

PURE MATHEMATICS
A level Practice Papers

PAPER N
MARK SCHEME

1	Attempts to write a differential equation. For example, $\frac{dF}{dt} \propto F$ or $\frac{dF}{dt} \propto -F$ is seen.	M1
	States $\frac{dF}{dt} = -kF$	A1
TOTAL: 2 marks		

2	Finds $ a = \sqrt{(4)^2 + (-1)^2 + (3)^2} = \sqrt{26}$	M1
	States $\cos \theta_y = -\frac{1}{\sqrt{26}}$	M1
	Solves to find $\theta_y = 101.309...^\circ$. Accept awrt 101.3°	A1
TOTAL: 3 marks		

3	Begins the proof by assuming the opposite is true. ‘Assumption: there do exist integers a and b such that $25a + 15b = 1$ ’	B1
	Understands that $25a + 15b = 1 \Rightarrow 5a + 3b = \frac{1}{5}$ ‘As both 25 and 15 are multiples of 5, divide both sides by 5 to leave $5a + 3b = \frac{1}{5}$ ’	M1
	Understands that if a and b are integers, then $5a$ is an integer, $3b$ is an integer and $5a + 3b$ is also an integer.	M1
	Recognises that this contradicts the statement that $5a + 3b = \frac{1}{5}$, as $\frac{1}{5}$ is not an integer. Therefore there do not exist integers a and b such that $25a + 15b = 1$	B1
TOTAL: 4 marks		

4	Selects $\cos 2x \equiv 2\cos^2 x - 1$ as the appropriate trigonometric identity.	M1
	Manipulates the identity to the question: $\cos 12x \equiv 2\cos^2 6x - 1$	M1
	States that $\int (\cos^2 6x) dx = \frac{1}{2} \int (1 + \cos 12x) dx$	M1
	Makes an attempt to integrate the expression, x and $\sin x$ are seen.	M1
	Correctly states $\frac{1}{2} \left(x + \frac{1}{12} \sin 12x \right) + C$	A1
TOTAL: 5 marks		

NOTES:

Student does not need to state '+C' to be awarded the third method mark.

Must be stated in the final answer.

5	Begins the proof by assuming the opposite is true. 'Assumption: there exists a product of two odd numbers that is even.'	B1
	Defines two odd numbers. Can choose any two different variables. 'Let $2m + 1$ and $2n + 1$ be our two odd numbers.'	B1
	Successfully multiplies the two odd numbers together: $(2m+1)(2n+1) \equiv 4mn + 2m + 2n + 1$	M1
	Factors the expression and concludes that this number must be odd. $4mn + 2m + 2n + 1 \equiv 2(2mn + m + n) + 1$ $2(2mn + m + n)$ is even, so $2(2mn + m + n) + 1$ must be odd.	M1
	Makes a valid conclusion. This contradicts the assumption that the product of two odd numbers is even, therefore the product of two odd numbers is odd.	B1
TOTAL: 5 marks		

NOTES: Alternative method

Assume the opposite is true: there exists a product of two odd numbers that is even. **(B1)**

If the product is even then 2 is a factor. **(B1)**

So 2 is a factor of at least one of the two numbers. **(M1)**

So at least one of the two numbers is even. **(M1)**

This contradicts the statement that both numbers are odd. **(B1)**

6	<p>States that:</p> $A(4-x)(x+5) + B(x-3)(x+5) + C(x-3)(4-x) \equiv 4x^2 + x - 23$	M1
	<p>Further states that:</p> $A(-x^2 - x + 20) + B(x^2 + 2x - 15) + C(-x^2 + 7x - 12) \equiv 4x^2 + x - 23$	M1
	<p>Equates the various terms. Equating the coefficients of x^2: $-A + B - C = 4$ Equating the coefficients of x: $-A + 2B + 7C = 1$ Equating constant terms: $20A - 15B - 12C = -23$</p>	M1*
	<p>Makes an attempt to manipulate the expressions in order to find A, B and C. Obtaining two different equations in the same two variables would constitute an attempt.</p>	M1*
	<p>Finds the correct value of any one variable: either $A = 2$, $B = 5$ or $C = -1$</p>	A1*
	<p>Finds the correct value of all three variables: $A = 2$, $B = 5$, $C = -1$</p>	A1
TOTAL: 6 marks		

NOTES: Alternative method

Uses the substitution method, having first obtained this equation:

$$A(4-x)(x+5) + B(x-3)(x+5) + C(x-3)(4-x) \equiv 4x^2 + x - 23 \quad (\text{M3 as above})$$

Substitutes $x = 4$ to obtain $9B = 45$ **(M1)**

Substitutes $x = 3$ to obtain $8A = 16$ **(M1)**

Substitutes $x = -5$ to obtain $-72C = 72$ **(A1)**

7	Demonstrates an attempt to find the vectors \overrightarrow{AB} , \overrightarrow{AC} and \overrightarrow{BC}	M1
	Finds $\overrightarrow{AB} = (0, 4, -2)$, $\overrightarrow{AC} = (5, 4, 8)$ and $\overrightarrow{BC} = (5, 0, 10)$	A1
	Demonstrates an attempt to find $ \overrightarrow{AB} $, $ \overrightarrow{AC} $ and $ \overrightarrow{BC} $	M1
	Finds $ \overrightarrow{AB} = \sqrt{(0)^2 + (4)^2 + (-2)^2} = \sqrt{20}$ Finds $ \overrightarrow{AC} = \sqrt{(5)^2 + (4)^2 + (8)^2} = \sqrt{105}$ Finds $ \overrightarrow{BC} = \sqrt{(5)^2 + (0)^2 + (10)^2} = \sqrt{125}$	A1
	States or implies in a right-angled triangle $c^2 = a^2 + b^2$	M1
	States that $ \overrightarrow{AB} ^2 + \overrightarrow{AC} ^2 = \overrightarrow{BC} ^2$	B1
TOTAL: 6 marks		

8a	Rearranges $x = 8(t + 10)$ to obtain $t = \frac{x - 80}{8}$	M1
	Substitutes $t = \frac{x - 80}{8}$ into $y = 100 - t^2$ For example, $y = 100 - \left(\frac{x - 80}{8}\right)^2$ is seen.	M1
	Finds $y = -\frac{1}{64}x^2 + \frac{5}{2}x$	A1
		(3 marks)
8b	Deduces that the width of the arch can be found by substituting $t = \pm 10$ into $x = 8(t + 10)$	M1
	Finds $x = 0$ and $x = 160$ and deduces the width of the arch is 160 m.	A1
		(2 marks)
8b	Deduces that the greatest height occurs when $\frac{dy}{dt} = 0 \Rightarrow -2t = 0 \Rightarrow t = 0$	M1
	Deduces that the height is 100 m.	A1
		(2 marks)
TOTAL: 7 marks		

9a	Finds $f(1.9) = -0.2188\dots$ and $f(2.0) = (+)0.1606\dots$	M1
	Change of sign and continuous function in the interval $[1.9, 2.0] \Rightarrow \text{root}$	A1
		(2 marks)
9b	Makes an attempt to differentiate $f(x)$	M1
	Correctly finds $f'(x) = -9\sin^2 x \cos x + \sin x$	A1
	Finds $f(1.95) = -0.0348\dots$ and $f'(1.95) = 3.8040\dots$	M1
	Attempts to find x_1 : $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_1 = 1.95 - \frac{-0.0348\dots}{3.8040\dots}$	M1
	Finds $x_1 = 1.959$	A1
		(5 marks)
TOTAL: 7 marks		

NOTES: 9a

Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval

10	Use Pythagoras' theorem to show that the length of $OB = 2\sqrt{3}$ or $OD = 2\sqrt{3}$ or states $BD = 4\sqrt{3}$	M1
	Makes an attempt to find $\angle DAB$ or $\angle DCB$. For example, $\cos \angle DAO = \frac{2}{4}$ is seen.	M1
	Correctly states that $\angle DAB = \frac{2\pi}{3}$ or $\angle DCB = \frac{2\pi}{3}$	A1
	Makes an attempt to find the area of the sector with a radius of 4 and a subtended angle of $\frac{2\pi}{3}$ For example, $A = \frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$ is shown.	M1
	Correctly states that the area of the sector is $\frac{16\pi}{3}$	A1
	Recognises the need to subtract the sector area from the area of the rhombus in an attempt to find the shaded area. For example, $\frac{16\pi}{3} - 8\sqrt{3}$ is seen.	M1
	Recognises that to find the total shaded area this number will need to be multiplied by 2 For example, $2 \times \left(\frac{16\pi}{3} - 8\sqrt{3} \right)$	M1
	Using clear algebra, correctly manipulates the expression and gives a clear final answer of $\frac{2}{3}(16\pi - 24\sqrt{3})$	A1
TOTAL: 8 marks		

11a	Differentiates $u = 4t^{\frac{2}{3}}$ obtaining $\frac{du}{dt} = \frac{8}{3}t^{-\frac{1}{3}}$ and differentiates $v = t^2 + 1$ obtaining $\frac{dv}{dt} = 2t$	M1
	Makes an attempt to substitute the above values into the product rule formula: $\frac{dH}{dt} = \frac{v \frac{du}{dt} - u \frac{dv}{dt}}{v^2}$	M1
	Finds $\frac{dH}{dt} = \frac{\frac{8}{3}t^{\frac{5}{3}} + \frac{8}{3}t^{-\frac{1}{3}} - 8t^{\frac{5}{3}}}{(t^2 + 1)^2}$	M1
	Fully simplifies using correct algebra to obtain $\frac{dH}{dt} = \frac{8(1 - 2t^2)}{3\sqrt[3]{t}(t^2 + 1)^2}$	A1
		(4 marks)
11b	Makes an attempt to substitute $t = 2$ into $\frac{dH}{dt} = \frac{8(1 - 2t^2)}{3\sqrt[3]{t}(t^2 + 1)^2} = \frac{8(1 - 2(2)^2)}{3\sqrt[3]{2}(2^2 + 1)^2}$	M1 ft
	Correctly finds $\frac{dH}{dt} = -0.592...$ and concludes that as $\frac{dH}{dt} < 0$ the toy soldier was decreasing in height after 2 seconds.	B1 ft*
		(2 marks)
11c	$\frac{dH}{dt} = \frac{8(1 - 2t^2)}{3\sqrt[3]{t}(t^2 + 1)^2} = 0$ or $8 - 16t^2 = 0$ at a turning point.	M1 ft
	Solves $8 - 16t^2 = 0$ to find $t = \frac{1}{\sqrt{2}}$ Can also state $t \neq -\frac{1}{\sqrt{2}}$	A1 ft
		(2 marks)
	TOTAL: 8 marks	

NOTES:

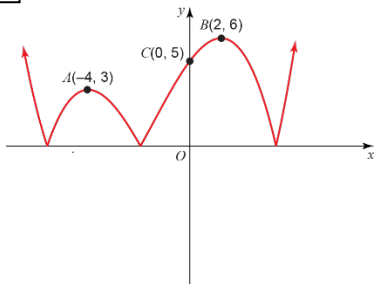
11b: Award ft marks for a correct answer using an incorrect answer from part a.

B1: Can also state $\frac{dH}{dt} < 0$ as the numerator of $\frac{dH}{dt}$ is negative and the denominator is positive.

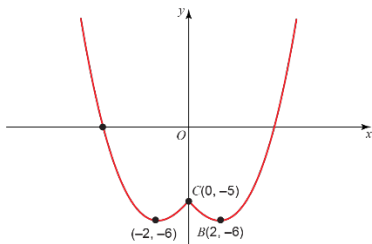
11c: Award ft marks for a correct answer using an incorrect answer from part a.

12a	Clear attempt to reflect the negative part of the original graph in the x -axis.	M1
	Labels all three points correctly.	A1
	Fully correct graph. (see below)	A1
		(3 marks)
12b	Clear attempt to reflect the positive x part of the original graph in the y -axis.	M1
	Labels all three points correctly.	A1
	Fully correct graph. (see below)	A1
		(3 marks)
12c	Clear attempt to move the graph to the left 3 spaces.	
	Clear attempt to stretch the graph vertically by a factor of 2.	
	Fully correct graph. (see below)	
		(3 marks)
TOTAL: 9 marks		

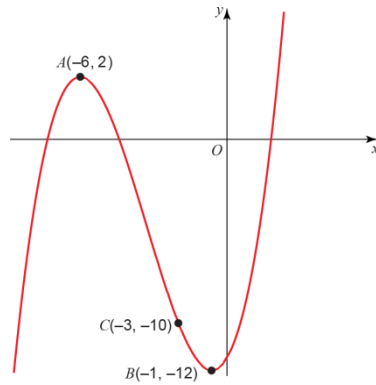
12a



12b



12c



13a	Recognises that it is a geometric series with a first term $a = 100$ and common ratio $r = 1.05$	M1
	Attempts to use the sum of a geometric series. For example, $S_9 = \frac{100(1-1.05^9)}{1-1.05}$ or $S_9 = \frac{100(1.05^9-1)}{1.05-1}$ is seen.	M1*
	Finds $S_9 = £1102.66$	A1
		(3 marks)
13b	States $\frac{100(1.05^n-1)}{1.05-1} > 6000$ or $\frac{100(1-1.05^n)}{1-1.05} > 6000$	M1
	Begins to simplify. $1.05^n > 4$ or $-1.05^n < -4$	M1
	Applies law of logarithms correctly $n \log 1.05 > \log 4$ or $-n \log 1.05 < -\log 4$	M1
	States $n > \frac{\log 4}{\log 1.05}$	A1
		(4 marks)
13c	Uses the sum of an arithmetic series to state $\frac{29}{2}[100 + (28)d] = 6000$	M1
	Solves for d . $d = £11.21$	A1
		(2 marks)
	TOTAL: 9 marks	

NOTES: 13a:

M1 Award mark if attempt to calculate the amount of money after 1, 2, 3, ..., 8 and 9 months is seen.

14a	Writes $(a+bx)^{\frac{1}{3}}$ as $a^{\frac{1}{3}}\left(1+\frac{b}{a}x\right)^{\frac{1}{3}}$	M1
	Expands $\left(1+\frac{b}{a}x\right)^{\frac{1}{3}}\left(1+\frac{b}{a}x\right)^{\frac{1}{3}} = 1 + \left(\frac{1}{3}\right)\left(\frac{b}{a}\right)x + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{b}{a}\right)^2 x^2}{2} + \dots$	M1
	<p>Simplifies:</p> $a^{\frac{1}{3}}\left(1 + \left(\frac{1}{3}\right)\left(\frac{b}{a}\right)x + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{b}{a}\right)^2 x^2}{2} + \dots\right)$ $= a^{\frac{1}{3}}\left(1 + \frac{b}{3a}x - \frac{b^2}{9a^2}x^2 \dots\right)$ $= a^{\frac{1}{3}} + \frac{b}{3a^{\frac{2}{3}}}x - \frac{b^2}{9a^{\frac{5}{3}}}x^2 \dots$ <p>Award mark even if x^2 term is not seen.</p>	M1
	Uses $a^{\frac{1}{3}} = 4$ to write $a = 64$.	A1
	Uses $\frac{b}{3a^{\frac{2}{3}}} = -\frac{1}{8}$ to write $b = -6$.	A1
		(5 marks)
14b	States expansion is valid for $\left -\frac{6}{64}x\right < 1$	B1 ft
	Solves to state $-\frac{32}{3} < x < \frac{32}{3}$	A1 ft
		(2 marks)
14c	Substitutes $a = 64$ and $b = -6$ into $-\frac{b^2}{9a^{\frac{5}{3}}}$	M1 ft
	Finds $c = -\frac{1}{256}$	A1 ft
		(2 marks)
	TOTAL: 9 marks	

NOTES:

14a: Note x^2 term is not necessary to answer part **a**, so is not required. Will be needed to answer part **c**.

14b: Award marks for a correct conclusion using incorrect values of a and b from part **a**.

14c: Award marks for a correct answer using incorrect values of a and b from part **a**.

15a	States $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$	M1
	Deduces that $V = \pi r^2 h = 1600\pi h$	M1
	Finds $\frac{dV}{dh} = 1600\pi$ and/or $\frac{dh}{dV} = \frac{1}{1600\pi}$	M1
	States $\frac{dV}{dt} = 4000\pi - 50\pi h$	M1
	Makes an attempt to find $\frac{dh}{dt} = (4000\pi - 50\pi h) \times \frac{1}{1600\pi}$	M1
	Shows a clear logical progression to state $160 \frac{dh}{dt} = 400 - 5h$	A1
		(6 marks)
15b	Separates the variables $\int \left(\frac{1}{400 - 5h} \right) dh = \int \frac{1}{160} dt$	M1
	Finds $-\frac{1}{5} \ln(400 - 5h) = \frac{t}{160} + C$	A1
	Uses the fact that $t = 0$ when $h = 50$ m to find C : $C = -\frac{1}{5} \ln(150)$	M1
	Substitutes $h = 60$ into the equation: $-\frac{1}{5} \ln(400 - 300) = \frac{t}{160} - \frac{1}{5} \ln(150)$	M1
	Uses law of logarithms to write: $\frac{1}{5} \ln(150) - \frac{1}{5} \ln(100) = \frac{t}{160}$ $\Rightarrow \frac{1}{5} \ln\left(\frac{150}{100}\right) = \frac{t}{160}$	M1
	States correct final answer: $t = 32 \ln\left(\frac{3}{2}\right)$ minutes.	A1
		(6 marks)
	TOTAL: 12 marks	

(TOTAL: 100 MARKS)