Pearson Edexcel
Level 3 Advanced Subsidiary GCE in Mathematics (8MA0)

Pearson Edexcel
Level 3 Advanced GCE in Mathematics (9MA0)

Sample Assessment Materials Model Answers – Pure Mathematics

First teaching from September 2017
First certification from June 2018

Sample Assessment Materials Model Answers – Pure Mathematics

Contents

Introduction

This booklet has been produced to support mathematics teachers delivering the new Pearson Edexcel Level 3 Advanced Subsidiary and Advanced GCE in Mathematics (8MAO and 9MAO) specifications for first teaching from September 2017.

This booklet looks at Sample Assessment Materials for AS and A level Mathematics qualifications, specifically at pure mathematics questions, and is intended to offer model solutions with different methods explored.

Content of Pure Mathematics

Content	AS level content	A level content
Proof	Proof by deduction Proof by exhaustion Disproof by counterexample.	Proof by contradiction
Algebra and functions	Algebraic expressions – basic algebraic manipulation, indices and surds Quadratic functions – factorising, solving, graphs and the discriminants Equations – quadratic/linear simultaneous Inequalities – linear and quadratic (including graphical solutions) Algebraic division, factor theorem and proof Graphs – cubic, quartic and reciprocal Transformations – transforming graphs – f(x) notation	Simplifying algebraic fractions Partial fractions Modulus function Composite and inverse functions Transformations Modelling with functions
Coordinate geometry in the (x, y) plane	Straight-line graphs, parallel/perpendicular, length and area problems Circles – equation of a circle, geometric problems on a grid	Definition and converting between parametric and Cartesian forms Curve sketching and modelling
Series and sequences	The binomial expansion	Arithmetic and geometric progressions (proofs of ‘sum formulae’) Sigma notation Recurrence and iterations
Trigonometry	Trigonometric ratios and graphs Trigonometric identities and equations	Radians (exact values), arcs and sectors Small angles Secant, cosecant and cotangent (definitions, identities and graphs); Inverse trigonometrical functions; Inverse trigonometrical functions Compound and double (and half) angle formulae

AS level

Content	AS level content	A level content
		$R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$ Proving trigonometric identities Solving problems in context (e.g. mechanics)
Exponentials and logarithms	Exponential functions and natural logarithms	
Differentiation	Definition, differentiating polynomials, second derivatives Gradients, tangents, normals, maxima and minima	Differentiating sin x and cos x from first principles Differentiating exponentials and logarithms Differentiating products, quotients, implicit and parametric functions. Second derivatives (rates of change of gradient, inflections) Rates of change problems (including growth and kinematics)
Integration	Definition as opposite of differentiation, indefinite integrals of x^n Definite integrals and areas under curves	Integrating x^n (including when $n = -1$), exponentials and trigonometric functions Using the reverse of differentiation, and using trigonometric identities to manipulate integrals Integration by substitution Integration by parts Use of partial fractions Areas under graphs or between two curves, including understanding the area is the limit of a sum (using sigma notation) The trapezium rule Differential equations (including knowledge of the family of solution curves)
Vectors	**(2D)** Definitions, magnitude/direction, addition and scalar multiplication Position vectors, distance between two points, geometric problems	**(3D)** Use of vectors in three dimensions; knowledge of column vectors and **i**, **j** and **k** unit vectors
Numerical methods		Location of roots Solving by iterative methods (knowledge of 'staircase and cobweb' diagrams) Newton-Raphson method Problem solving

AS Level

Question 1

The line l passes through the points A (3, 1) and B (4, – 2).

Find an equation for l.

(3)

Gradient	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	Gradient = $\dfrac{\text{Change in } y}{\text{Change in } x}$	
	$m = \dfrac{-2-1}{4-3} = -3$		M1
Using	$y - y_1 = m(x - x_1)$	Equation of a line, gradient m through a known point (x_1, y_1)	
		Or	
	$y - 1 = -3(x - 3)$	$y + 2 = -3(x - 4)$	A1
	$y - 1 = -3x + 9$	$y + 2 = -3x + 12$	
	$y = -3x + 10$	$y = -3x + 10$	A1
Alternatives	$y = mx + c$		
	$y = -3x + c$		M1
	$1 = -3 \times 3 + c$, at (3, 1)		
	$c = 10$		A1
	$y = -3x + 10$		A1
	$\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$		
	$\dfrac{y-1}{-2-1} = \dfrac{x-3}{4-3}$		M1
	$y - 1 = -3(x - 3)$		A1
	$y = -3x + 10$		A1

Question 2

The curve C has equation

$$y = 2x^2 - 12x + 16.$$

Find the gradient of the curve at the point $P(5, 6)$.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

	$y = 2x^2 - 12x + 16$	
Differentiate	$\frac{dy}{dx} = 4x - 12$	M1 A1
When $x = 5$ at P	$\frac{dy}{dx} = 4 \times 5 - 12$	M1
	$= 8$	A1

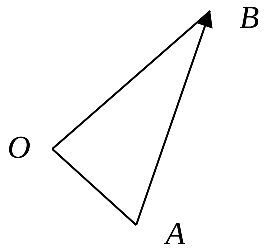
AS level

Question 3

Given that the point A has position vector $3\mathbf{i} - 7\mathbf{j}$ and the point B has position vector $8\mathbf{i} + 3\mathbf{j}$,

(a) find the vector $\overrightarrow{AB}$.

(2)



$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 8\mathbf{i} + 3\mathbf{j} - (3\mathbf{i} - 7\mathbf{j})$ M1

$= 5\mathbf{i} + 10\mathbf{j}$ A1

(b) Find $|\overrightarrow{AB}|$. Give your answer as a simplified surd.

(2)

$|\overrightarrow{AB}| = \sqrt{5^2 + 10^2}$ M1

$|\overrightarrow{AB}| = \sqrt{125}$

$|\overrightarrow{AB}| = 5\sqrt{5}$ A1

Question 4

$$f(x) = 4x^3 - 12x^2 + 2x - 6$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

$f(x) = 4x^3 - 12x^2 + 2x - 6$	If $(x - 3)$ is a factor of $f(x)$ then $f(3) = 0$	M1
$f(3) = 4\times3^3 - 12\times3^2 + 2\times3 - 6$		
$f(3) = 0$ as required		A1

(b) Hence show that 3 is the only real root of the equation $f(x) = 0$.

(4)

$f(x) = 4x^3 - 12x^2 + 2x - 6$

$f(x) = (x - 3)(4x^2 + 2)$ M1

A1

$f(x) = 2(x - 3)(2x^2 + 1)$

For $f(x) = 0$

Require $x = 3$ or $x^2 = -\frac{1}{2}$ M1

Since $x \neq \sqrt{-\frac{1}{2}}$

There is only one root $x = 3$ A1

Or

For $x - 3 = 0$ $x = 3$ one real root	For $ax^2 + bx + c = 0$	M1
For $2x^2 + 1 = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
$a = 2$, $b = 0$, $c = 1$		
So, '$b^2 - 4ac$' $= 0^2 - 4 \times 2 \times 1 = -8$	Has real roots when $b^2 - 4ac \geq 0$	
Therefore no real roots		A1

Question 5

Given that $f(x) = 2x + 3 + \frac{12}{x^2}, \quad x > 0,$

show that $\int_{2}^{2\sqrt{2}} f(x)\,dx = 16 + 3\sqrt{2}$

(5)

$f(x) = 2x + 3 + 12x^{-2}$ B1

Integrate $\int_{2}^{2\sqrt{2}} \left(2x + 3 + 12x^{-2}\right)dx = \left[\frac{2x^2}{2} + 3x + \frac{12x^{-1}}{-1}\right]_1^{2\sqrt{2}}$ M1

$= \left[x^2 + 3x - \frac{12}{x}\right]_1^{2\sqrt{2}}$ A1

Top limit – bottom limit

$\left(\left(2\sqrt{2}\right)^2 + 3\times 2\sqrt{2} - \frac{12}{2\sqrt{2}}\right) - \left(1 + 3 - 12\right)$

$\left(8 + 6\sqrt{2} - \frac{6}{\sqrt{2}}\right) - (-8)$

Rationalise

$\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

$= \left(8 + 6\sqrt{2} - 3\sqrt{2}\right) + 8$ M1

$= 16 + 3\sqrt{2}$ A1

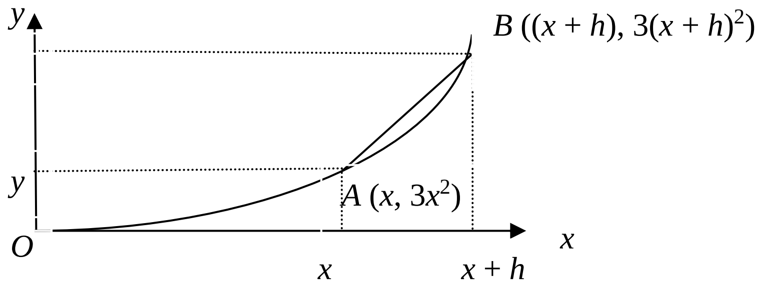
Question 6

Prove, from first principles, that the derivative of $3x^2$ is $6x$.

(4)

Consider the graph of $y = 3x^2$, and the chord AB

h or δx is a small increment in x



Gradient of the chord AB, $\frac{\delta y}{\delta x} \to \frac{3(x+h)^2 - 3x^2}{x+h-x}$ or $\frac{3(x+\delta x)^2 - 3x^2}{x+\delta x - x}$ B1

$= \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$

$= \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$ M1

$= \frac{6xh + 3h^2}{h}$

$= 6x + 3h$ or $6x + 3\delta x$ A1

As $h \to 0$, or as $\delta x \to 0$, $\frac{\delta y}{\delta x} \to \frac{dy}{dx}$ $= 6x$ A1

Pearson

AS level

Question 7

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2-\frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

From the formula Booklet

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n \quad (n \in \mathbb{N})$$

Where $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

$$\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6\left(-\frac{x}{2}\right) + \binom{7}{2}2^5\left(-\frac{x}{2}\right)^2 + \cdots$$ M1

$= 128$ B1

$-224x$ A1

$+168x^2 + \ldots$ A1

Alternative

(From the A level section of the formula Booklet)

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x| < 1,\ n \in \mathbb{R})$$

$$\left(2-\frac{x}{2}\right)^7 = \left(2\left(1-\frac{x}{4}\right)\right)^7$$

$$= 2^7\left[1 + 7\left(-\frac{x}{4}\right) + \frac{7\times 6\left(-\frac{x}{4}\right)^2}{2!} + \ldots\right]$$ M1

$$= 128\left(1 + 7\left(-\frac{x}{4}\right) + \frac{7\times 6\left(-\frac{x}{4}\right)^2}{2!} + \cdots\right)$$

$= 128$ B1

$-224x$ A1

$+168x^2 + \ldots$ A1

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7.

(1)

Require $2 - \frac{x}{2} = 1.995$

$\frac{x}{2} = 0.005$

$x = 0.01$ B1

Therefore substituting $x = 0.01$ into $128 - 224x + 168x^2$ would give an approximation for 1.995^7

Question 8

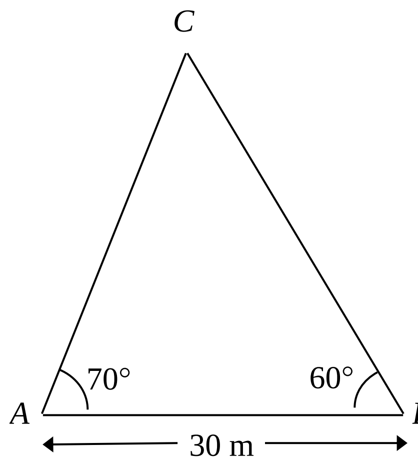


Figure 1

A triangular lawn is modelled by the triangle *ABC*, shown in Figure 1. The length *AB* is to be 30 m long.

Given that angle *BAC* = 70° and angle *ABC* = 60°,

(a) calculate the area of the lawn to 3 significant figures.

(4)

	Angle *ACB* = 50°	Angle *BCA* = 50°	
Sine Rule	$\frac{AC}{\sin 60} = \frac{30}{\sin 50}$	$\frac{BC}{\sin 70} = \frac{30}{\sin 50}$	M1
	$AC = \frac{30 \sin 60}{\sin 50}$	$BC = \frac{30 \sin 70}{\sin 50}$	
	AC = 33.915	*BC* = 36.800	A1
Area = $\frac{1}{2}ab\sin C$	Area = 0.5 × 33.915 × 30 × sin 70	Area = 0.5 × 36.800 × 30 × sin 60	M1
	= 478 m^2	= 478 m^2	A1

(b) Why is your answer unlikely to be accurate to the nearest square metre?

(1)

Plausible reason e.g.	Angles and sides are not given to 4 s.f. Lawn may not be flat.	B1

AS level

Question 9

Solve, for $360° \leq x < 540°$,

$$12 \sin^2 x + 7 \cos x - 13 = 0.$$

Give your answers to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

$12\sin^2 x + 7\cos x - 13 = 0$

Using $\sin^2 x + \cos^2 x = 1$

$12(1 - \cos^2 x) + 7\cos x - 13 = 0$ — M1

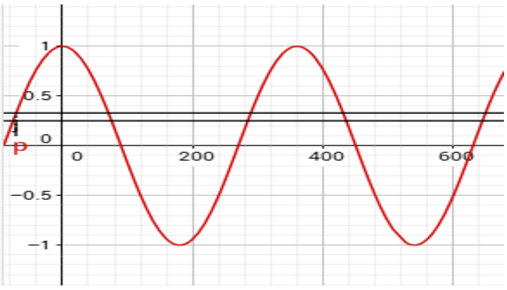
$12 - 12\cos^2 x + 7\cos x - 13 = 0$

$12\cos^2 x - 7\cos x + 1 = 0$ — A1

Factorising $(4\cos x - 1)(3\cos x - 1) = 0$ or quadratic formula

$\cos x = {}^1/_3$ or ¼ — M1

$x = 70.5°$ and $75.5°$ using a calculator — M1



Using the diagram, or otherwise, solutions in the range $360° \leq x \leq 540°$ are — A1

$360° + 70.5°$ and $360° + 75.5°$

$430.5°$ and $435.5°$

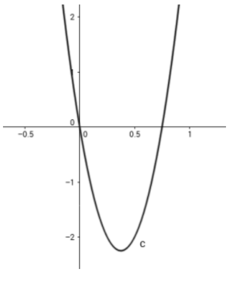
AS level

Question 10

The equation $kx^2 + 4kx + 3 = 0$, where k is a constant, has no real roots.

Prove that $0 \leq k < \dfrac{3}{4}$.

(4)

	$kx^2 + 4kx + 3 = 0$		
	For no real roots, $b^2 - 4ac < 0$		
	$a = k,\ b = 4k,\ c = 3$		
Therefore	$(4k)^2 - 4 \times k \times 3 < 0$		M1
	$16k^2 - 12k < 0$		
	$4k(4k - 3) < 0$		
Critical values	$k = 0$ and $k = \dfrac{3}{4}$	Sketch graph of y against k	M1
	Curve below the axis		
	$0 < k < \dfrac{3}{4}$		
	However, in the equation $kx^2 + 4kx + 3 = 0$, $k = 0$ requires $3 = 0$		
	So, $k = 0$ is not a possible value		B1
	$0 \leq k < \dfrac{3}{4}$, as required		A1

AS level

Question 11

(a) Prove that for all positive values of x and y,

$$\sqrt{xy} \leq \frac{x+y}{2}.$$

(2)

Way 1 Consider	$\left(\sqrt{x}-\sqrt{y}\right)^2 \geq 0$	Since x and y are positive, their square roots are real and any squared term is positive	
Expanding	$x - 2\sqrt{xy} + y \geq 0$		M1
	$x + y \geq 2\sqrt{xy}$		
Hence	$\sqrt{xy} \leq \dfrac{x+y}{2}$	As required.	A1
Way 2	$(x-y)^2 \geq 0$		
	$x^2 - 2xy + y^2 \geq 0$		
	$x^2 + 2xy + y^2 \geq 4xy$	Add $4xy$ to each side	M1
	$(x+y)^2 \geq 4xy$		
	$x + y \geq 2\sqrt{xy}$		
Hence	$\sqrt{xy} \leq \dfrac{x+y}{2}$	As required	A1

Way 2 can be derived from a diagram

Area of square > sum of rectangles

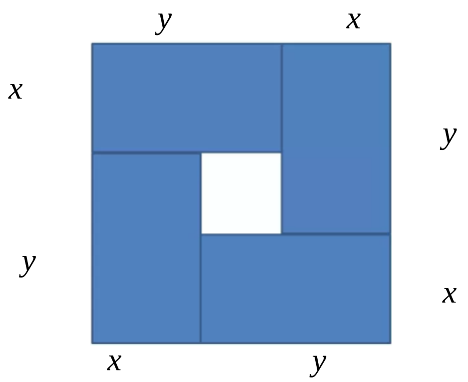
$(x+y)^2 > xy + xy + xy + xy$

$(x+y)^2 > 4xy$

Then as way 2

The square at the centre has area $(x-y)^2$

Leading to equality of the inequality when $x = y$



M1

A1

(b) Prove by counterexample that this is not true when x and y are both negative.

(1)

For any two negative values e.g. $x = -8$, $y = -2$

$\sqrt{16} = 4$

$\frac{-8+-2}{2} = -5$

Since 4 is not smaller or equal to –5 the result is not true for x and y both negative. B1

Question 12

A student was asked to give the exact solution to the equation $2^{2x+4} - 9(2^x) = 0$.

The student's attempt is shown below:

$$2^{2x+4} - 9(2^x) = 0$$
$$2^{2x} + 2^4 - 9(2^x) = 0$$
$$\text{Let } 2^x = y$$
$$y^2 - 9y + 8 = 0$$
$$(y - 8)(y - 1) = 0$$
$$y = 8 \text{ or } y = 1$$
$$\text{So } x = 3 \text{ or } x = 0$$

(a) Identify the two errors made by the student.

(2)

Error 1, line 2	$2^{2x+4} - 9(2^x) = 0$ $2^{2x}\,2^4 - 9(2^x) = 0$	The 1st error is writing $2^{2x+4} = 2^{2x} + 2^4$, instead of writing $2^{2x+4} = 2^{2x} \times 2^4$	B1
Error 2, line 4	$2^4 \neq 8$, $2^4 = 16$	The 2nd error is writing $2^4 = 8$ and using this as an added constant rather than $2^4 = 16$ used as a multiplying coefficient.	B1

(b) Find the exact solution to the equation.

(2)

Way 1 $16(2^{2x}) - 9(2^x) = 0$

$16(2^x)^2 - 9(2^x) = 0$

Let $y = 2^x$ $16y^2 - 9y = 0$ M1

$y(16y - 9) = 0$

$y = 0$ or $y = \frac{9}{16}$

Hence $2^x = \frac{9}{16}$ $2^x = 0$ has no solutions

$\log_2 2^x = \log_2 \frac{9}{16}$ $x = \log_2 \frac{9}{16}$ exact solution A1

$x = \log_2 9 - \log_2 16$

$x = \log_2 9 - 4$ (equivalent exact solution)

Way 2 $2^{2x+4} - 9(2^x) = 0$

$2^{2x+4} = 9(2^x)$

$\log 2^{2x+4} = \log 9(2^x)$

$(2x+4)\log 2 = \log 9 + \log(2^x)$

$(2x+4)\log 2 = \log 9 + x\log 2$ M1

$2x\log 2 + 4\log 2 = \log 9 + x\log 2$

$2x\log 2 - x\log 2 = \log 9 - 4\log 2$

$x = \frac{\log 9}{\log 2} - 4$ exact solution A1

Question 13

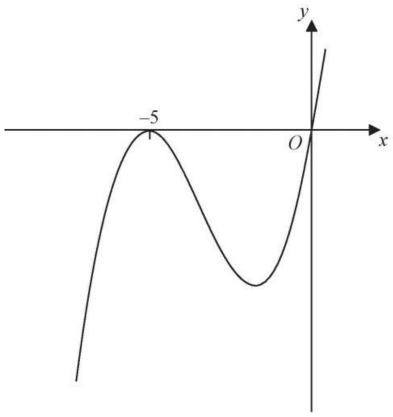
(a) Factorise completely $x^3 + 10x^2 + 25x$

(2)

$x^3 + 10x^2 + 25x$	
$x(x^2 + 10x + 25)$	M1
$x(x + 5)(x + 5)$ or $x(x + 5)^2$	A1

(b) Sketch the curve with equation $y = x^3 + 10x^2 + 25x$, showing the coordinates of the points at which the curve cuts or touches the x-axis.

(2)



A cubic with correct orientation	M1
Curve passes through the origin (0, 0) and touches at (–5, 0)	A1

The point with coordinates (–3, 0) lies on the curve with equation

$$y = (x + a)^3 + 10(x + a)^2 + 25(x + a), \text{ where } a \text{ is a constant.}$$

(c) Find the two possible values of a.

(3)

$(x + a)$ translates a units to the left	M1
To translate –5, to –3, require $a = -2$	A1
To translate 0 to –3, require $a = 3$	A1

Question 14

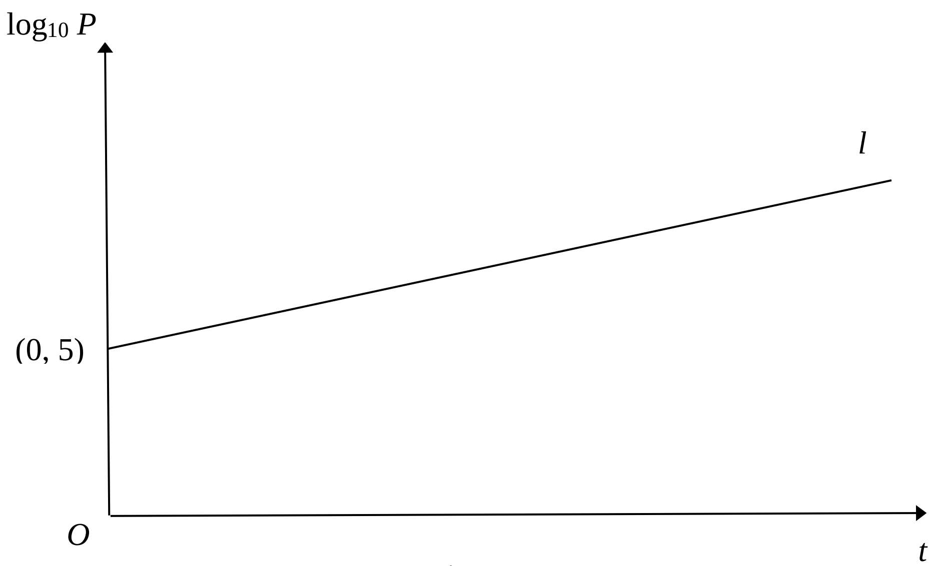


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded. The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

Using $y = mx + c$

$\log_{10} P = mt + c$ — M1

$m = \frac{1}{200}$, $c = 5$

$\log_{10} P = \frac{1}{200}t + 5$ — A1

(b) Find the value of *a* and the value of *b*.

(4)

$P = ab^t$

$\log_{10} P = \log_{10} ab^t$

$\log_{10} P = \log_{10} a + \log_{10} b^t$

$\log_{10} P = \log_{10} a + t\log_{10} b$ M1

A graph of $\log_{10} P$ against t will be a straight line , gradient $\log_{10} b$, intercept $\log_{10} a$

From part (a)

$\log_{10} a = 5$ (Intercepts) M1

$a = 10^5$

$a = 100000$ A1

(gradients)

$\log_{10} b = \frac{1}{200}$

$b = 10^{\frac{1}{200}}$

$b = 1.01$ A1

(c) With reference to the model, interpret

(i) the value of the constant *a*,

(ii) the value of the constant *b*.

(2)

(i)	The initial population when $t = 0$	B1
(ii)	The rate of increase of the population each year.	B1

(d) Find

(i) the population predicted by the model when t = 100, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(i)

$$P = 100000 \times 10^{\frac{t}{200}}$$

When t = 100

$$P = 100000 \times 10^{\frac{100}{200}}$$

$P = 300000$ to the nearest one hundred thousand. B1

(ii)

$$200000 = 100000 \times 10^{\frac{t}{200}}$$

$$2 = 10^{\frac{t}{200}}$$

$$\log_{10} 2 = \frac{t}{200}$$

$t = 200 \times \log_{10} 2$ M1

$t = 60.2$ years A1

(e) State two reasons why this may not be a realistic population model.

(2)

- 100 years is a long time and population may be affected by wars and disease
- Inaccuracies in measuring gradient may result in widely different estimates
- Population growth may not be proportional to population size
- The model predicts unlimited growth

B2

Question 15

Diagram not drawn to scale

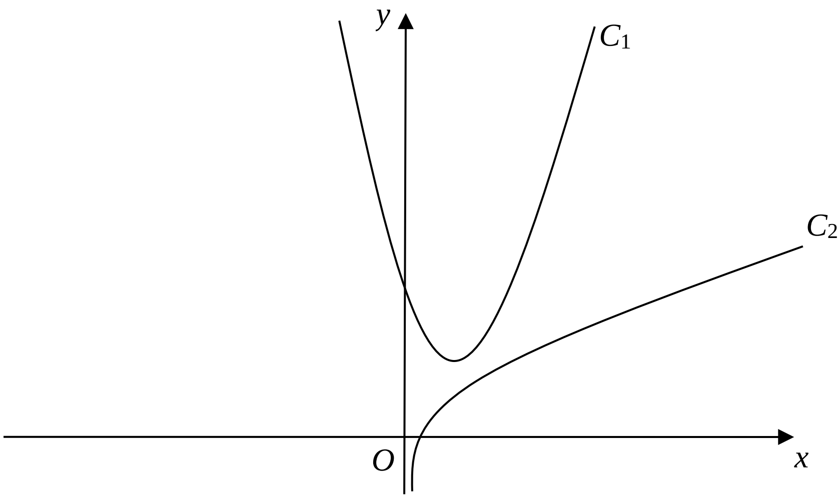


Figure 3

The curve C_1, shown in Figure 3, has equation $y = 4x^2 - 6x + 4$.

The point $P\left(\frac{1}{2}, 2\right)$ lies on C_1.

The curve C_2, also shown in Figure 3, has equation $y = \frac{1}{2}x + \ln(2x)$. The normal to C_1 at the point P meets C_2 at the point Q.

Find the exact coordinates of Q.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(8)

$$y = 4x^2 - 6x + 4$$

$$\frac{dy}{dx} = 8x - 6$$ M1

At P, $x = \frac{1}{2}$

$$\frac{dy}{dx} = 8 \times \frac{1}{2} - 6 = -2$$ M1

	For two lines with gradients m_1 and m_2 perpendicular to each other: $$m_1 m_2 = -1$$ Therefore, the normal at P has gradient ½		M1
At $P(\frac{1}{2}, 2)$	Equation of a line, gradient m through a known point (x_1, y_1) $$y - y_1 = m(x - x_1)$$ $$y - 2 = \frac{1}{2}(x - \frac{1}{2})$$ $$y - 2 = \frac{1}{2}x - \frac{1}{4}$$	Or equivalent	
	$$y = \frac{1}{2}x + \frac{7}{4}$$		A1
At Q	The normal to $C1$ meets $C2$, therefore the y-values are equal: $$\frac{1}{2}x + \frac{7}{4} = \frac{1}{2}x + \ln 2x$$		M1
	$$\ln 2x = \frac{7}{4}$$ $$2x = e^{\frac{7}{4}}$$		M1
	$$x = \frac{1}{2}e^{\frac{7}{4}}$$		
When $x = \frac{1}{2}e^{\frac{7}{4}}$	$$y = \frac{1}{2} \times \frac{1}{2}e^{\frac{7}{4}} + \ln 2 \times \frac{1}{2}e^{\frac{7}{4}}$$ $$y = \frac{1}{4}e^{\frac{7}{4}} + \ln e^{\frac{7}{4}}$$ $$y = \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4}$$		M1
Hence	$$Q(\frac{1}{2}e^{\frac{7}{4}}, \frac{1}{4}e^{\frac{7}{4}} + \frac{7}{4})$$	Exact Coordinates	A1

Question 16

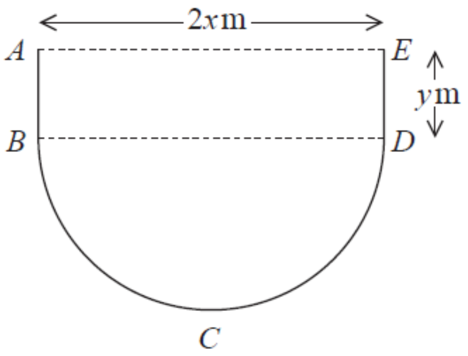


Figure 4

Figure 4 shows the plan view of the design for a swimming pool.

The shape of this pool *ABCDEA* consists of a rectangular section *ABDE* joined to a semicircular section *BCD* as shown in Figure 4.

Given that *AE* = 2*x* metres, *ED* = *y* metres and the area of the pool is 250 m^2,

(a) show that the perimeter, *P* metres, of the pool is given by

$$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$$

(4)

	Length of arc *BCD* = $\frac{\pi \times 2x}{2} = \pi x$	Using $\left(\frac{\pi \times \text{Diameter}}{2}\right)$	
	Perimeter *ABCDE*, $P = 2x + 2y + \pi x$		
Area *ABCDE*	$250 = 2xy + \frac{\pi x^2}{2}$	Area = rectangle + semi-circle	B1
	$y = \frac{250}{2x} - \frac{\pi x}{4}$	Making *y* the subject	M1
Substitute for *y*	$P = 2x + 2(\frac{250}{2x} - \frac{\pi x}{4}) + \pi x$		M1
	$P = 2x + \frac{250}{x} - \frac{\pi x}{2} + \pi x$		
	$P = 2x + \frac{250}{x} + \frac{\pi x}{2}$	As required.	A1

(b) Explain why $0 < x < \sqrt{\frac{500}{\pi}}$ **(2)**

For the shape given, we require $x > 0$ and $y > 0$

$$\frac{250}{2x} - \frac{\pi x}{4} > 0$$ M1

$$\frac{250}{2x} > \frac{\pi x}{4}$$

$$500 > \pi x^2$$

$$\frac{500}{\pi} > x^2$$

Since x is positive A1

$$0 < x < \sqrt{\frac{500}{\pi}}$$ As required.

(c) Find the minimum perimeter of the pool, giving your answer to 3 significant figures. **(4)**

For the minimum perimeter, require $\frac{dP}{dx} = 0$

$$P = 2x + 250x^{-1} + \frac{\pi x}{2}$$

$$\frac{dP}{dx} = 2 - \frac{250}{x^2} + \frac{\pi}{2}$$ M1 A1

For min $\frac{dP}{dx} = 0$

$$2 - \frac{250}{x^2} + \frac{\pi}{2} = 0$$

$$2 + \frac{\pi}{2} = \frac{250}{x^2}$$

$$\frac{4+\pi}{500} = \frac{1}{x^2}$$

$$x^2 = \frac{500}{4+\pi}$$

$$x = \sqrt{\frac{500}{4+\pi}} = 8.36734...$$ M1

When $x = \sqrt{\frac{500}{4+\pi}}$

$$P = 2\sqrt{\frac{500}{4+\pi}} + \frac{250}{\sqrt{\frac{500}{4+\pi}}} + \frac{\pi\sqrt{\frac{500}{4+\pi}}}{2}$$

Or
$x = 8.36734\ldots$ $P = 2\times 8.36734... + \dfrac{250}{8.36734...} + \dfrac{\pi \times 8.36734...}{2}$

$P = 59.75614\ldots$ A1
$P = 59.8$ m to 3 s.f. (including units)

Question 17

A circle C with centre at (- 2, 6) passes through the point (10, 11).

(a) Show that the circle C also passes through the point (10, 1).

(3)

	$(x-a)^2+(y-b)^2=r^2$	The equation of a circle, radius r, centre (a, b)	
Point (10,11) is on the circle.	$(x--2)^2+(y-6)^2=r^2$ $(10--2)^2+(11-6)^2=r^2$ $r=\sqrt{144+25}=13$		
Hence full equation	$(x+2)^2+(y-6)^2=13^2$		M1
At (10,1)	$(10+2)^2+(1-6)^2=144+25=13^2$ Hence (10,1) is on the circle C		M1 A1

The tangent to the circle C at the point (10, 11) meets the y axis at the point P and the tangent to the circle C at the point (10, 1) meets the y axis at the point Q.

(b) Show that the distance PQ is 58 explaining your method clearly.

(7)

	At (10,11) the gradient of the radius is $m=\frac{11-6}{10--2}=\frac{5}{12}$	Using . $m=\frac{y_2-y_1}{x_2-x_1}$	M1
	The tangent is perpendicular to the radius. $m=\frac{-12}{5}$	Using $m_1 m_2=-1$	M1
At (10,11)	$y-y_1=m(x-x_1)$ $y-11=\frac{-12}{5}(x-10)$ $y=\frac{-12}{5}x+35$	Equation of a line, gradient m through a known point (x_1, y_1) Intercept $P(0,35)$	M1 A1

At (10,1) the gradient of the radius is

$$m = \frac{1-6}{10--2} = \frac{-5}{12}$$

the gradient of the tangent is M1

$$m = \frac{12}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{12}{5}(x - 10)$$

$$y = \frac{12}{5}x - 23$$ Intercept $Q(0, -23)$ M1

Distance *PQ* between the intercepts is 35 + 23 = 58 as required. A1

A level Pure 1

Question 1

The curve C has equation $y = 3x^4 - 8x^3 - 3$.

(a) Find (i) $\dfrac{dy}{dx}$,

(ii) $\dfrac{d^2y}{dx^2}$.

(3)

$y = 3x^4 - 8x^3 - 3$

$\dfrac{dy}{dx} = 12x^3 - 24x^2$ M1 A1

$\dfrac{d^2y}{dx^2} = 36x^2 - 48x$ A1

(b) Verify that C has a stationary point when $x = 2$.

(2)

For stationary points, gradient, $\dfrac{dy}{dx} = 0$

$12x^3 - 24x^2 = 0$

$12x^2(x-2) = 0$

$x = 0, 2$ M1 A1

OR

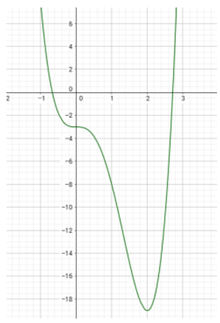
Substitute $x = 2$ into $\dfrac{dy}{dx} = 12x^3 - 24x^2$

$\dfrac{dy}{dx} = 12(2)^3 - 24(2)^2$ M1

$\dfrac{dy}{dx} = 0$

Therefore there is a stationary point when $x = 2$ A1

Graph for illustration.



Minimum at $x = 2$, point of inflexion at $x = 0$

(c) Determine the nature of this stationary point, giving a reason for your answer. **(2)**

(c)	When $x = 2$, $\frac{d^2y}{dx^2} = 36 \times 4 - 48 \times 2$ $\frac{d^2y}{dx^2} = 48$ Since $\frac{d^2y}{dx^2} > 0$, $x = 2$ is a minimum point. Alternatively use $\frac{dy}{dx}$ either side of $x = 2$ and show $x < 2$ is negative and $x > 2$ is positive. E.g. $x = 1.9$ gives -4.332, $x = 2.1$ gives 5.292	M1 A1

Question 2

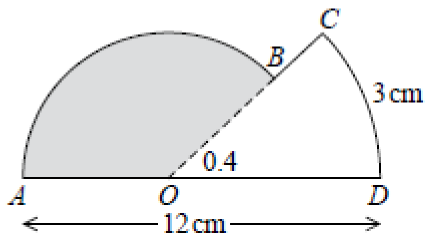


Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, $\angle COD = 0.4$ radians and *AOD* is a straight line of length 12 cm, find

(a) the length of *OD*,

(2)

Arc length = $r\theta$	
$3 = 0.4r$	M1
$r = 7.5$ cm	A1

(b) the area of the shaded sector *AOB*.

(3)

Area of sector = $\frac{1}{2}r^2\theta$	$r = 12 - 7.5 = 4.5$ $\theta = \pi - 0.4$	M1
Area of sector = $\frac{1}{2}4.5^2(\pi - 0.4)$		M1
$= 27.8$ cm^2		A1

Question 3

A circle C has equation $x^2 + y^2 - 4x + 10y = k$, where k is a constant.

(a) Find the coordinates of the centre of C.

(2)

$x^2 + y^2 - 4x + 10y = k$

A circle with equation: $(x-a)^2 + (y-b)^2 = r^2$ Has centre (a, b), radius r

Complete the square on the x terms and the y terms

$(x-2)^2 - 4 + (y+5)^2 - 25 = k$ M1

$(x-2)^2 + (y+5)^2 = k + 29$

Centre $(2, -5)$ A1

(b) State the range of possible values for k.

(2)

The radius must be positive

$k + 29 > 0$ M1

$k > -29$ A1

Question 4

Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7,$$

show that $a = \ln k$, where k is a constant to be found.

(4)

$\int_a^{2a} \frac{t+1}{t} dt = \ln 7$		
$\int_a^{2a} 1+\frac{1}{t} dt = \ln 7$	$\frac{1}{t} = t^{-1}$ for integrating	
Integrate		M1
$[t + \ln t]_a^{2a} = \ln 7$		M1
Substitute, (top limit) – (bottom limit)		
$(2a + \ln 2a) - (a + \ln a) = \ln 7$		M1
$a + \ln 2 = \ln 7$		
$a = \ln 7 - \ln 2$		
$a = \ln \frac{7}{2}$		
Therefore $k = \frac{7}{2}$		A1

Question 5

A curve C has parametric equations

$$x = 2t - 1,\ \ y = 4t - 7 + \frac{3}{t}, \quad t \neq 0.$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x+1}, \quad x \neq -1,$$

where a and b are integers to be found.

(3)

$$x = 2t - 1,\ y = 4t - 7 + \frac{3}{t}$$

Eliminate t between the parametric equations

$$t = \frac{x+1}{2}$$

$$y = 4\left(\frac{x+1}{2}\right) - 7 + \frac{3}{\frac{x+1}{2}}$$ M1

$$y = 2x - 5 + \frac{6}{x+1}$$

Common denominator

$$y = \frac{(2x-5)(x+1)+6}{x+1}$$ M1

$$y = \frac{2x^2 - 3x + 1}{x+1}$$ A1

Therefore $a = -3$, $b = 1$

Question 6

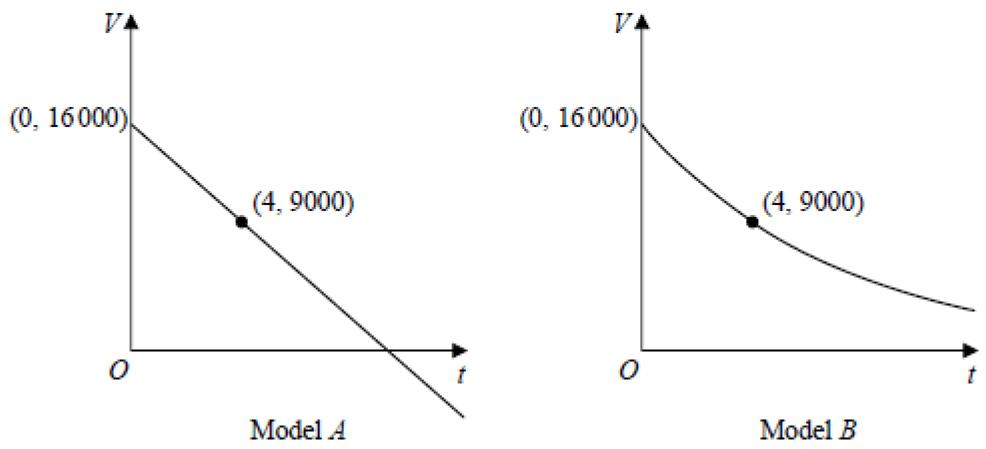
A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oilfield depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(i)

Gradient $= \dfrac{9000 - 16000}{4} = -\dfrac{7000}{4}$

Gradient = −1750

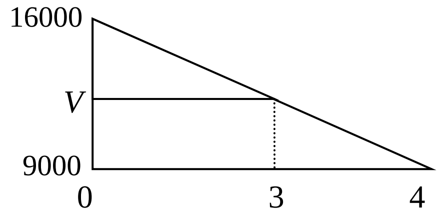
Equation of the line using $y = mx + c$

$V = -1750t + 16000$ B1

When $t = 3$, $V = -1750(3) + 16000$

= 10750 barrels

OR By similar triangles



$$\frac{16000-V}{16000-9000}=\frac{3}{4}$$
$$64000-4V=21000$$
$$4V=43000$$
$$V=10750 \text{ barrels.}$$

B1

(ii) This (linear) model predicts that the daily volume of oil would become negative as t increases which is impossible
An example: E.g. $t = 10$ gives $V = -1500$ which is impossible, or similar.
Valid range for t: When $V = 0$, $t=\frac{16000}{1750}=\frac{64}{7}$, therefore $0 \le t \le \frac{64}{7}$ B1

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model *B*.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

(i) Let $V = Ae^{kt}$ or equivalent. M1

When $t = 0$, $16000 = A$ — $e^{0k}=1$

When $t = 4$, $9000=16000e^{4k}$ dM1

$$\frac{9}{16}=e^{4k}$$

$$\ln\frac{9}{16}=4k$$

$$k=\frac{1}{4}\ln\frac{9}{16}$$ Awrt −0.144 M1

Therefore, $V=16000e^{\frac{1}{4}\ln\frac{9}{16}t}$ or A1

$V=16000e^{-0.144t}$

(ii) When $t = 3$, $V=16000e^{\frac{1}{4}\ln\frac{9}{16}\times 3}$ B1

$V = 10400$

A level Paper 1

Question 7

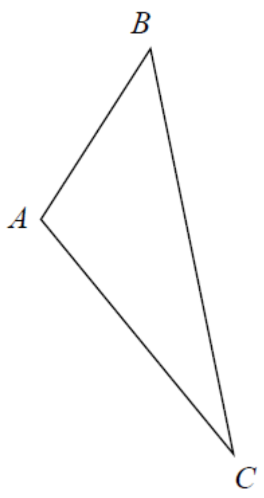


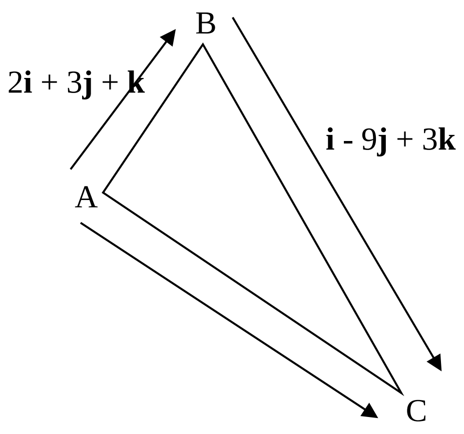
Figure 2

Figure 2 shows a sketch of a triangle *ABC*.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$\overrightarrow{AC} = 3\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$ M1

	$AB = \sqrt{14}$, $AC = \sqrt{61}$, $BC = \sqrt{91}$	Use Pythagoras Find all lengths	M1 A1
	Using the cosine rule $\cos\theta = \dfrac{14+61-91}{2\sqrt{14}\sqrt{61}}$		M1
	$\theta = 105.9$		A1
OR	Let $\angle BAC = \theta$ $\overrightarrow{AC} = 3\mathbf{i} - 6\mathbf{j} + 4\mathbf{k}$ as before		M1
	Using the scalar (dot) product $\cos\theta = \dfrac{\overrightarrow{AB}\overrightarrow{AC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert}$	Use Pythagoras Find lengths	M1 A1
	$\cos\theta = \dfrac{2\times3+3\times-6+1\times4}{\sqrt{2^2+3^2+1^2}\sqrt{3^2+(-6)^2+4}}$		M1
	$\cos\theta = \dfrac{-8}{\sqrt{14}\sqrt{61}}$ $\theta = 105.9$		A1

Question 8

$$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5.$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4].

(2)

If f(x) has a root in the interval [3.5, 4] there will be a change of sign between f(3.5) and f(4).

$f(3.5) = -4.8$ M1

$f(4) = 3.1$

Hence there is a root in the interval [3.5, 4] A1

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

From the formula booklet

Numerical solution of equations
The Newton-Raphson iteration for solving $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 4 - \frac{3.099}{16.67}$$ M1

$x_1 = 3.81$ A1

(c) Show that α is the only root of f(x) = 0.

(2)

Require f(x) = 0

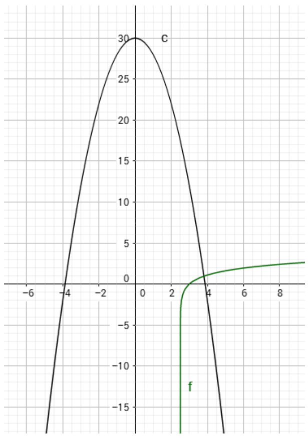
$\ln(2x-5)+2x^2-30=0$

$\ln(2x-5)=30-2x^2$

M1

Graphically, the intersection(s) of $y=\ln(2x-5)$ and $y=30-2x^2$

(sketch)



The graph illustrates that there is only one root, because there is only one intersection.

$y=\ln(2x-5)$ crosses the x-axis when:

$0=\ln(2x-5)$

$1=2x-5$ since $e^0=1$

$x=3$

Or $y=\ln(2x-5)$ is a translation of $y = \ln x$ by the vector $\begin{pmatrix}5\\0\end{pmatrix}$ followed by a horizontal stretch, scale factor ½

A1

A level Paper 1

Pearson

Question 9

(a) Prove that $\tan\theta + \cot\theta \equiv 2\operatorname{cosec} 2\theta$, $\theta \neq \dfrac{n\pi}{2}$, $n \in \mathbb{Z}$.

(4)

Prove $\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta$

LHS $\equiv \tan\theta + \cot\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ M1

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$ A1

$= \dfrac{1}{\cos\theta\sin\theta}$ Using $\sin^2\theta + \cos^2\theta = 1$

$= \dfrac{2}{2\cos\theta\sin\theta}$

$= \dfrac{2}{\sin 2\theta}$ Using $\sin 2\theta = 2\cos\theta\sin\theta$ M1

$= 2\operatorname{cosec}2\theta$

$\equiv$ RHS A1

(b) Hence explain why the equation $\tan\theta + \cot\theta = 1$ does not have any real solutions.

(1)

Require $\dfrac{2}{\sin 2\theta} = 1$

$\sin 2\theta = 2$

Since $-1 \leq \sin 2\theta \leq 1$

$\sin 2\theta = 2$ is not possible and there are no real solutions. B1

Question 10

Given that θ is measured in radians, prove, from first principles, that the derivative of sin θ is cos θ

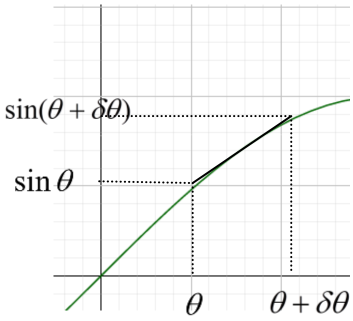
You may assume the formula for sin $(A \pm B)$ and that, as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

Let $y = \sin\theta$

Let $\delta\theta$ (or h) be a small increment in θ

Gradient of the chord



B1

$$\frac{\delta y}{\delta\theta} = \frac{\sin(\theta+\delta\theta) - \sin\theta}{\theta + \delta\theta - \theta}$$

$$\frac{\delta y}{\delta\theta} = \frac{\sin(\theta+\delta\theta) - \sin\theta}{\delta\theta}$$

$$\frac{\delta y}{\delta\theta} = \frac{\sin\theta\cos\delta\theta + \cos\theta\sin\delta\theta - \sin\theta}{\delta\theta}$$ M1 A1

$$\frac{\delta y}{\delta\theta} = \frac{\sin\delta\theta\cos\theta}{\delta\theta} + \sin\theta\left(\frac{\cos\delta\theta - 1}{\delta\theta}\right)$$ M1

For small $\delta\theta$ in radians, $\sin\delta\theta \approx \delta\theta$ and $\cos\delta\theta \approx 1$

Therefore as $\delta\theta \to 0$

$$\frac{\delta y}{\delta x} \to \frac{\mathrm{d}x}{\mathrm{d}\theta}$$

$$\frac{\sin\delta\theta}{\delta\theta} \to 1$$

$$\frac{\cos\delta\theta - 1}{\delta\theta} \to 0$$

Or in the notation of the question

$$\frac{\sin h}{h} \to 1$$

$$\frac{\cos h - 1}{h} \to 0$$

Hence $\frac{\mathrm{d}x}{\mathrm{d}\theta} = \cos\theta$ A1

Question 11

An archer shoots an arrow. The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \quad d \geq 0,$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.

(3)

When $H = 0$	
$1.8 + 0.4d - 0.002d^2 = 0$	M1
$d = \dfrac{-0.4 \pm \sqrt{0.4^2 - 4 \times (-0.002) \times 1.8}}{2 \times (-0.002)}$	dM1
$d = 204$ m to 3.s.f.	A1

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.

(1)

1.8 is the initial height of the arrow above the ground.	B1

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form $A - B(d - C)^2$, where A, B and C are constants to be found.

(3)

$1.8 - (0.002d^2 - 0.4d)$		
$= 1.8 - 0.002(d^2 - 200d)$		M1
$= 1.8 - 0.002((d - 100)^2 - 10000)$	Completing the square.	M1
$= 1.8 - 0.002(d - 100)^2 + 20$		
$= 21.8 - 0.002(d - 100)^2$		
Hence $A = 21.8$, $B = 0.002$ and $C = 100$		A1

It is decided that the model should be adapted for a different archer. The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \quad d \geq 0.$$

Hence, or otherwise, find, for the adapted model,

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.

(2)

(i)	$H = 22.1 - 0.002(d - 100)^2$ When $d = 100$, maximum height = 22.1 m	Allowing for 2.1m initial height.	B1
(ii)	100 m		B1

Question 12

In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment, were counted.

N and T are expected to satisfy a relationship of the form $N = aT^b$, where a and b are constants.

(a) Show that this relationship can be expressed in the form $\log_{10} N = m \log_{10} T + c$, giving m and c in terms of the constants a and/or b.

(2)

$N = aT^b$

Taking $\log_{10}$ of both sides

$\log_{10} N = \log_{10} aT^b$		
$\log_{10} N = \log_{10} a + \log_{10} T^b$	Using $\log xy = \log x + \log y$	M1
$\log_{10} N = \log_{10} a + b \log_{10} T$	Using $\log x^n = n\log x$	
Therefore $m = b$ and $c = \log_{10} a$		A1

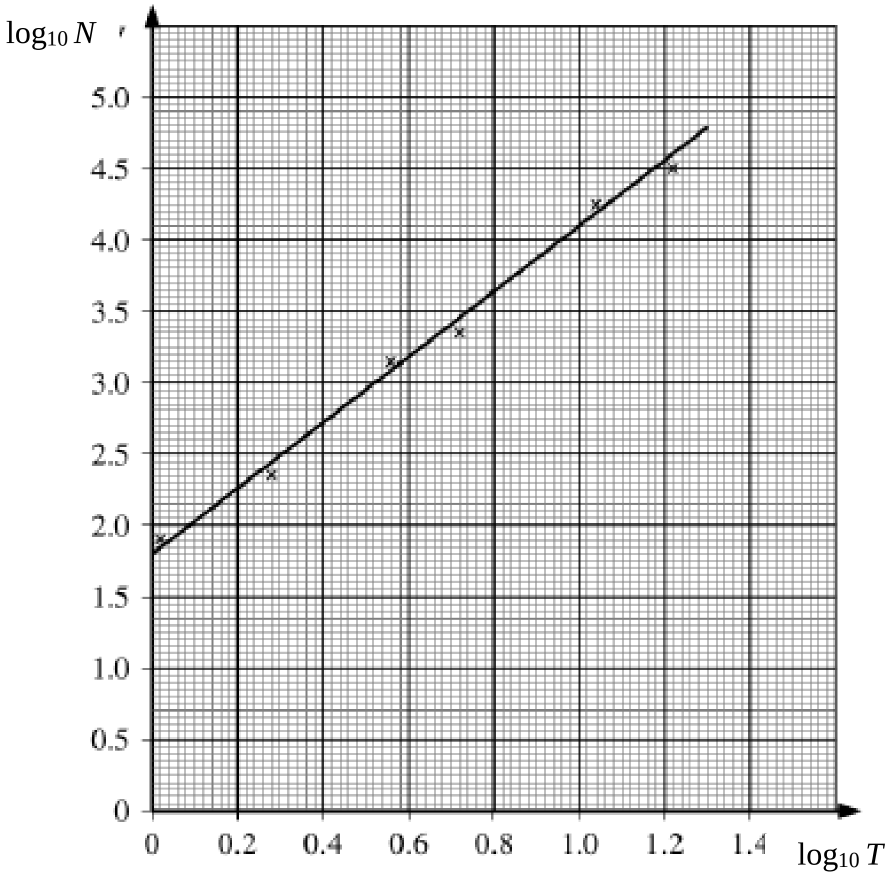


Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$.

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

Plotting $\log_{10} N$ versus $\log_{10} T$ gives a straight line,
intercept $\log_{10} a$ and gradient b

Intercept $\log_{10} a = 1.8$

$a = 10^{1.8}$ M1

$a = 63$ to nearest whole number (2 s.f.)

Gradient $b = \dfrac{4.6 - 1.8}{1.2} = 2.3$ to 1.d.p. M1

Hence

$N = 63T^{2.3}$

After 3 days M1

$N = 63(3)^{2.3} \approx 800$ A1

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

$\log_{10} 1000000 = 6$ which is outside the evidence presented on the graph. We cannot extrapolate the data and assume that the model still holds.

M1

A1

(d) With reference to the model, interpret the value of the constant *a*.

(1)

a is the number of microbes 1 day after the start of the experiment as seen by putting $T = 1$ into $N = aT^b$

B1

Question 13

The curve C has parametric equations

$$x = 2\cos t,\ \ y = \sqrt{3}\cos 2t, \quad 0 \le t \le \pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(2)

$x = 2\cos t,\ y = \sqrt{3}\cos 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ — 'Chain rule'

$\frac{dx}{dt} = -2\sin t,\ \frac{dy}{dt} = -2\sqrt{3}\sin 2t$ — $\sin 2t = 2\sin t\cos t$

$\frac{dy}{dx} = \frac{-2\sqrt{3}\sin 2t}{-2\sin t}$ — M1

$\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t}$ — A1

OR $\frac{dy}{dx} = \frac{\sqrt{3} \times 2\sin t\cos t}{\sin t}$

$\frac{dy}{dx} = 2\sqrt{3}\cos t$ — (alternative)

The point P lies on C where $t = \dfrac{2\pi}{3}$.

The line l is the normal to C at P.

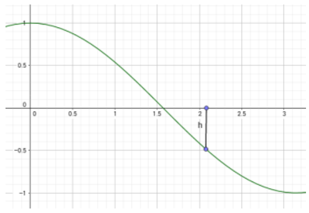
(b) Show that an equation for l is $2x - (2\sqrt{3})y - 1 = 0$.

(5)

At P

$$\frac{dy}{dx} = 2\sqrt{3}\cos\frac{2\pi}{3}$$

$$\cos\frac{2\pi}{3} = -\frac{1}{2}$$



$$\frac{dy}{dx} = -\sqrt{3}$$

M1

The tangent at P has gradient $-\sqrt{3}$

The gradient of the normal at P is $\dfrac{1}{\sqrt{3}}$

M1

When $t = \dfrac{2\pi}{3}$, $x = 2\cos\dfrac{2\pi}{3}$,

$y = \sqrt{3}\cos\dfrac{4\pi}{3}$

The equation of a line gradient m through the point (x_1, y_1)

$x = -1$, $y = -\dfrac{\sqrt{3}}{2}$, hence $P(-1, -\dfrac{\sqrt{3}}{2})$

B1

Using $y - y_1 = m(x - x_1)$

$$y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$$

M1

$2\sqrt{3}y + 3 = 2(x+1)$

$2\sqrt{3}y + 3 = 2x + 2$

$2x - 2\sqrt{3}y - 1 = 0$ as required.

A1

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.
You must show clearly how you obtained your answers.

(6)

$2x-2\sqrt{3}y-1=0$		
$x=2\cos t,\ y=\sqrt{3}\cos 2t$		
Therefore $4\cos t-6\cos 2t-1=0$	Identity	M1
$4\cos t-6\cos 2t-1=0$	'$\cos 2t=2\cos^2 t-1$'	
$4\cos t-12\cos^2 t+5=0$		M1
$12\cos^2 t-4\cos t-5=0$		A1
$(6\cos t-5)(2\cos t+1)=0$	Factorise, use of the quadratic formula or otherwise	
$\cos t=-\frac{1}{2}$ at P, or $\cos t=\frac{5}{6}$ at Q (rejected)		M1
$x=2\cos t,\ y=\sqrt{3}\cos 2t$		
Therefore at Q		
$x=2\times\frac{5}{6}$, $\quad y=\sqrt{3}\cos 2t$		
$x=\frac{5}{3}$, $\quad y=\sqrt{3}(2\cos^2 t-1)$		
$y=\sqrt{3}(2\times\frac{25}{36}-1))$		M1
$y=\sqrt{3}(\frac{25}{18}-1))$		
$y=\frac{7\sqrt{3}}{18}$		
Therefore $Q(\frac{5}{3},\frac{7\sqrt{3}}{18})$, exact values.		A1

Question 14

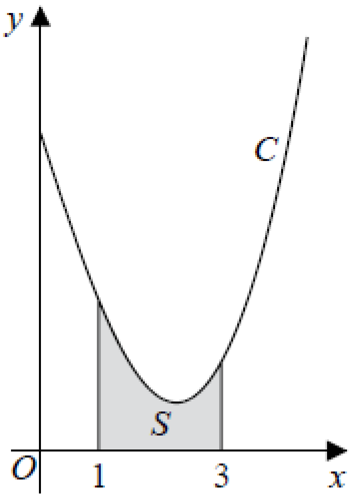


Figure 4

Figure 4 shows a sketch of part of the curve *C* with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0.$$

The finite region *S*, shown shaded in Figure 4, is bounded by the curve *C*, the line with equation $x = 1$, the *x*-axis and the line with equation $x = 3$.

The table below shows corresponding values of *x* and *y*, with the values of *y* given to 4 decimal places as appropriate.

x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958

(a) Use the trapezium rule, with all the values of *y* in the table, to obtain an estimate for the area of *S*, giving your answer to 3 decimal places.

(3)

From the formula book:

Numerical Integration
The trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + ... + y_{n-1})\}$ where $h = \frac{b-a}{n}$

Area $S \approx \frac{1}{2}0.5\{(3 + 2.2958) + 2(2.3041 + 1.9242 + 1.9089)\}$ B1 M1

Area $S \approx 4.393$ to 3 d.p. A1

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

e.g. B1

- Increase the number of strips
- Decrease the width of the strips
- Use more trapezia

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(*In part (c), solutions based entirely on graphical or numerical methods are not acceptable.*)
(6)

From the formula book, integration by parts:

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$y = \frac{x^2 \ln x}{3} - 2x + 5 \ dx$$

$$\text{Area } S = \int_1^3 \frac{x^2 \ln x}{3} - 2x + 5 \ dx$$

By parts:

$$u = \ln x \qquad \frac{dv}{dx} = \frac{x^2}{3}$$

$$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{x^3}{9}$$

$$\text{Area } S = \left[\frac{x^3}{9} \ln x\right]_1^3 - \int_1^3 \frac{x^3}{9} \frac{1}{x} dx + \left[-x^2 + 5x\right]_1^3$$ M1 A1 B1

$$= 3\ln 3 - \left[\frac{x^3}{27}\right]_1^3 + ((-9+15) - (-1+5))$$ M1

$$= \ln 3^3 - \frac{26}{27} + \frac{54}{27}$$

$$= \ln 27 + \frac{28}{27}$$ M1

$a = 28$, $b = 27$, $c = 27$ A1

Question 15

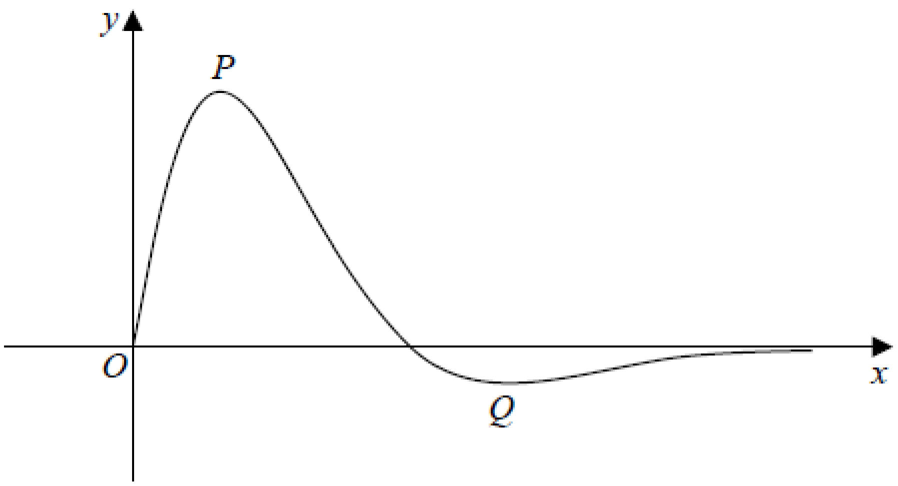


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{4\sin 2x}{\mathrm{e}^{(\sqrt{2})x-1}}, \quad 0 \leq x \leq \pi.$$

The curve has a maximum turning point at P and a minimum turning point at Q, as shown in Figure 5.

(a) Show that the x-coordinates of point P and point Q are solutions of the equation $\tan 2x = \sqrt{2}$.
(4)

Quotient rule:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{u}{v}\right) = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

From the formula book, quotient rule:

f(*x*)	**f ’(*x*)**
$\frac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

$u = 4\sin 2x$ $\quad$ $v = \mathrm{e}^{(\sqrt{2})x-1}$

$\frac{\mathrm{d}u}{\mathrm{d}x} = 8\cos 2x$ $\quad$ $\frac{\mathrm{d}v}{\mathrm{d}x} = \sqrt{2}\mathrm{e}^{(\sqrt{2})x-1}$

$$\mathrm{f}'(x) = \frac{e^{(\sqrt{2})x-1} 8\cos 2x - 4\sin 2x\sqrt{2}e^{(\sqrt{2})x-1}}{(\mathrm{e}^{(\sqrt{2})x-1})^2}$$ M1

$$\mathrm{f}'(x) = \frac{8\cos 2x - 4\sqrt{2}\sin 2x}{(\mathrm{e}^{(\sqrt{2})x-1})}$$ A1

For turning points f '(x) = 0, therefore

$8\cos 2x - 4\sqrt{2}\sin 2x = 0$ M1

$\frac{\sin 2x}{\cos 2x} = \frac{8}{4\sqrt{2}}$

$\tan 2x = \sqrt{2}$ A1

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \mathrm{f}(2x)$,

(ii) $y = 3 - 2\mathrm{f}(x)$.

(4)

(i)	$\tan 4x = \sqrt{2}$ $4x = 0.9553\ldots$ or $\pi + 0.9553\ \ldots$ $x = 0.239$ or 1.02	$x \rightarrow 2x$ The minimum is the 2nd solution.	M1
	$x = 0.239$ corresponds to the maximum $x = 1.02$ corresponds to the minimum required.		A1
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2\mathrm{f}'(x)$ so for $\mathrm{f}'(x) = 0$ $\tan 2x = \sqrt{2}$	$-2\mathrm{f}(x)$ is a reflection of $\mathrm{f}(x)$ and a $\times$ 2 vertical stretch. The minimum will correspond to the	M1
	$2x = 0.9553\ldots$ $x = 0.478$	transformed x-value of P.	A1

Pearson

A level Pure 2

Question 1

$$f(x) = 2x^3 - 5x^2 + ax + a.$$

Given that $(x + 2)$ is a factor of f (x), find the value of the constant a.

(3)

If $(x+2)$ is a factor ,then $f(-2)=0$ $f(-2)=2(-2)^3-5(-2)^2-2a+a\ =0$ $f(-2)=-16-20-a\ =0$	Or by long division gives a remainder of $-36-a$ which must be zero if $(x+2)$ is a factor.	M1
Therefore $-a = 36$		dM1
So $a = -36$		A1

A level Paper 2

Question 2

Some A level students were given the following question.

Solve, for $-90° < \theta < 90°$, the equation $\cos \theta = 2 \sin \theta$.

The attempts of two of the students are shown below.

Student A	Student B
$\cos \theta = 2 \sin \theta$	$\cos \theta = 2 \sin \theta$
$\tan \theta = 2$	$\cos^2 \theta = 4 \sin^2 \theta$
$\theta = 63.4°$	$1 - \sin^2 \theta = 4 \sin^2 \theta$
	$\sin^2 \theta = \frac{1}{5}$
	$\sin \theta = \pm \frac{1}{\sqrt{5}}$
	$\theta = \pm 26.6°$

(a) Identify an error made by student A.

(1)

To get $\tan \theta$, student A has divided both sides by $\cos \theta$. But this should give

$1 = 2 \tan \theta$

$\tan \theta = \frac{1}{2}$ B1

Or student A makes the mistake $\frac{\cos \theta}{\sin \theta} = \tan \theta$

Student B gives $\theta = -26.6°$ as one of the answers to $\cos\theta = 2\sin\theta$.

(b) (i) Explain why this answer is incorrect.

(ii) Explain how this incorrect answer arose.

(2)

(i)	Part (a) only gives one solution $\theta = \tan^{-1} 0.5$ $\theta = 26.6°$ $\cos 26.6 = 2\sin 26.6$ $\cos(-26.6) \neq 2\sin(-26.6)$	B1
(ii)	The incorrect answer is introduced by squaring.	B1

Pearson

A level Paper 2

Question 3

Given $y = x(2x + 1)^4$, show that $\frac{dy}{dx} = (2x + 1)^n (Ax + B)$, where n, A and B are constants to be found.

(4)

$y = x(2x+1)^4$

Product rule:

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$u = x \qquad v = (2x+1)^4$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = 8(2x+1)^3$

$\frac{dy}{dx} = 8x(2x+1)^3 + (2x+1)^4$	M1 A1
$\frac{dy}{dx} = (2x+1)^3(8x+(2x+1))$	M1
$\frac{dy}{dx} = (2x+1)^3(10x+1)$	A1

A level Paper 2

Question 4

Given

$$f(x) = e^x, \quad x \in \mathbb{R},$$

$$g(x) = 3 \ln x, \qquad x > 0, x \in \mathbb{R},$$

(a) find an expression for gf (x), simplifying your answer.

(2)

$$\text{gf}(x) = 3\ln e^x$$ M1

$$\text{gf}(x) = 3x$$ A1

(b) Show that there is only one real value of x for which $\text{gf}(x) = \text{fg}(x)$.

(3)

$$\text{fg}(x) = e^{3\ln x}$$

$$\text{fg}(x) = e^{\ln x^3}$$

$$\text{fg}(x) = x^3$$

For $\text{gf}(x) = \text{fg}(x)$ M1

$$3x = x^3$$

$$x^3 - 3x = 0$$

$$x(x^2 - 3) = 0$$

$$x = 0 \text{ or } \pm\sqrt{3}$$ M1

e^x is defined for $x = 0$ or $\pm\sqrt{3}$, however, $\ln x$ is not defined for $x = 0$ or $-\sqrt{3}$.

Therefore, $x = \sqrt{3}$ is the only real value of x for which $\text{gf}(x) = \text{fg}(x)$. A1

A level Paper 2

Question 5

The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}.$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

When $t = 0.5$

$m = 25e^{-0.05\times0.5}$ M1

$m = 25e^{-0.025}$

$m = 24.4$ g A1

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

$m = 25e^{-0.05t}$

$\frac{dm}{dt} = -0.05 \times 25e^{-0.05t}$ M1

$\frac{dm}{dt} = -0.05m$ i.e. $\frac{dm}{dt} = km$

Where $k = -0.05$ A1

A level Paper 2

Pearson

Question 6

For each statement below you must state if it is always true, sometimes true or never true, giving a reason in each case. The first one has been done for you.

The quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) has 2 real roots.

Sometimes true.

It only has 2 real roots when $b^2 - 4ac > 0$.
When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.

(i) When a real value of x is substituted into $x^2 - 6x + 10$, the result is positive.

(2)

Always	Sometimes	Never
✓		

$x^2 - 6x + 10 = (x-3)^2 - 9 + 10$

$x^2 - 6x + 10 = (x-3)^2 + 1$ M1

This has a minimum value of 1 for all real x so is always positive. A1

(ii) If $ax > b$ then $x > \frac{b}{a}$.

(2)

Always	Sometimes	Never
	✓	

Only if $a > 0$ otherwise if $a < 0, x < \frac{b}{a}$ M1 A1 (iii)The difference between consecutive square numbers is odd.

(2)

Always	Sometimes	Never
✓		

$(n+1)^2 - n^2$

$= n^2 + 2n + 1 - n^2$ M1

$= 2n + 1$

Since $2n$ is a multiple of 2 it must be even.

Therefore $2n + 1$ must be odd. A1

A level Paper 2

Question 7

(a) Use the binomial expansion, in ascending powers of x, to show that

$$\sqrt{(4-x)} = 2 - \frac{1}{4}x + kx^2 + \ldots$$

where k is a rational constant to be found.

(4)

From the formula book:

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x| < 1,\ n \in \mathbb{R})$$

$$(4-x)^{\frac{1}{2}} = (4(1-\frac{x}{4}))^{\frac{1}{2}}$$

$$(4-x)^{\frac{1}{2}} = 2\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$$ M1

$$2\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 2(1+\left(\frac{1}{2}\right)\left(\frac{-x}{4}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{1\times 2}\left(\frac{-x}{4}\right)^2+\ldots$$ M1

$$2\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 2\left(1-\frac{x}{8}-\frac{x^2}{128}+\ldots\right)$$ A1

$$2\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 2-\frac{x}{4}-\frac{x^2}{64}$$

$$k = -\frac{1}{64}$$ A1

A level Paper 2

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

(b) State, giving a reason, if the expansion is valid for this value of x.

(1)

The expression is valid for

$\left|\frac{x}{4}\right| < 1$, or $-1 < \frac{x}{4} < 1$

$|x| < 4$, or $-4 < x < 4$

B1

Therefore valid for $x = 1$

A level Paper 2

Question 8

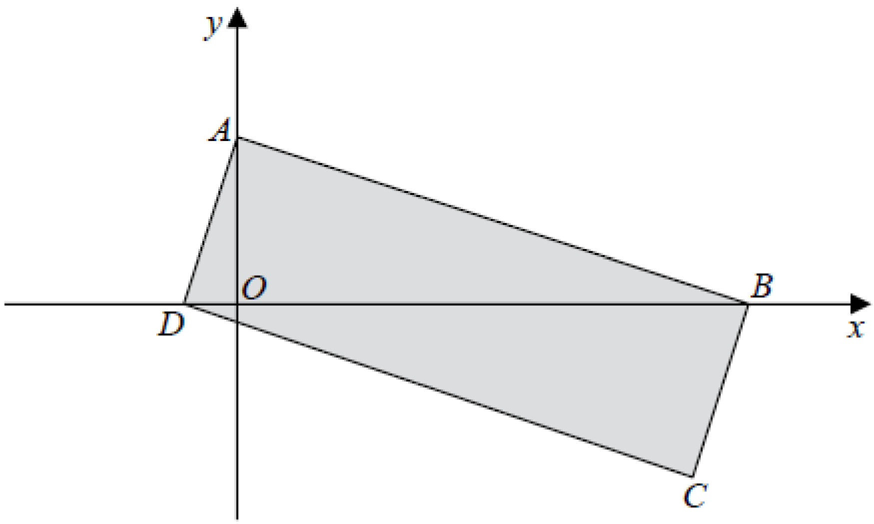


Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$,

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$,

(4)

$AB \quad 5y+2x=10$

$5y=10-2x$

$y=2-\frac{2}{5}x$

The gradient of *AB* is $-\frac{2}{5}$ or -0.4 B1

Therefore the gradient of *AD* is $\frac{5}{2}$ or 2.5

$AB \quad 5y+2x=10$

When $x=0, y=2$, when $y=0, x=5$ B1

Therefore *A*(0,2) and *B*(5,0)

Using $y-y_1=m(x-x_1)$ at point A   Or $y=mx+c$

$y-2=\frac{5}{2}(x-0)$

$y-2=\frac{5}{2}x$

$2y-4=5x$

$2y-5x=4$ as required

$2=\frac{5}{2}\times 0+c$ M1

$c=2$

$y=\frac{5}{2}x+2$ A1

$2y = 5x + 4$

$2y-5x=4$ as required

(b) find the area of the rectangle *ABCD*.

(3)

AD $2y-5x=4$

When $y=0, x=-\frac{4}{5}$

Therefore D$\left(-\frac{4}{5},0\right)$

Using $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|AB|=\sqrt{(5-0)^2+(0-2)^2}$ M1

$|AB|=\sqrt{29}$

$|AD|=\sqrt{\left(0-\frac{4}{5}\right)^2+(2-0)^2}$

$|AD|=\sqrt{\frac{16}{25}+4}$

$|AD|=\sqrt{\frac{116}{25}}=\frac{\sqrt{116}}{5}$

Area = $\sqrt{29}\times\frac{\sqrt{116}}{5}=11.6$ (exact answer) M1 A1

OR BD = 5 + $^4/_5$ = 5.8, AO = 2 M1

Area $\Delta ABD=\frac{1}{2}\times 5.8\times 2=5.8$

Total rectangle =11.6 (exact answer) M1 A1

A level Paper 2

Question 9

Given that A is constant and

$$\int_1^4 (3\sqrt{x} + A)\ \mathrm{d}x = 2A^2,$$

show that there are exactly two possible values for A.

(5)

$\int_1^4 \left(3\sqrt{x} + A\right) dx = 2A^2$		
$\int_1^4 \left(3x^{\frac{1}{2}} + A\right) dx = 2A^2$		
$\left[2x^{\frac{3}{2}} + Ax\right]_1^4 = 2A^2$		M1 A1
$(16 + 4A) - (2 + A) = 2A^2$		M1
$2A^2 - 3A - 14 = 0$	Or quadratic formula.	
$(2A - 7)(A + 2) = 0$		M1
The two roots are: $A = \frac{7}{2}$ or -2		A1

Question 10

In a geometric series the common ratio is r and sum to n terms is S_n.

Given $S_\infty = \frac{8}{7} \times S_6$, show that $r = \pm\frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

From the formula booklet:

Geometric Series

$$S_n = \frac{a(1-r^n)}{1-r} \qquad S_\infty = \frac{a}{1-r} \text{ for } |r|<1$$

$$S_6 = \frac{a(1-r^6)}{1-r}$$

$$S_\infty = \frac{8}{7} \times S_6$$

$$\frac{a}{1-r} = \frac{8}{7} \times \frac{a(1-r^6)}{1-r}$$ M1

$$7 = 8(1-r^6)$$ M1

$$r^6 = \frac{1}{8}$$

$$r^2 = \frac{1}{2}$$

$$r = \pm\frac{1}{\sqrt{2}}$$ M1 A1

Therefore $k = 2$

A level Paper 2

Question 11

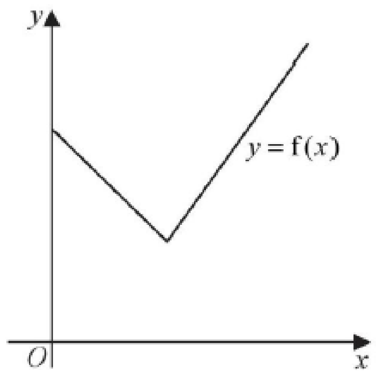


Figure 2

Figure 2 shows a sketch of part of the graph $y = \text{f}(x)$ where $\text{f}(x) = 2|3 - x| + 5, \quad x \geq 0$.

(a) State the range of f.

(1)

Equation of the left section
$y = 6 - 2x + 5$
$y = 11 - 2x$

Equation of the right section
$y = -6 + 2x + 5$
$y = 2x - 1$

When they cross
$2x - 1 = 11 - 2x$
$4x = 12$
$x = 3$
When $x = 3, y = 5$ lowest point.

Range $\text{f}(x) \geq 5$

Or
$|3 - x|$ has a minimum value when $x = 3$
When $x = 3, y = 5$

Range $\text{f}(x) \geq 5$ B1

A level Paper 2

(b) Solve the equation $f(x) = \frac{1}{2}x + 30$.

(3)

$2(3 - x) + 5 = \frac{1}{2}x + 30$

$11 - 2x = \frac{1}{2}x + 30$

$-19 = 2.5x$

$x = -7.6$ Not in the range.

$-2(3 - x) + 5 = \frac{1}{2}x + 30$ M1

$2x - 1 = \frac{1}{2}x + 30$

$1.5x = 31$ M1

$x = 20\frac{2}{3}$ only. A1

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k.

(2)

$f(x) = k$ is where $f(x)$ meets the horizontal line $y = k$
A horizontal line crosses in two places, i.e. 2 solutions. M1

The minimum value will be as part (a) when $f(x) > 5$. It cannot equal 5 as $f(x)$ has only one solution at this point.

The maximum value for two solutions will be when $f(x)$ crosses the y-axis at $y = 11$. Therefore:

$5 < f(x) \leq 11$

$5 < k \leq 11$ A1

A level Paper 2

Question 12

(a) Solve, for $-180° \leq x < 180°$, the equation $3\sin^2 x + \sin x + 8 = 9\cos^2 x$, giving your answers to 2 decimal places.

(6)

$3\sin^2 x + \sin x + 8 = 9\cos^2 x$

Using $\sin^2 x + \cos^2 x = 1$

$3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$ — M1

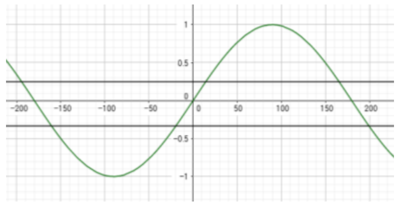
$12\sin^2 x + \sin x - 1 = 0$ — A1

$(4\sin x - 1)(3\sin x + 1) = 0$ — M1

$\sin x = \frac{1}{4}$ or $-\frac{1}{3}$ — A1

$x = 14.48°, 165.52°, -19, 47°, -160.53°$ — M1 A1

$\sin x = 0.25 \rightarrow x = 14.48, 165.52$



$\sin x = -0.3 \rightarrow x = -19.47, -160.53$

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°),$$

giving your answer to 2 decimal places.

(2)

$2\theta - 30 = 165.52$

$2\theta = 195.52$

$\theta = 97.76$

$2\theta - 30 = -19.47$ — M1

$2\theta = 10.53$

$\theta = 5.26$ Smallest. — A1

A level Paper 2

Question 13

(a) Express 10 cos θ – 3 sin θ in the form R cos (θ + α), where $R > 0$ and $0 < \alpha < 90°$.
Give the exact value of R and give the value of α, in degrees, to 2 decimal places.

(3)

Let $R\cos(\theta+\alpha)=10\cos\theta-3\sin\theta$

$R\cos\theta\cos\alpha-R\sin\theta\sin\alpha\equiv10\cos\theta-3\sin\theta$

Equating $\cos\theta$ and $\sin\theta$

$$R\cos\alpha=10$$
$$R\sin\alpha=3$$

Squaring and adding

$$R^2\cos^2\alpha+R^2\sin^2\alpha=10^2+3^2$$
$$R^2(\cos^2\alpha+\sin^2\alpha)=109$$

Using $\cos^2 x+\sin^2 x=1$

$$R=\sqrt{109}$$ B1

Dividing

$$\tan\alpha=\frac{3}{10}$$ M1

$$\alpha=16.70° \text{ to 2 d.p.}$$ A1

Hence

$10\cos\theta-3\sin\theta\equiv\sqrt{109}\cos(\theta+16.70)$

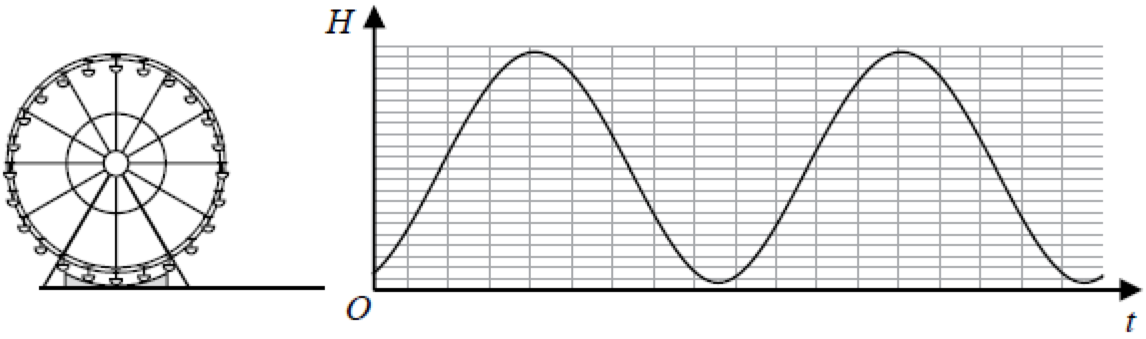


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation $H = a - 10\cos(80t)° + 3\sin(80t)°$, where a is a constant. Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,
(b) (i) find a complete equation for the model.

Using (a)
$H = a - \sqrt{109}\cos(80t + 16.70)$
When $t = 0$, $H = 1$
$1 = a - \sqrt{109}\cos(16.70)$
$a = 11$
Therefore
$H = 11 - \sqrt{109}\cos(80t + 16.70)$ B1
Or
$H = 11 - 10\cos 80t + 3\sin 80t$

(ii) Hence find the maximum height of the passenger above the ground.

(2)

(ii) For maximum H, require $\cos(80t + 16.70)$ to take its minimum value of -1, then B1

$$H_{max} = 11 + \sqrt{109}$$
$$H_{max} = 21.4 \text{ to 3 s.f.}$$

(c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(3)

For $\cos(80t + 16.70) = -1$

$80t + 16.70 = 180$ or 540

540° for the second cycle — M1

$$t = \frac{540 - 16.70}{80}$$ M1

t = 6.54125 minutes — A1

t = 6 minutes 32 seconds

It is decided that, to increase profits, the speed of the wheel is to be increased.

(d) How would you adapt the equation of the model to reflect this increase in speed? **(1)**

Increase the 80 in the formula

$H = 11 - 10\cos 80t + 3\sin 80t$

For example

$H = 11 - 10\cos 90t + 3\sin 90t$

This has the effect of more cycles in the same time period so the wheel would travel faster. — B1

A level Paper 2

Question 14

A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm. In the model they assume that the can is made from a metal of negligible thickness.

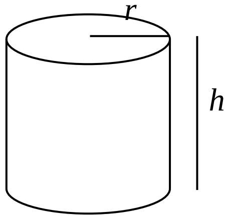
(a) Prove that the total surface area, S cm^2, of the can is given by $S = 2\pi r^2 + \frac{1000}{r}$.

(3)

Volume

$$V = \pi r^2 h$$

$$500 = \pi r^2 h$$



B1

Surface area

$$S = 2\pi r^2 + 2\pi rh$$

Eliminate h between the two equations

$$h = \frac{500}{\pi r^2}$$

1ml = 1 cm^3

$$S = 2\pi r^2 + 2\pi r\frac{500}{\pi r^2}$$

M1

$$S = 2\pi r^2 + \frac{1000}{r}$$

A1

As required.

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area.

(5)

$$S = 2\pi r^2 + 1000r^{-1}$$

$$\frac{dS}{dr} = 4\pi r - 1000r^{-2}$$ M1 A1

$$\frac{dS}{dr} = 4\pi r - \frac{1000}{r^2}$$

For minimum S, $\frac{dS}{dr} = 0$, therefore

$$4\pi r - \frac{1000}{r^2} = 0$$

$$r^3 = \frac{1000}{4\pi}$$ M1

$$r = \frac{10}{\sqrt[3]{4\pi}} \text{ or } 4.30$$ A1

From (a)

$$h = \frac{500}{\pi r^2}$$

Therefore

$$h = \frac{500}{\pi\left(\frac{10}{\sqrt[3]{4\pi}}\right)^2} \text{ or } 8.60$$ A1

For minimum S, the can needs radius 4.30 cm and height 8.60 cm.

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

Possible valid reasons such as

- The radius is too big for the size of our hands — B1
- If $r = 4.3$cm and $h = 8.6$cm the can is square in profile. All drinks cans are taller than they are wide
- The radius is too big for us to drink from
- They have different dimensions to other drinks cans and would be difficult to stack on shelves with other drinks cans

A level Paper 2

Question 15

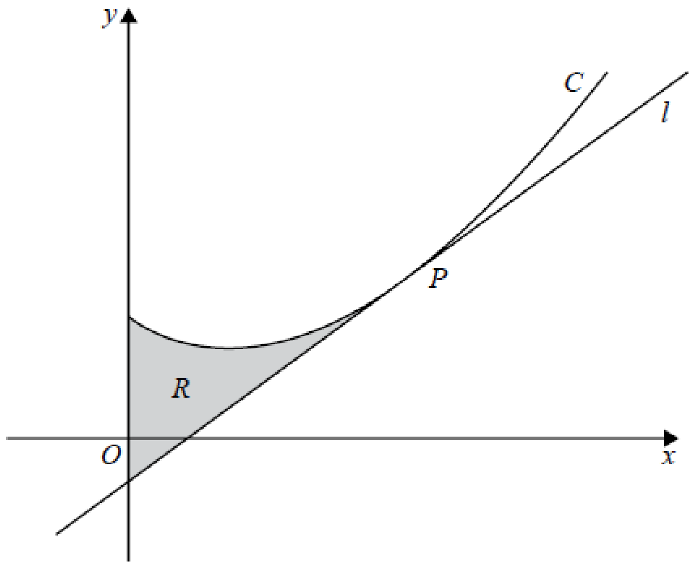


Figure 4

Figure 4 shows a sketch of the curve C with equation $y = 5x^{\frac{3}{2}} - 9x + 11, \quad x \geq 0$.

The point P with coordinates (4, 15) lies on C. The line l is the tangent to C at the point P.

The region R, shown shaded in Figure 4, is bounded by the curve C, the line l and the y-axis.

Show that the area of R is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

$$y = 5x^{\frac{3}{2}} - 9x + 11$$

Find the equation of the line l.

$$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$$ M1 A1

When $x = 4$ at P

$$\frac{dy}{dx} = \frac{15}{2}4^{\frac{1}{2}} - 9 = 6$$ M1

The gradient of the line l is 6

Using $y - y_1 = m(x - x_1)$ at P

$$y - 15 = 6(x - 4)$$ M1

$$y - 15 = 6x - 24$$

$$y = 6x - 9$$ A1

Shaded area

$$\int_0^4 (\text{top curve}) - (\text{bottom curve})\,dx$$

$$\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) - (6x - 9)\,dx$$ M1

$$\int_0^4 \left(5x^{\frac{3}{2}} - 15x + 20\right) dx$$

$$= \left[2x^{\frac{5}{2}} - \frac{15}{2}x^2 + 20x\right]_0^4$$ A1

= (64 – 120 + 80) – (0) M1

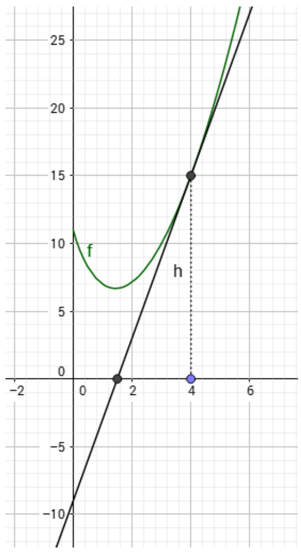
= 24 square units A1

Correct notation with good explanation. A1

Alternative: First 5 marks as before.

$$y = 6x - 9$$

The tangent crosses the x-axis when $y = 0$ and $x = 1.5$



Area beneath the curve and the *x*-axis

$$\int_0^4 \left(5x^{\frac{3}{2}} - 9x + 11\right) dx$$

$$= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 20x\right]_0^4$$ M1

= 64 – 72 + 44

= 36 square units A1

Area of the upper triangle

$$= \frac{2.5 \times 15}{2} = 18.75$$

Area of the lower triangle

$$\frac{1.5 \times 9}{2} = 6.75$$ M1

Total shaded area

= 36 – 18.72 + 6.75 A1

= 24 square units A1

Correct notation with good explanation.

Question 16

(a) Express $\dfrac{1}{P(11-2P)}$ in partial fractions.

(3)

Two linear factors

$$\frac{1}{P(11-2P)} \equiv \frac{A}{P} + \frac{B}{11-2P}$$ B1

Common denominator

$$\frac{1}{P(11-2P)} \equiv \frac{A(11-2P)+BP}{P(11-2P)}$$

Equating the numerator

$$1 \equiv A(11-2P)+BP$$

When $P = 0$

$$1 = 11A$$

$$A = \frac{1}{11}$$ M1

When $P = \dfrac{11}{2}$

$$1 = \frac{11}{2}B$$

$$B = \frac{2}{11}$$

Therefore

$$\frac{1}{P(11-2P)} \equiv \frac{1}{11P} + \frac{2}{11(11-2P)}$$ A1

A level Paper 2

Pearson

A population of meerkats is being studied. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

$$\frac{dP}{dt} = \frac{1}{22}P(11-2P)$$

Separating the variables

$$\int \frac{22}{P(11-2P)}\,dP = \int dt \qquad \text{B1}$$

Using (a)

$$\int \frac{22}{11P} + \frac{44}{11(11-2P)}\,dt = t$$

$$\int \frac{2}{P} + \frac{4}{(11-2P)}\,dt = t \qquad \text{M1}$$

$$2\ln P - 2\ln(11-2P) = t + c \qquad \text{A1}$$

When $t = 0$, $P = 1$

$$2\ln 1 - 2\ln(11-2) = 0 + c \qquad \text{M1}$$

$$c = -2\ln 9$$

$$2\ln P - 2\ln(11-2P) = t - 2\ln 9$$

When $P = 2$

$$2\ln 2 - 2\ln(11-4) = t - 2\ln 9$$

$$2\ln 2 - 2\ln 7 = t - 2\ln 9 \qquad \text{M1}$$

$$t = 1.89 \text{ to 3 s.f.} \qquad \text{A1}$$

(c) show that

$$P = \frac{A}{B + C\,\mathrm{e}^{-\frac{1}{2}t}},$$

where A, B and C are integers to be found.

(3)

$$2\ln P - 2\ln(11-2P) = t - 2\ln 9$$

$$\frac{t}{2} = \ln P - \ln(11-2P) + \ln 9$$

$$\frac{t}{2} = \ln\frac{9P}{11-2P}$$

$$\mathrm{e}^{\frac{t}{2}} = \frac{9P}{11-2P}$$

Laws of logarithms

$\ln xy = \ln x + \ln y$

$\ln\frac{x}{y} = \ln x - \ln y$

M1

Make P the subject

$$11\mathrm{e}^{\frac{t}{2}} - 2P\mathrm{e}^{\frac{t}{2}} = 9P$$

$$9P + 2P\mathrm{e}^{\frac{t}{2}} = 11\mathrm{e}^{\frac{t}{2}}$$

$$P(9 + 2\mathrm{e}^{\frac{t}{2}}) = 11\mathrm{e}^{\frac{t}{2}}$$

$$P = \frac{11\mathrm{e}^{\frac{t}{2}}}{(9 + 2\mathrm{e}^{\frac{t}{2}})}$$

M1

Divide numerator and denominator by $\mathrm{e}^{\frac{t}{2}}$

$$P = \frac{11}{(9\mathrm{e}^{-\frac{t}{2}} + 2)}$$

$A = 11$, $B = 2$ and $C = 9$

A1

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