solutions in each of $[-\pi, 0], [\pi, 2\pi], [3\pi, 4\pi], [5\pi, 6\pi], [7\pi, 8\pi], [9\pi, 10\pi]$ and $[11\pi, 12\pi]$	B1ft	2.4
		(2)	
			(6 marks)

Question 4 Notes:	
(a)	
M1:	Deduces $(x-2)$ is a factor of $\mathrm{f}(x)$ and attempts to find a quadratic factor of $\mathrm{f}(x)$ by either equating coefficients or by algebraic long division
A1:	$(x-2)(x^2-4x-1)$
(b)	
(i), (ii)	
M1:	Correct method (i.e. completing the square or applying the quadratic formula) to solve a 3TQ.
	Note: M1 can be given here for at least one of either $2-\sqrt{5}$ or $2+\sqrt{5}$ written down in part (b).
A1:	Finds and identifies the correct exact x coordinate of P and the correct exact x coordinate of R
(c)	
B1:	Correct deduction of 14 (real solutions)
B1:	See scheme

Question	Scheme	Marks	AOs
5	Let x_P be the positive solution and x_N be the negative solution of $\mathrm{f}(x) = 0$ **Note:** $y = \mathrm{f}(x)$ is symmetrical about the line $x = \frac{5}{3}$		
(a)	$\mathrm{f}(x) = 7 - \lvert 3x - 5 \rvert = 0 \Rightarrow \lvert 3x - 5 \rvert = 7$ at least one of either…	M1	2.1
	• $3x - 5 = 7 \Rightarrow x_P = 4$ • $3x - 5 = -7 \Rightarrow x_N = -\frac{2}{3}$	A1	1.1b
	Area $(R) = \frac{1}{2}\left(4 - -\frac{2}{3}\right)(7)$ **or** $2\left(\frac{1}{2}\left(4 - \frac{5}{3}\right)(7)\right)$ **or** $2\left(\frac{1}{2}\left(\frac{5}{3} - -\frac{2}{3}\right)(7)\right)$	M1	3.1a
	$= \frac{49}{3}$ or $16\frac{1}{3}$ (units)^2	A1	1.1b
		(4)	
(b)	$7 - \lvert 3x - 5 \rvert = k$, k is a constant, has two distinct real solutions		
	Deduces that $k < 7$	B1	2.2a
		(1)	
	(5 marks)		

Question 5 Notes:

(a)	
M1:	Complete process of using the modulus function $y = \mathrm{f}(x)$ to find at least one of the x coordinates where $y = \mathrm{f}(x)$ cuts through the x-axis.
A1:	At least one of either $x = 4$ or $x = -\frac{2}{3}$
M1:	Finds at least one value where $y = \mathrm{f}(x)$ cuts through the x-axis together with a complete process to find the Area (R); e.g. • $\frac{1}{2}(\text{their } x_P - \text{their } x_N)(7)$ • $2\left(\frac{1}{2}\left(\text{their } x_P - \frac{5}{3}\right)(7)\right)$, where their $x_P > \frac{5}{3}$ • $2\left(\frac{1}{2}\left(\frac{5}{3} - \text{their } x_N\right)(7)\right)$, where their $x_N < 0$
A1:	See scheme
(b)	
B1:	Uses Figure 3 and the equation $y = \mathrm{f}(x)$ to deduce the correct answer. E.g. • $k < 7$ • $\{k : k < 7\}$

Question	Scheme	Marks	AOs
6	$\left\{(2+kx)^{-4} = 2^{-4}\left(1+\frac{kx}{2}\right)^{-4} = \frac{1}{16}\left(1+(-4)\left(\frac{kx}{2}\right)+\frac{(-4)(-5)}{2!}\left(\frac{kx}{2}\right)^2+...\right)\right\}$		
(a)	For the x^2 term: $\left(\frac{1}{16}\right)\frac{(-4)(-5)}{2!}\left(\frac{k}{2}\right)^2$ $\left\{=\frac{5}{32}k^2\right\}$	M1	1.1b
		A1	1.1b
	$\frac{1}{16}\frac{(-4)(-5)}{2!}\left(\frac{k}{2}\right)^2 = \frac{125}{32} \Rightarrow \frac{5}{32}k^2 = \frac{125}{32} \Rightarrow k^2 = 25 \Rightarrow k = ... \Rightarrow A = ...$	dM1	3.1a
	$\left\{A = -\frac{4}{32}k \Rightarrow\right\}$ $A = -\frac{4}{32}(5)$	M1	2.2a
	$A = -\frac{5}{8}$ or -0.625	A1	1.1b
		(5)	
(b)	$\mathrm{f}(x)$ is valid when $\left\lvert\frac{kx}{2}\right\rvert < 1 \Rightarrow \left\lvert\frac{5x}{2}\right\rvert < 1 \Rightarrow \lvert x\rvert < \frac{2}{5}$		
	E.g. • As $x = \frac{1}{10}$ lies in the interval $\lvert x\rvert < \frac{2}{5}$, the binomial expansion is valid • As $\left\lvert\left(\frac{5}{2}\right)\left(\frac{1}{10}\right)\right\rvert = \frac{1}{4} < 1$, the binomial expansion is valid	B1ft	2.3
		(1)	
		(6 marks)	

Question 6 Notes:

(a)	
M1:	For either $\frac{(-4)(-5)}{2!}$ or $\left(\frac{k}{2}\right)^2$ or $\left(\frac{kx}{2}\right)^2$ or $\frac{(-4)(-5)}{2}$ or 10 as part of their x^2 coefficient
A1:	For $\left(\frac{1}{16}\right)\frac{(-4)(-5)}{2!}\left(\frac{k}{2}\right)^2$ or $\frac{5}{32}k^2$ or equivalent as part of their x^2 coefficient
dM1:	**dependent on the previous M mark** A complete strategy to find a value for *k* and use their *k* to find a value for *A*
M1:	Deduces and applies $A = -\frac{4}{32}(\text{their } k)$ or $A = -\frac{1}{8}(\text{their } k)$
A1:	$A = -\frac{5}{8}$ or -0.625
(b)	
B1ft:	See scheme **Note:** Allow follow through for applying either $\lvert x\rvert < \frac{2}{\text{their } k}$ or $\left\lvert\left(\frac{\text{their } k}{2}\right)\left(\frac{1}{10}\right)\right\rvert$

Question	Scheme	Marks	AOs
7	$f(x) = \frac{2}{x} - e^x + 2x^2,\ x \in \mathbb{R}, x \neq 0$		
(a)	Evaluates both $f(-1.5)$ and $f(-1)$	M1	1.1b
	$f(-1.5) = 2.943536507...$ and $f(-1) = -0.3678794412...$ Sign change and as $f(x)$ is continuous α lies between -1.5 and -1	A1	2.4
		(2)	
(b)	(i) $\{x_3 =\}\ -1.0428$	B1	1.1b
	(ii) $\{\alpha =\}\ -1.06$ (2 dp)	B1	2.2a
		(2)	
(c)	$\{x_2 =\}\ 3 - \left(\frac{-1.4189}{-8.3078}\right)$	M1	1.1b
	{= 2.829208695...} = 2.83 (2 dp)	A1	1.1b
		(2)	
(d)	• Draws a tangent to the curve at $x = 1.5$ and identifies (possibly by writing x_2) where the tangent cuts the x-axis	M1	1.1b
	and concludes either • second approximation is not good because it is not in the interval [1.5, 3] • x_2 (which is indicated on Figure 3) is nowhere near the root β	A1	2.4
		(2)	
			(8 marks)

Question 7 Notes:	
(a)	
M1:	Evaluates both $\mathrm{f}(-1.5)$ and $\mathrm{f}(-1)$
A1:	$\mathrm{f}(-1.5) =$ awrt 3 or $\mathrm{f}(-1.5) = 2$ (truncated) **and** $\mathrm{f}(-1) =$ awrt -0.4 or $\mathrm{f}(-1) = -0.3$ (truncated) **and** a correct conclusion
(b)(i)	
B1:	See scheme
(b)(ii)	
B1:	Deduces (e.g. using further iterations) that $\alpha = -1.06$ accurate to 2 dp
(c)	
M1:	Attempts $x_2 = x_1 - \frac{\mathrm{f}(x_1)}{\mathrm{f}'(x_1)}$; $x_1 = 3$; which can be evidenced by $3 - \left(\frac{-1.4189}{-8.3078}\right)$
A1:	2.83
(d)	
M1:	See scheme
A1:	See scheme

Question	Scheme	Marks	AOs
8 (a)	Deduces that the radius of the circle is 6	B1	2.2a
	$(x-9)^2+(y+6)^2=36$	M1	1.1b
		A1	1.1b
		(3)	
(b)	Let d be the distance from $(9,-6)$ to l		
	$d^2+4^2=6^2 \Rightarrow d^2=...$	M1	3.1a
	$d=\sqrt{20}$ or $2\sqrt{5}$	A1	1.1b
	$\{l:\}\quad y=-6+2\sqrt{5},\ y=-6-2\sqrt{5}$	dM1	2.2a
		A1	1.1b
		(4)	
(b) Alt 1	**Either** $\left\{x=9+\frac{8}{2}=13\Rightarrow\right\}\ (13-9)^2+(y+6)^2=36\Rightarrow y=...$ **or** $\left\{x=9-\frac{8}{2}=5\Rightarrow\right\}\ (5-9)^2+(y+6)^2=36\Rightarrow y=...$	M1	3.1a
	$\{l:\}\quad y=-6+2\sqrt{5}$	A1	1.1b
	$\{l:\}\quad y=-6-2\sqrt{5}$	dM1	2.2a
		A1	1.1b
		(4)	
			(7 marks)

Question 8 Notes:	
(a)	
B1:	Deduces that the radius of the circle is 6. This can be achieved by either • Stating that $r = 6$ or radius = 6 or $r^2 = 36$ • Writing $(x \pm \alpha)^2 + (y \pm \beta)^2 = 36$ or 6^2; $\alpha, \beta \neq 0$
M1	$(x \pm 9)^2 + (y \pm 6)^2 = k$; $k > 0$
A1:	$(x-9)^2 + (y+6)^2 = 36$ or $(x-9)^2 + (y+6)^2 = 6^2$ o.e.
(b)	
M1:	Uses the circle property "the perpendicular from the centre to a chord bisects the chord" in a complete strategy of writing an equation of the form $d^2 + \left(\frac{8}{2}\right)^2 = (\text{their } r)^2$ and progresses as far as $d^2 = \ldots$
A1:	$d = \sqrt{20}$ or $2\sqrt{5}$
dM1:	**depends on the previous M mark** Deduces the horizontal line l is d units from the line $y = -6$ and so writes both • $y = -6 + (\text{their } d)$ **and** $y = -6 - (\text{their } d)$
A1:	For either: • $y = -6 + 2\sqrt{5}$ and $y = -6 - 2\sqrt{5}$ • $y = -6 + \sqrt{20}$ and $y = -6 - \sqrt{20}$
(b)	
Alt 1	
M1:	Uses the circle property "the perpendicular from the centre to a chord bisects the chord" in a complete strategy of substituting either $x = 13$ or $x = 5$ into their circle equation and progresses as far as $y = \ldots$
A1:	For $y = -6 + 2\sqrt{5}$ or $y = -6 + \sqrt{20}$
dM1:	**depends on the previous M mark** Finds y in the form $y = -6 + (\text{their } d)$, deduces the other horizontal line l is d units below the line $y = -6$ and so writes $y = -6 - (\text{their } d)$
A1:	For either: • $y = -6 + 2\sqrt{5}$ and $y = -6 - 2\sqrt{5}$ • $y = -6 + \sqrt{20}$ and $y = -6 - \sqrt{20}$

Question	Scheme		Marks	AOs
9 (a)	$\{y = ab^t \Rightarrow\}$ $\log_{10} y = \log_{10}(ab^t) \Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^t$		M1	1.1b
	$\Rightarrow \log_{10} y = t\log_{10} b + \log_{10} a$, {where $c = \log_{10} a$}		A1	2.1
			(2)	
(b)	$c = \log_{10} a = 2.23$,	$m = \log_{10} b = -0.076$		
	$\Rightarrow a = 10^{2.23}$ $\{= 169.8243652...\}$	$\Rightarrow b = 10^{-0.076}$ $\{= 0.8394599865...\}$	M1	1.1b
	$a = 170$ (2 sf) and $b = 0.84$ (2 sf)		A1	1.1b
			(2)	
(c)	$y = (170)(0.84)^t$			
	(i) $\{a = "170" \Rightarrow\}$ e.g. • "170" milligrams of antibiotic were initially given to the patient • the initial dose of the antibiotic is estimated as "170" milligrams		B1ft	3.4
	(ii) $\{b = "0.84" \Rightarrow\}$ e.g. after the antibiotic is first given the amount of antibiotic in the patient's bloodstream reduces by approximately "16%" per hour		B1ft	3.4
			(2)	
(d)	$30 = (170)(0.84)^t \Rightarrow \frac{30}{170} = (0.84)^t \Rightarrow \ln\left(\frac{30}{170}\right) = \ln(0.84)^t$ $\Rightarrow \ln\left(\frac{30}{170}\right) = t\ln(0.84) \Rightarrow t = \frac{\ln\left(\frac{30}{170}\right)}{\ln(0.84)}$		M1	3.4
	$\{t = 9.948766031... \Rightarrow\}$ $t = 9.9$(hours) (1 dp)		A1	1.1b
			(2)	
(d) Alt 1	$\log_{10} 30 = -0.076t + 2.23 \Rightarrow t = \frac{2.23 - \log_{10} 30}{0.076}$		M1	3.4
	$\{t = 9.90629928... \Rightarrow\}$ $t = 9.9$ (hours) (1 dp)		A1	1.1b
			(2)	
(e)	e.g. As $t = 9.9$ is outside of the experimental data $0 \leqslant t \leqslant 5$, we do not have enough evidence to deduce that the model $y = (170)(0.84)^t$ is still valid. So, the estimate in part (d) should be treated with caution.		B1	3.5b
			(1)	
				(9 marks)

Question 9 Notes:	
(a)	
M1:	Starting from $y = ab^t$, takes logs of both sides and uses the addition law of logarithms to progress as far as $\log_{10} y = \log_{10} a + \log_{10} b^t$
A1:	Starting from $y = ab^t$, correctly shows that $\log_{10} y = t\log_{10} b + \log_{10} a$ with no errors seen
Note:	M1 (special case) can be given in part (a) for stating $c = \log_{10} a$
(b)	
M1:	For either $a = 10^{2.23}$ or $b = 10^{-0.076}$
A1:	$a = 170$ and $b = 0.84$
(c)(i)	
B1ft:	Correct practical interpretation of their *a*, where their $a > 0$
(c)(ii)	
B1ft:	Correct practical interpretation of their *b*, where their *b*: $0 < b < 1$
(d)	
M1:	Substitutes $y = 2.5$ into the model $y = (\text{their } a)(\text{their } b)^t$ and rearranges their equation to give $t = ...$
A1:	9.9 (hours) (1 dp)
(d) Alt 1	
M1:	Substitutes $y = 30$, $m = -0.076$ and $c = 2.23$ into the model $\log_{10} y = mt + c$ and rearranges their equation to give $t = ...$
A1:	9.9 (hours) (1 dp)
(e)	
B1:	E.g. Estimate should be treated with caution because $t = 9.9$ is outside the range of times, i.e. $0 \leqslant t \leqslant 5$, for which the model $y = (170)(0.84)^t$ is valid

Question	Scheme	Marks	AOs
10 (a)	$\frac{dA}{dt} \propto \sqrt{A} \Rightarrow \frac{dA}{dt} = k\sqrt{A}$ or $\frac{dA}{dt} = kA^{\frac{1}{2}}$	B1	3.1b
	$\int \frac{1}{A^{\frac{1}{2}}}\, dA = \int k\, dt$	M1	1.1b
	$\left\{ \int A^{-\frac{1}{2}}\, dA = \int k\, dt \Rightarrow \right\} \frac{A^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\ \{+c\}$ or $2A^{\frac{1}{2}} = kt\ \{+c\}$	A1	1.1b
	$\{t = 0,\ A = 9 \Rightarrow\}\ 2\sqrt{9} = k(0) + c$	M1	3.4
	$\Rightarrow c = 6 \Rightarrow 2A^{\frac{1}{2}} = kt + 6$ $\{t = 6,\ A = 56.25 \Rightarrow\}\ 2\sqrt{56.25} = k(6) + 6$	dM1	1.1b
	$\Rightarrow 15 = 6k + 6 \Rightarrow k = \frac{9}{6} \Rightarrow k = \frac{3}{2}$ $\Rightarrow 2A^{\frac{1}{2}} = \frac{3}{2}t + 6 \Rightarrow A^{\frac{1}{2}} = \frac{3}{4}t + 3 \Rightarrow A = \left(\frac{3}{4}t + 3\right)^2$ *	A1*	2.1
		(6)	
(b) **(i), (ii)**	**Either** • $t = 12,\ A = \left(\frac{3}{4}(12) + 3\right)^2 = 144\ \{\approx 143.78\}$ $t = 18,\ A = 272.25\ \{\approx\ 271.19\}$ $t = 24,\ A = 441\ \{> 334.81\}$ $\{t = 30,\ A = 650.25\ \{> 337.33\}\}$ **or** • $A = 143.78 \Rightarrow 143.78 = \left(\frac{3}{4}t + 3\right)^2 \Rightarrow t = 11.98777...\ \{\approx 12\}$ $A = 271.19 \Rightarrow t = 17.95713...\ \{\approx 18\}$ $A = 334.81 \Rightarrow t = 20.39709...\ \{< 24\}$ $\{A = 337.33 \Rightarrow t = 20.48873...\ \{< 30\}\}$	M1	3.4
	Biologist's model works well for $t = 12$ and $t = 18$ but appears to give an overestimate for A (or does not work well) when $t = 24$ and $t = 30$	A1	3.5a
	E.g. • The biologist's model appears to break down for large values of t. This may be because the biologist's model predicts values for A which are greater than the total surface area of the piece of bread used in the experiment. • The biologist's results indicate an upper limit for A, but the biologist's model does not give an upper limit for A.	B1	3.2a
		(3)	
			(9 marks)

Question 10 Notes:	
(a)	
B1:	Translates the biologist's model regarding proportionality into a differential equation, which involves a constant of proportionality. E.g. $\frac{\mathrm{d}A}{\mathrm{d}t} \propto \sqrt{A} \Rightarrow \frac{\mathrm{d}A}{\mathrm{d}t} = k\sqrt{A}$
M1:	Correct method of separating the variables A and t in their differential equation
A1:	$\frac{A^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt$ or $2A^{\frac{1}{2}} = kt,$ with or without a constant of integration
M1:	Some evidence of applying the measurements $t = 0,\ A = 9$ or $A = 9.00$ to a changed equation containing a constant of integration. e.g. c
dM1:	**dependent on the previous M mark** Applies $t = 6,\ A = 56.25$ and their value of c to their changed equation which contains their constant of proportionality
A1*:	Shows that $A = \left(\frac{3}{4}t + 3\right)^2$, with no errors in their working
(b)	
(i), (ii)	
M1:	Uses the model found in part (a) to find • **either** values for A when $t = 12$, $t = 18$ and $t = 24$ • **or** values for t when $A = 143.78,\ A = 271.19$ and $A = 334.81$
A1:	• **Either** $t = 12 \Rightarrow A = 144,\ t = 18 \Rightarrow A =$ awrt 272 **and** $t = 24 \Rightarrow A = 441$ • **or** $A = 143.78 \Rightarrow t =$ awrt 12, $A = 271.19 \Rightarrow t =$ awrt 18 **and** $A = 334.81 \Rightarrow t =$ awrt 20 **and** evaluates (see scheme) the outcomes of the model
B1:	See scheme

Question	Scheme	Marks	AOs
11	$\mathrm{f}(x) = \dfrac{\sin 2x}{-3+\cos 2x}, \quad 0 \leqslant x \leqslant \pi$		
(a)	$\left\{ u = \sin 2x \quad v = -3+\cos 2x \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = 2\cos 2x \quad \dfrac{\mathrm{d}v}{\mathrm{d}x} = -2\sin 2x \right\}$		
	$\mathrm{f}'(x) = \dfrac{(-3+\cos 2x)(2\cos 2x) - (\sin 2x)(-2\sin 2x)}{(-3+\cos 2x)^2}$	M1	2.1
		A1	1.1b
	$\mathrm{f}'(x) = 0 \Rightarrow (-3+\cos 2x)(2\cos 2x) - (\sin 2x)(-2\sin 2x) = 0$	M1	1.1b
	$-6\cos 2x + 2\cos^2 2x + 2\sin^2 2x = 0 \Rightarrow -6\cos 2x + 2 = 0$ $\Rightarrow \cos 2x = \dfrac{1}{3}$ *	A1*	2.1
		(4)	
(b)	Maximum turning point for (i) $y = \mathrm{f}(3x) + 5$, (ii) $y = -\mathrm{f}\left(\dfrac{1}{4}x\right)$		
	$\{ \cos 2x = \dfrac{1}{3} \Rightarrow$ Principal Value = 1.2309593... $\}$		
	(i) For either • $\left\{\cos 6x = \dfrac{1}{3} \text{ (2nd sol}^{\text{n}}) \Rightarrow \right\}$ $x = \dfrac{2\pi - 1.2309593...}{6}$ • $\left\{\cos 2x = \dfrac{1}{3} \text{ (2nd sol}^{\text{n}}) \Rightarrow \right\}$ $3x = \dfrac{2\pi - 1.2309593...}{2} \Rightarrow x = ...$	M1	3.1a
	$\Rightarrow x = 0.842037... = 0.84$ (2 dp)	A1	1.1b
	(ii) For either • $\left\{\cos\left(\dfrac{1}{2}x\right) = \dfrac{1}{3} \text{ (1st sol}^{\text{n}}) \Rightarrow \right\}$ $x = 2(1.2309593...)$ • $\left\{\cos 2x = \dfrac{1}{3} \text{ (2nd sol}^{\text{n}}) \Rightarrow \right\}$ $\dfrac{1}{4}x = \dfrac{1.2309593...}{2} \Rightarrow x = ...$	M1	3.1a
	$\Rightarrow x = 2.461918... = 2.46$ (2 dp)	A1	1.1b
		(4)	
		(8 marks)	

Question 11 Notes:	
(a)	
M1:	Attempts to differentiate by using the quotient rule with $u = \sin 2x$ and $v = -3 + \cos 2x$ or attempts to differentiate by using the product rule with $u = \sin 2x$ and $v = (-3 + \cos 2x)^{-1}$
A1:	Correct $f'(x)$, which can be un-simplified or simplified
M1:	Sets $f'(x) = 0$ and proceeds with their working to set the numerator of $f'(x)$ equal to 0
A1*:	Shows $\cos 2x = \frac{1}{3}$ with no errors seen in their working
(b)(i)	
M1:	• Attempts to find the second solution for $\cos 6x = \frac{1}{3}$ E.g. $x = \frac{2\pi - 1.2309593...}{6} = \frac{5.052225...}{6}$ • Attempts to find the second solution of $\cos 2x = \frac{1}{3}$ and proceeds to divide their result by 3 E.g. $3x = \frac{2\pi - 1.2309593...}{2} \Rightarrow x = \frac{2.526112...}{3}$
A1:	0.84 (2 dp) or anything that rounds to 0.84
(b)(ii)	
M1:	• Attempts to find the first solution for $\cos\left(\frac{1}{2}x\right) = \frac{1}{3}$ E.g. $x = 2(1.2309593...)$ • Attempts to find the first solution of $\cos 2x = \frac{1}{3}$ and proceeds to multiply the result by 4 E.g. $\frac{1}{4}x = \frac{1.2309593...}{2} \Rightarrow x = 4(0.615479...)$
A1:	2.46 (2 dp) or anything that rounds to 2.46

Question	Scheme	Marks	AOs
12 (a)	Total mass $= \dfrac{4500(1-(0.98)^{23})}{1-0.98}$ or $\dfrac{4500((0.98)^{23}-1)}{0.98-1}$	M1	3.1b
	$= 83621.86152... = 83600$ (tonnes) (3 sf)	A1	1.1b
		(2)	
(b)	Expected mass in the year 2040 $= 4500(0.98)^{23-1}$	M1	3.4
	$= 2885.268132... = 2890$ (tonnes) (3 sf)	A1	1.1b
		(2)	
(c)	Total cost $= 800(1500(23)) + 600(83621.86152... - 1500(23))$	M1	3.1b
		M1	1.1b
	$= 800(34500) + 600(49121.86152...)$ $= 27600000 + 29473116.91$ $= 57073116.91$		
	$\Rightarrow x =$ (£) 57.1 (million) (3 sf)	A1	3.2a
		(3)	
(c) Alt 1	Total cost $= 200(1500(23)) + 600(83621.86152...)$	M1	3.1b
		M1	1.1b
	$= 200(34500) + 600(49121.86152...)$ $= 6900000 + 50173116.91$ $= 57073116.91$		
	$\Rightarrow x =$ (£) 57.1 (million) (3 sf)	A1	3.2a
		(3)	
			(7 marks)

Question 12 Notes:	
(a)	
M1:	Complete method of applying the correct geometric series summation formula with either $n = 22$ or $n = 23$, $a = 4500$ and $r = 0.98$
A1:	Correct answer to 3 significant figures of $83\,600$ (tonnes)
(b)	
M1:	Uses the geometric series model to apply the correct *n*th term formula with either $n = 22$ or $n = 23$, $a = 4500$ and $r = 0.98$
A1:	Correct answer to 3 significant figures of $2\,890$ (tonnes)
(c)	
M1:	A ***complete strategy*** to find the total cost
M1:	For either • $800(1500(23))$ $\{= 27\,600\,000\}$ • $600(83621.86152... - 1500(23))$ $\{= 29\,473\,116.91\}$ • $800(1500(22))$ $\{= 26\,400\,000\}$ • $600(\text{"}80\,736.59338...\text{"} - 1500(22))$ $\{= 28\,641\,956.03\}$
A1:	Correct answer of $x = (£)57.1$ (million) (3 sf) **Note:** Using rounded answer from part (a) gives • $x = 27\,600\,000 + 29\,460\,000 = 57\,060\,000 = (£)57.1$ (million) (3 sf)
(c)	
Alt 1	
M1:	A ***complete strategy*** to find the total cost
M1:	For either • $200(1500(23))$ $\{= 6\,900\,000\}$ • $600(83621.86152...)$ $\{= 50\,173\,116.91\}$ • $200(1500(22))$ $\{= 6\,600\,000\}$
A1:	Correct answer to 3 significant figures of $x = (£)57.1$ (million) **Note:** Using rounded answer in part (a) gives • $6\,900\,000 + 50\,160\,000 = 57\,060\,000 \Rightarrow x = (£)57.1$ (million) (3 sf)
	Note: Using $n = 22$ throughout gives (a) $80\,736.59338...$ (b) $2944.151155...$ (c) $55.04195603...$

Question	Scheme	Marks	AOs
13	$x = 6\cos t\,,\ y = 5\sin 2t;\ \ 0 \leqslant t \leqslant \frac{\pi}{2}$		
	$\left\{\int y\frac{dx}{dt}\{dt\}\right\} = \int (5\sin 2t)(-6\sin t)\{dt\}$	M1	2.1
		A1	1.1b
	$= \int (5(2\sin t\cos t))(-6\sin t)\{dt\}$	M1	1.1b
	$= -60\int \sin^2 t\cos t\{dt\}$		
	$= -60\left[\frac{1}{3}\sin^3 t\right]\ \ \left\{= -20\left[\sin^3 t\right]\right\}$	M1	3.1a
		A1	1.1b
	$\left\{\text{Limits: } x=0 \Rightarrow 0 = 6\cos t \Rightarrow t = \frac{\pi}{2};\ \ x=3 \Rightarrow 3 = 6\cos t \Rightarrow t = \frac{\pi}{3}\right\}$		
	$\text{Area } (R) = \int_0^3 y\,dx = -20\left[\sin^3 t\right]_{\frac{\pi}{2}}^{\frac{\pi}{3}} = -20\left(\sin^3\left(\frac{\pi}{3}\right) - \sin^3\left(\frac{\pi}{2}\right)\right)$	M1	1.1b
	$= -20\left(\left(\frac{\sqrt{3}}{2}\right)^3 - 1\right) = -20\left(\frac{3}{8}\sqrt{3} - 1\right) = 20 - \frac{15}{2}\sqrt{3}$ *	A1*	2.1
		(7)	
		(7 marks)	

Question 13 Notes:

M1:	Begins proof by applying a full method of $\int y\frac{dx}{dt}\{dt\}$ to give $\int (5\sin 2t)\left(\text{their } \frac{dx}{dt}\right)\{dt\}$.
A1:	$\int (5\sin 2t)(-6\sin t)\{dt\}$.
M1:	Applies $\sin 2t \equiv 2\sin t\cos t$ to achieve an integral of the form $\pm K\int \sin^2 t\cos t\{dt\};\ K \neq 0$, which may be un-simplified or simplified
M1:	Applies parametric integration to achieve an integral of the form $\pm K\int \sin^2 t\cos t\{dt\};\ K \neq 0$, followed by a correct integration strategy of "reverse chain rule" or "integration by substitution" to give $\int \sin^2 t\cos t\{dt\}$ in the form $\pm\lambda\sin^3 t;\ \lambda \neq 0$ or $\pm\lambda u^3;\ \lambda \neq 0$ where $u = \sin t$
A1:	$\sin^2 t\cos t \to \frac{1}{3}\sin^3 t$ or $\sin^2 t\cos t \to \frac{1}{3}u^3$ where $u = \sin t$
M1:	Applies limits of $t = \frac{\pi}{3}$ and $t = \frac{\pi}{2}$ to an integrated expression of the form $\pm\alpha\sin^3 t;\ \alpha \neq 0$ and subtracts either way round
A1*:	Correctly uses their limits to show that the area of R is $20 - \frac{15}{2}\sqrt{3}$

Question	Scheme	Marks	AOs
14	$y = kx^2$ and $y = \sqrt{kx}$, $x \geqslant 0$		
	E.g. • $kx^2 = \sqrt{kx} \Rightarrow k^2x^4 = kx \Rightarrow k^2x^4 - kx = 0 \Rightarrow kx(kx^3 - 1) = 0$ $\{\Rightarrow kx = 0 \Rightarrow x = 0\} \Rightarrow kx^3 - 1 = 0 \Rightarrow x^3 = \frac{1}{k} \Rightarrow x = ...$ • $kx^2 = \sqrt{kx} \Rightarrow k^2x^4 = kx \Rightarrow kx^3 = 1 \Rightarrow x = ...$ • $kx^2 = \sqrt{kx} \Rightarrow k^{\frac{1}{2}}x^{\frac{3}{2}} = 1 \Rightarrow x^{\frac{3}{2}} = k^{-\frac{1}{2}} \Rightarrow x = ...$	M1	2.1
	$x = \sqrt[3]{\frac{1}{k}}$ or $x = k^{-\frac{1}{3}}$	A1	1.1b
	$\text{Area}(R) = \int_0^{k^{-\frac{1}{3}}} \left(\sqrt{kx} - kx^2\right) \mathrm{d}x = \left[\frac{\sqrt{k}\, x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{1}{3}kx^3 \right]_0^{k^{-\frac{1}{3}}}$	M1	1.1b
		B1	1.1b
	$= \left(\frac{2}{3}\sqrt{k}\frac{1}{\sqrt{k}} - \frac{k}{3}.\frac{1}{k}\right) - (0 - 0) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ *	A1*	2.1
		(5)	
			(5 marks)

Question 14 Notes:

M1:	Equates the two curves and solves $kx^2 = \sqrt{kx}$ to give $x = ...$
A1:	$x = \sqrt[3]{\frac{1}{k}}$ or $x = k^{-\frac{1}{3}}$
M1:	Evidence of attempting $\int \left(\sqrt{kx} - kx^2\right) \mathrm{d}x$ or $\left(\int \sqrt{kx}\, \mathrm{d}x - \int kx^2\, \mathrm{d}x\right)$ with at least one of either $\sqrt{kx} \to \pm \alpha x^{\frac{3}{2}}$ or $kx^2 \to \pm \beta x^3$; $\alpha, \beta \neq 0$. You can ignore the limits for this mark
B1:	At least one of either $\sqrt{kx} \to \frac{\sqrt{k}\, x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$ or $kx^2 \to \frac{1}{3}kx^3$, which can be un-simplified or simplified
A1*:	Correct use of integration and limits to show that, for all values of k, the area of R is $\frac{1}{3}$

Question	Scheme	Marks	AOs
15	$a_{n+1} = k - \dfrac{3k}{a_n},\ \ n \in \mathbb{Z}^+;\ k$ is a constant Sequence $a_1, a_2, a_3, \ldots$ where $a_2 = 2$ is periodic of order 3		
(a)	$a_3 = k - \dfrac{3k}{2} = -\dfrac{1}{2}k\ ;\qquad a_4 = k - \dfrac{3k}{\left(-\frac{1}{2}k\right)} = k + 6$	M1	1.1b
	$\{a_5 = a_2 \Rightarrow\}\ a_5 = k - \dfrac{3k}{k+6} = 2$	M1	3.1a
	$\Rightarrow k(k+6) - 3k = 2(k+6) \Rightarrow k^2 + 6k - 3k = 2k + 12$ $\Rightarrow k^2 + k - 12 = 0\ *$	A1*	2.1
		(3)	
(b)	$(k+4)(k-3) = 0 \Rightarrow k = -4, 3$ $k = 3;\ \ \{a_2 = 2,\}\ a_3 = -\dfrac{3}{2},\ a_4 = 9$	M1	3.1a
	$\{k = -4;\ \{a_2 = 2,\}\ a_3 = 2\ \ \{\Rightarrow a_4 = 2,\ a_1 = 2\ ;$ so reject as $a_1 = a_2\}\}$	A1	1.1b
	Note: $k = 3;\ \ a_1 = 9, a_2 = 2,\ a_3 = -\dfrac{3}{2},\ a_4 = 9$, etc.		
	$\displaystyle\sum_{r=1}^{121} a_r = 40\left(2 - \frac{3}{2} + 9\right) + 9$	M1	2.2a
	$= 40(9.5) + 9 = 380 + 9 = 389$	A1	1.1b
		(4)	
			(7 marks)

Question 15 Notes:

(a)	
M1:	Uses $a_2 = 2$ to find **both** a_3 in terms of *k* (which can be un-simplified or simplified) **and** a_4 in terms of *k* (which can be un-simplified or simplified)
M1:	Shows understanding that the sequence is periodic of order 3 by applying complete strategy of finding a_5 in terms of *k* and setting the result equal to 2 (which is the same as a_2)
A1*:	Shows that $k^2 + k - 12 = 0$ with no errors in their working
(b)	
M1:	Complete process of finding and using $k = 3$ to find ***the values*** of either a_3 and a_4 **or** a_1 and a_3
A1:	Uses $k = 3$ to find $a_3 = -\dfrac{3}{2}$ and $a_4 = 9$ **or** $a_1 = 9$ and $a_3 = -\dfrac{3}{2}$
M1:	Deduces $\displaystyle\sum_{r=1}^{121} a_r = 40(2 + \text{"}-1.5\text{"} + \text{"}9\text{"}) + \text{"}9\text{"}$
A1:	389