PURE MATHEMATICS
A level Practice Papers

PAPER L
MARK SCHEME

1a	Makes an attempt to substitute $k = 1$, $k = 2$ and $k = 4$ into $a_k = 2^k + 1, k$ Ö 1	**M1**
	Shows that $a_1 = 3$, $a_2 = 5$ and $a_4 = 17$ and these are prime numbers.	**A1**
		(2 marks)
1b	Substitutes a value of k that does not yield a prime number. For example, $a_3 = 9$ or $a_5 = 33$	**A1**
	Concludes that their number is not prime. For example, states that 9 = 3 × 3, so 9 is not prime.	**B1**
		(2 marks)
	TOTAL: 4 marks	

2	Recognises the need to write $\sin^3 x \equiv \sin x\left(\sin^2 x\right)$	**M1**
	Selects the correct trigonometric identity to write $\sin x\left(\sin^2 x\right) \equiv \sin x\left(1 - \cos^2 x\right)$. Could also write $\sin x - \sin x \cos^2 x$	**M1**
	Makes an attempt to find $\int\left(\sin x - \sin x \cos^2 x\right) \mathrm{d}x$	**M1**
	Correctly states answer $-\cos x + \frac{1}{3}\cos^3 x + C$	**A1**
	TOTAL: 4 marks	**(4 marks)**

Question	Scheme	Marks
3	Begins the proof by assuming the opposite is true. 'Assumption: given a rational number a and an irrational number b, assume that $a - b$ is rational.'	**B1**
	Sets up the proof by defining the different rational and irrational numbers. The choice of variables does not matter. Let $a = \frac{m}{n}$ As we are assuming $a - b$ is rational, let $a - b = \frac{p}{q}$ So $a - b = \frac{p}{q} \Rightarrow \frac{m}{n} - b = \frac{p}{q}$	**M1**
	Solves $\frac{m}{n} - b = \frac{p}{q}$ to make b the subject and rewrites the resulting expression as a single fraction: $\frac{m}{n} - b = \frac{p}{q} \Rightarrow b = \frac{m}{n} - \frac{p}{q} = \frac{mq - pn}{nq}$	**M1**
	Makes a valid conclusion. $b = \frac{mq - pn}{nq}$, which is rational, contradicts the assumption b is an irrational number. Therefore the difference of a rational number and an irrational number is irrational.	**B1**
	TOTAL: 4 marks	**(4 marks)**

Question	Scheme	Marks
4a	Differentiates $x = \sec 4y$ to obtain $\frac{\mathrm{d}x}{\mathrm{d}y} = 4\sec 4y \tan 4y$	**M1**
	Writes $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4\sec 4y \tan 4y}$	**A1**
		(2 marks)
4b	Use the identity $\tan^2 A + 1 = \sec^2 A$ to write $\tan 4y = \sqrt{\sec^2 4y - 1} = \sqrt{x^2 - 1}$	**M1**
	Attempts to substitute $\sec 4y = x$ and $\tan 4y = \sqrt{x^2 - 1}$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4\sec 4y \tan 4y}$	**M1**
	Correctly substitutes to find $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4x\sqrt{x^2 - 1}}$ and states $k = \frac{1}{4}$	**A1**
		(3 marks)
	TOTAL: 5 marks	

5a	Writes tan$x$ and sec$x$ in terms of sin x and cos x. eg $\frac{\tan x - \sec x}{1 - \sin x} = (\frac{\sin x}{\cos x} - \frac{1}{\cos x}) \div (1 - \sin x)$	**M1**
	Manipulates the expression to find $\left(\frac{\sin x - 1}{\cos x}\right) \times \left(\frac{1}{1 - \sin x}\right)$	**M1**
	Simplifies to find $-\frac{1}{\cos x} = -\sec x$	**A1**
		(3 marks)
5b	States that $-\sec x = \sqrt{2}$ or $\sec x = -\sqrt{2}$	**B1**
	Writes that $\cos x = -\frac{1}{\sqrt{2}}$ or $x = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$	**M1**
	Finds $x = \frac{3\pi}{4}, \frac{5\pi}{4}$	**A1**
		(3 marks)
	TOTAL: 6 marks	

6a	States $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x + h - x}$	**M1**
	Makes correct substitutions: $f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$	**M1**
	Uses the appropriate trigonometric addition formula to write: $f'(x) = \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$	**M1**
	Groups the terms appropriately $f'(x) = \lim_{h \to 0}\left(\left(\frac{\cos h - 1}{h}\right)\sin x + \left(\frac{\sin h}{h}\right)\cos x\right)$	**A1**
		(4 marks)
6b	Explains that as $h \to 0$, $\frac{\cos h - 1}{h} \to 0$ and $\frac{\sin h}{h} \to 1$	**M1**
	Concludes that this leaves $0 \times \sin x + 1 \times \cos x$ So if $f(x) = \sin x, f'(x) = \cos x$	**A1**
		(2 marks)
	TOTAL: 6 marks	

Q	Scheme	Marks
7	Makes an attempt to set up a long division. For example: $x^2-2x-15\overline{\big)x^4+2x^3-29x^2-48x+90}$ is seen.	**M1**
	Award 1 accuracy mark for each of the following: x^2 seen, $4x$ seen, -6 seen. $\begin{array}{r} x^2+4x-6 \\ x^2-2x-15\overline{\big)x^4+2x^3-29x^2-47x+77} \\ \underline{x^4-2x^3-15x^2} \\ 4x^3-14x^2-47x \\ \underline{4x^3-8x^2-60x} \\ -6x^2+13x+77 \\ \underline{-6x^2+12x+90} \\ x-13 \end{array}$	**A3**
	Equates the various terms to obtain the equation: $x-13=V(x-5)+W(x+3)$ Equating the coefficients of x: $V+W=1$ Equating constant terms: $-5V+3W=-13$	**M1**
	Multiplies one or or both of the equations in an effort to equate one of the two variables.	**M1**
	Finds $W=-1$ and $V=2$.	**A1**
	TOTAL: 7 marks	

8a	States the range is y Ö -5 or $\mathrm{f}(x)$ Ö -5	**B1**
		(1 mark)
8b	Recognises that $3(x-4)-5=-\frac{1}{3}x+k$ and $-3(x-4)-5=-\frac{1}{3}x+k$	**M1**
	Makes an attempt to solve both of these equations.	**M1**
	Correctly states $\frac{10}{3}x=k+17$. Equivalent version is acceptable.	**A1**
	Correctly states $-\frac{8}{3}x=k-7$. Equivalent version is acceptable.	**A1**
	Makes an attempt to substitute one equation into the other in an effort to solve for *k*. For example, $x=\frac{3}{10}(k+17)$ and $-\left(\frac{8}{3}\right)\left(\frac{3}{10}\right)(k+17)=k-7$ is seen.	**M1 ft**
	Correctly solves to find $k=-\frac{11}{3}$	**A1 ft**
	States the correct range for *k*. $k>-\frac{11}{3}$	**B1**
		(2 marks)
	TOTAL: 8 marks	

NOTES 8b: Award ft marks for a correct method using an incorrect answer from earlier in the question.

Alternative Method

Draws line with gradient $-\frac{1}{3}$ passing through vertex and calculates $k=-\frac{11}{3}$, so answer is $k>-\frac{11}{3}$

M1: States the *x*-coordinate of the vertex of the graph is 4

M1: States the *y*-coordinate of the vertex of the graph is −5

M1: Writes down the gradient of $-\frac{1}{3}$ or implies it later in the question.

M1: Attempts to use $y-y_1=m(x-x_1)$ with $(x_1,y_1)=(4,-5)$ and $m=-\frac{1}{3}$

A1: Finds $y=-\frac{1}{3}x-\frac{11}{3}$ o.e.

B1: States the correct range for *k*: $k>-\frac{11}{3}$

9a

Makes an attempt to rearrange $x = \frac{1+4t}{1-t}$ to make t the subject. For example, $x - xt = 1 + 4t$ is seen.	**M1**
Correctly states $t = \frac{x-1}{4+x}$	**A1**
Makes an attempt to substitute $t = \frac{x-1}{4+x}$ into $y = \frac{2+bt}{1-t}$ For example, $y = \frac{2 + \frac{bx-b}{x+4}}{1 - \frac{x-1}{x+4}} = \frac{\frac{2x+8+bx-b}{x+4}}{\frac{x+4-x+1}{x+4}}$ is seen.	**M1**
Simplifies the expression showing all steps. For example, $y = \frac{2x+8+bx-b}{5} = \left(\frac{2+b}{5}\right)x + \left(\frac{8-b}{5}\right)$	**A1**
	(4 marks)
9b Interprets the gradient of line being −1 as $\frac{2+b}{5} = -1$ and finds $b = -7$	**M1**
Substitutes $t = -1$ to find $x = -\frac{3}{2}$ and $y = \frac{9}{2}$ And substitutes $t = 0$ to find $x = 1$ and $y = 2$	**M1**
Makes an attempt to use Pythagoras' Theorem to find the length of the line: $\sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2}$	**M1**
Correctly finds the length of the line segment, $\frac{5\sqrt{2}}{2}$ or states $a = \frac{5}{2}$	**A1**
	(4 marks)
TOTAL: 8 marks	

10	Makes an attempt to find $\int \left(\frac{e^{2x}}{e^{2x}-1} \right) dx$ Writing $\int \frac{f'(x)}{f(x)} dx = \ln[f(x)]$ or writing $\ln(e^{2x}-1)$ constitutes an attempt.	**M1**
	Correctly states $\int \left(\frac{e^{2x}}{e^{2x}-1} \right) dx = \frac{1}{2}\ln(e^{2x}-1)(+C)$	**A1**
	Makes an attempt to substitute the limits $x = \ln b$ and $x = \ln 2$ into $\frac{1}{2}\ln(e^{2x}-1)$ For example, $\frac{1}{2}\ln(e^{2\ln b}-1)$ and $\frac{1}{2}\ln(e^{2\ln 2}-1)$ is seen.	**M1 ft**
	Uses laws of logarithms to begin to simplify the expression. Either $\frac{1}{2}\ln(b^2-1)$ or $\frac{1}{2}\ln(2^2-1)$ is seen.	**M1 ft**
	Correctly states the two answers as $\frac{1}{2}\ln(b^2-1)$ and $\frac{1}{2}\ln 3$	**A1 ft**
	States that $\frac{1}{2}\ln(b^2-1) - \frac{1}{2}\ln 3 = \ln 4$	**M1 ft**
	Makes an attempt to solve this equation. For example, $\ln\left(\frac{b^2-1}{3}\right) = 2\ln 4$ is seen.	**M1 ft**
	Correctly states the final answer $b = 7$	**A1 ft**
	TOTAL: 8 marks	

NOTES:

Student does not need to state '+C' in an answer unless it is the final answer to an indefinite integral.

Award ft marks for a correct answer using an incorrect initial answer.

11a	Recognises that it is a geometric series with a first term $a = 100$ and common ratio $r = 1.05$	**M1**
	Attempts to use the sum of a geometric series. For example, $S_9 = \dfrac{100(1-1.05^9)}{1-1.05}$ or $S_9 = \dfrac{100(1.05^9-1)}{1.05-1}$ is seen.	**M1***
	Finds $S_9 = £1102.66$	**A1**
		(3 marks)
11b	States $\dfrac{100(1.05^n-1)}{1.05-1} > 6000$ or $\dfrac{100(1-1.05^n)}{1-1.05} > 6000$	**M1**
	Begins to simplify. $1.05^n > 4$ or $-1.05^n < -4$	**M1**
	Applies law of logarithms correctly $n\log 1.05 > \log 4$ or $-n\log 1.05 < -\log 4$	**M1**
	States $n > \dfrac{\log 4}{\log 1.05}$	**A1**
		(4 marks)
11c	Uses the sum of an arithmetic series to state $\dfrac{29}{2}\left[100+(28)d\right] = 6000$	**M1**
	Solves for d. $d = £11.21$	**A1**
		(2 marks)
	TOTAL: 9 marks	

NOTES 11a:

M1

Award mark if attempt to calculate the amount of money after 1, 2, 3,….,8 and 9 months is seen.

12a	Makes an attempt to find the resultant force by adding the three force vectors together.	**M1**
	Finds $R = (6\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})\text{N}$	**A1**
		(2 marks)
12b	States $F = ma$ or writes $(6\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}) = 3(a)$	**M1**
	Finds $a = (2\mathbf{i} + \mathbf{j} + \mathbf{k})\text{ms}^{-2}$	**A1**
		(2 marks)
12c	Demonstrates an attempt to find $\lvert a \rvert$ For example, $\lvert a \rvert = \sqrt{(2)^2 + (1)^2 + (1)^2}$	**M1**
	Finds $\lvert a \rvert = \sqrt{6}\text{ m s}^{-2}$	**A1**
		(2 marks)
12d	States $s = ut + \frac{1}{2}at^2$	**M1**
	Makes an attempt to substitute values into the equation. $s = (0)(10) + \frac{1}{2}(\sqrt{6})(10)^2$	**M1 ft**
	Finds $s = 50\sqrt{6}\text{ m}$	**A1 ft**
		(3 marks)
	TOTAL: 9 marks	

NOTES: 12d

Award ft marks for a correct answer to part **d** using their incorrect answer from part **c**.

13a	Finds $h(19.3) = (+)0.974...$ and $h(19.4) = -0.393...$	**M1**
	Change of sign and continuous function in the interval $[19.3, 19.4] \Rightarrow$ root	**A1**
		(2 marks)
13b	Makes an attempt to differentiate $h(t)$	**M1**
	Correctly finds $h'(t) = \frac{40}{t+1} + 8\cos\left(\frac{t}{5}\right) - \frac{1}{2}t$	**A1**
	Finds $h(19.35) = 0.2903...$ and $h'(19.35) = -13.6792...$	**M1**
	Attempts to find x_1 $x_1 = x_0 - \frac{h(x_0)}{h'(x_0)} \Rightarrow x_1 = 19.35 - \frac{0.2903...}{-13.6792...}$	**M1**
	Finds $x_1 = 19.371$	**A1**
		(5 marks)
13c	Demonstrates an understanding that $x = 19.3705$ and $x = 19.3715$ are the two values to be calculated.	**M1**
	Finds $h(19.3705) = (+)0.0100...$ and $h(19.3715) = -0.00366...$	**M1**
	Change of sign and continuous function in the interval $[19.3705, 19.3715] \Rightarrow$ root	**A1**
		(3 marks)
	TOTAL: 10 marks	

NOTES: 13a

Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.

14a Correctly writes $6(2+3x)^{-1}$ as: $6\left(2^{-1}\left(1+\frac{3}{2}x\right)^{-1}\right)$ or $3\left(1+\frac{3}{2}x\right)^{-1}$	**M1**
Completes the binomial expansion: $3\left(1+\frac{3}{2}x\right)^{-1}=3\left(1+(-1)\left(\frac{3}{2}\right)x+\frac{(-1)(-2)\left(\frac{3}{2}\right)^2x^2}{2}+\ldots\right)$	**M1**
Simplifies to obtain $3-\frac{9}{2}x+\frac{27}{4}x^2+\ldots$	**A1**
Correctly writes $4(3-5x)^{-1}$ as: $4\left(3^{-1}\left(1-\frac{5}{3}x\right)^{-1}\right)$ or $\frac{4}{3}\left(1-\frac{5}{3}x\right)^{-1}$	**M1**
Completes the binomial expansion: $\frac{4}{3}\left(1-\frac{5}{3}x\right)^{-1}=\frac{4}{3}\left(1+(-1)\left(-\frac{5}{3}\right)x+\frac{(-1)(-2)\left(-\frac{5}{3}\right)^2x^2}{2}+\ldots\right)$	**M1**
Simplifies to obtain $\frac{4}{3}+\frac{20}{9}x+\frac{100}{27}x^2+\ldots$	**A1**
Simplifies by subtracting to obtain $\frac{5}{3}-\frac{121}{18}x+\frac{329}{108}x^2+\ldots$ The need to subtract, or the subtracting shown, must be seen in order to award the mark.	**A1**
	(7 marks)
14b Makes an attempt to substitute $x = 0.01$ into f(x). For example, $\frac{6}{2+3(0.01)}-\frac{4}{3-5(0.01)}$ is seen.	**M1**
States the answer 1.5997328	**A1**
	(2 marks)
14c Makes an attempt to substitute $x = 0.01$ into $\frac{5}{3}-\frac{121}{18}x-\frac{329}{108}x^2+\ldots$	**M1 ft**
States the answer 1.59974907… Accept awrt 1.60.	**M1 ft**
Finds the percentage error: 0.0010%	**A1 ft**
	(3 marks)
TOTAL: 12 marks	

NOTES:

14a

If one expansion is correct and one is incorrect, or both are incorrect, award the final accuracy mark if they are subtracted correctly.

14c

Award all 3 marks for a correct answer using their incorrect answer from part **a**.

(TOTAL: 100 MARKS)