NAME:

Date to be handed in:

MARK (out of 100):

PAPER N

Qu	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Pure Mathematics

A Level: Practice Paper

Time: 2 hours

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Questions to revise:

1 In a rainforest, the area covered by trees, F, has been measured every year since 1990.
It was found that the rate of loss of trees is proportional to the remaining area covered by trees.

Write down a differential equation relating F to t, where t is the numbers of years since 1990. **(2 marks)**

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2 Find the angle that the vector $\mathbf{a} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ makes with the positive y-axis. **(3 marks)**

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3 Use proof by contradiction to show that there exist no integers a and b for which $25a + 15b = 1$ **(4 marks)**

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4 Find $\int \cos^2 6x \, dx$ **(5 marks)**

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5 Use proof by contradiction to prove the statement: 'The product of two odd numbers is odd.' **(5 marks)**

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6 $f(x) = \dfrac{4x^2 + x - 23}{(x-3)(4-x)(x+5)}, \quad x > 4$

Given that $f(x)$ can be expressed in the form $\dfrac{A}{x-3} + \dfrac{B}{4-x} + \dfrac{C}{x+5}$ find the values of A, B and C.

(6 marks)

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7 A triangle has vertices $A(-2, 0, -4)$, $B(-2, 4, -6)$ and $C(3, 4, 4)$.

By considering the side lengths of the triangle, show that the triangle is a right-angled triangle.

(6 marks)

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8 A large arch is planned for a football stadium.
The parametric equations of the arch are $x = 8(t + 10), \; y = 100 - t^2, \; -10 \leq t \leq 10$
where x and y are distances in metres.

a Find the cartesian equation of the arch. **(3 marks)**

b Find the width of the arch. **(2 marks)**

c Find the greatest possible height of the arch. **(2 marks)**

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9 $f(x) = 2 - 3\sin^3 x - \cos x$, where x is in radians.

a Show that $f(x) = 0$ has a root α between $x = 1.9$ and $x = 2.0$. **(2 marks)**

b Using $x_0 = 1.95$ as a first approximation, apply the Newton–Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places. **(5 marks)**

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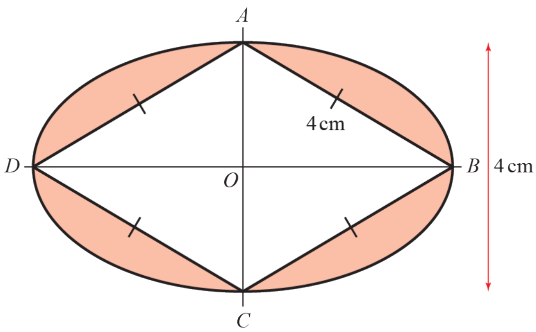
10 The diagram shows a logo comprised of a rhombus surrounded by two arcs.
Arc BAD has centre C and arc BCD has centre A.
Some of the dimensions of the logo are shown in the diagram.



Prove that the shaded area of the logo is $\frac{2}{3}\left(16\pi - 24\sqrt{3}\right)$ **(8 marks)**

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11 A toy soldier is connected to a parachute. The soldier is thrown into the air from ground level.
The height, in metres, of the soldier above the ground can be modelled by the equation

$H = \dfrac{4t^{\frac{2}{3}}}{t^2+1}$, 0 Ñ t Ñ 6 s, where H is height of the soldier above the ground and t is the time since the soldier was thrown.

a Show that $\dfrac{dH}{dt} = \dfrac{8\left(1-2t^2\right)}{3\sqrt[3]{t}\left(t^2+1\right)^2}$ **(4 marks)**

b Using the differentiated function, explain whether the soldier was increasing or decreasing in height after 2 seconds. **(2 marks)**

c Find the exact time when the soldier reaches a maximum height. **(2 marks)**

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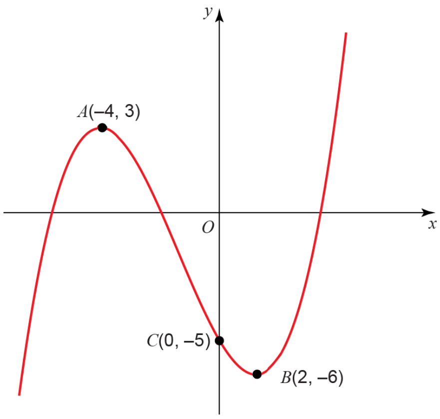
12 The diagram shows the graph of h(x).



The points $A(-4, 3)$ and $B(2, -6)$ are turning points on the graph and $C(0, -5)$ is the y-intercept.

Sketch on separate diagrams, the graphs of

a $y = |f(x)|$ **(3 marks)**

b $y = f(|x|)$ **(3 marks)**

c $y = 2f(x + 3)$ **(3 marks)**

Where possible, label clearly the transformations of the points A, B and C on your new diagrams and give their coordinates.

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13 At the beginning of each month Kath places £100 into a bank account to save for a family holiday. Each subsequent month she increases her payments by 5%.

a Assuming the bank account does not pay interest, find the amount of money in the account after 9 months. **(3 marks)**

Month n is the first month in which there is more than £6000 in the account.

b Show that $n > \dfrac{\log 4}{\log 1.05}$ **(4 marks)**

Maggie begins saving at the same time as Kath.
She initially places £50 into the same account and plans to increase her payments by a constant amount each month.

c Given that she would like to reach a total of £6000 in 29 months, by how much should Maggie increase her payments each month?

(2 marks)

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14 The first three terms in the binomial expansion of $(a+bx)^{\frac{1}{3}}$ are $4 - \frac{1}{8}x + cx^2 + \ldots$

a Find the values of a and b. **(5 marks)**

b State the range of values of x for which the expansion is valid. **(2 marks)**

c Find the value of c. **(2 marks)**

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15 A large cylindrical tank has radius 40 m.
Water flows into the cylinder from a pipe at a rate of 4000π m^3 min^{-1}.

At time t, the depth of water in the tank is h m.
Water leaves the bottom of the tank through another pipe at a rate of $50\pi h$ m^3 min^{-1}.

a Show that t minutes after water begins to flow out of the bottom of the cylinder,

$$160\frac{\mathrm{d}h}{\mathrm{d}t} = 400 - 5h$$ **(6 marks)**

b When $t = 0$ min, $h = 50$ m.
Find the exact value of t when $h = 60$ m. **(6 marks)**

(TOTAL: 100 MARKS)