<u>*3.4 Mechanics and Materials*</u>

3.4.1 Force, Energy and Momentum

3.4.1.1 Scalars and Vectors

Content

- Nature of scalars and vectors.
- Examples should include:
    - Velocity/speed, mass, force/weight, acceleration, displacement/distance.
- Addition of vectors by calculation or scale drawing.
- Calculations will be limited to two vectors at right angles. Scale drawings may involve vectors at angles other than 90°.
- Resolution of vectors into two components at right angles to each other.
- Examples should include components of forces along and perpendicular to an inclined plane.
- Problems may be solved either by the use of resolved forces or the use of a closed triangle.
- Conditions for equilibrium for two or three coplanar forces acting at a point.
- Appreciation of the meaning of equilibrium in the context of an object at rest or moving with constant velocity.

Opportunities for Skills Development

- Investigation of the conditions for equilibrium for three coplanar forces acting at a point using a force board.

Scalars and Vectors

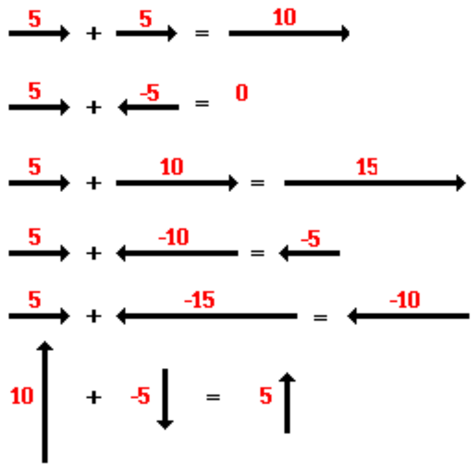
Scalar: A quantity that has only magnitude, no direction i.e. speed.
Vector: A quantity with magnitude and direction i.e. velocity.

Examples

Velocity is an example of a vector; it is the rate of change of displacement. v = s/t where v = velocity, s = displacement and t = time. Since displacement is a vector quantity also, as it has direction as well as magnitude i.e. 10m in the north direction, velocity is also a vector as it is the rate of change of this vector. However speed is the rate of change of the distance travelled, therefore since distance is a scalar quantity, speed is a scalar quantity.
Other examples include force/weight and mass. Pretty much everything has a quantity called mass, which is a scalar quantity. Weight is a force caused by gravity. You multiply somethings mass by the force of gravity (9.81/'g') to find its weight. The units for force/weight are in 'Newton's' whereas mass is 'kilograms'. If you had an object that had a

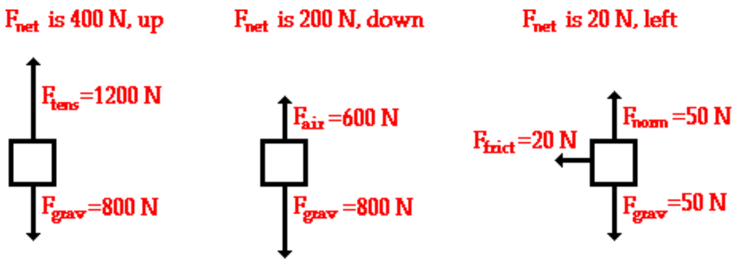
mass of 10kg, then the weight would be 10 multiplied by 9.81. This works the other way and you can convert the weight into mass, as you may need to do this for certain questions. Acceleration is also a vector quantity, and is the rate of change of the velocity. Displacement and distance have already been touched on, where displacement is a vector quantity and distance a scalar. You may be given other examples which you will be required to know however you will pick them up as you go through the course. Some questions in the multiple choice section may test your knowledge on scalars and vectors.

Addition of vectors



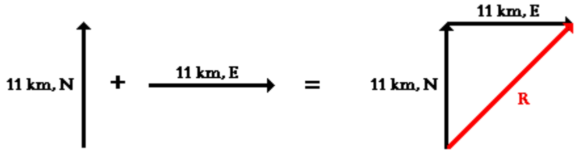
Addition of vectors in the same plane is fairly simple as you can see. If the vectors are working together in the same plane you simply add them up, if not it is the difference between the two, in the direction of the larger vector.

If we apply the rules to actual situations with real forces it stays fairly simple. In the picture on the far right, the 50N's cancel each other out, therefore the resultant force is 20N left.



Resolving Vectors

Resolution of vectors is most commonly done by Pythagoras and Trigonometry. If you have two vectors at a right angle to each other, you can use Pythagoras to find the missing side length.
For example, if a car moves 11km north, then 11km east, what is its displacement from its original position? Well if you drew a triangle for its path, with a straight line from the original position to its final position, this would be the hypotenuse, and also its displacement.

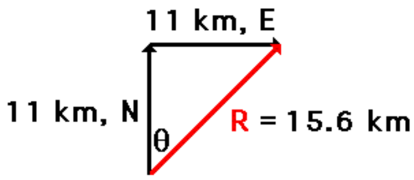


$$11^2 + 11^2 = R^2$$

$$242 = R^2$$

$$15.6 = R$$

The diagram above illustrates this concept, where R is the displacement. Ultimately, using $a^2=b^2+c^2$ you can calculate the displacement. You may also be asked to find the angle of displacement, in which case you can use trigonometry to find this.

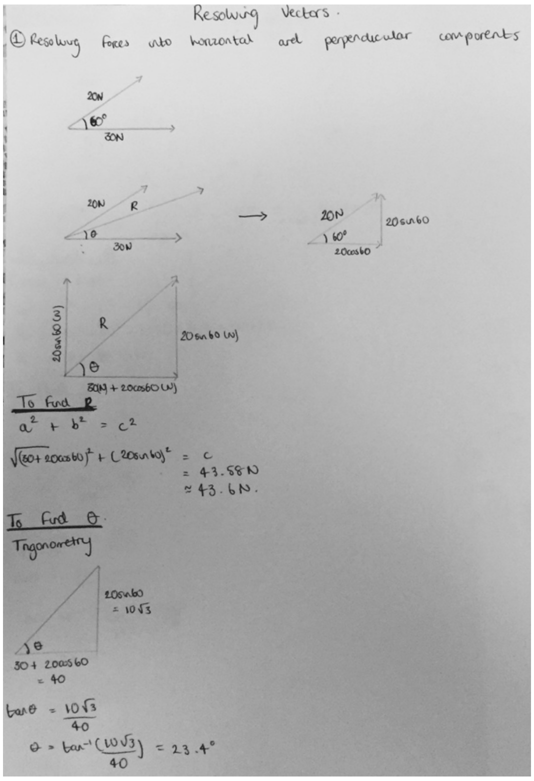


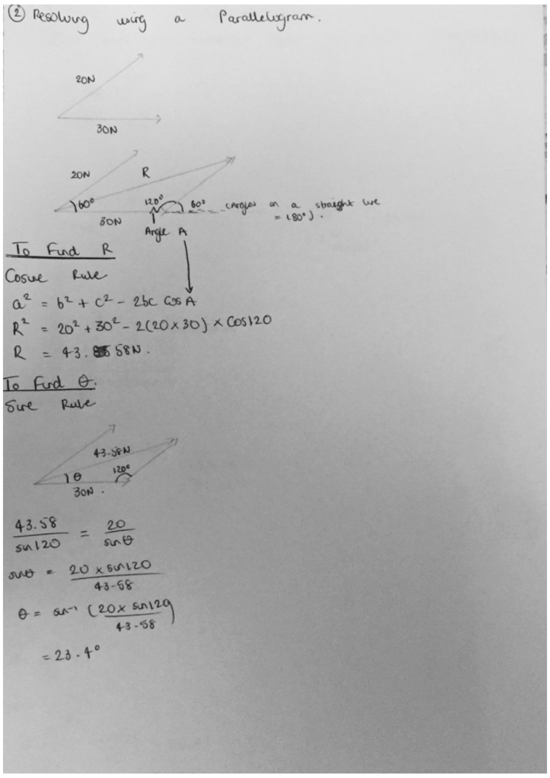
Here you can see that θ is the missing angle. To find it, you can call on your knowledge from GCSE to find the angle used SOHCAHTOA. So sinθ = 11/15.6 so θ = 45 degrees.

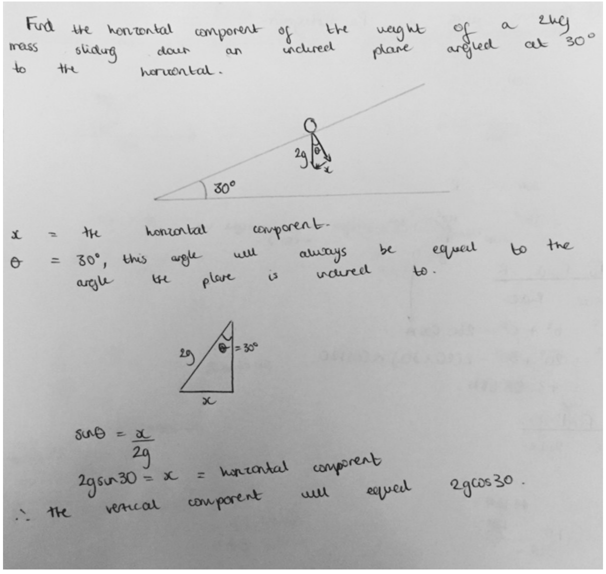
$$\sin\Theta = \frac{11\text{ km}}{15.6\text{ km}} = 0.7051$$

$$\Theta = \sin^{-1}(0.7051) = 45°$$

Resolving vectors not at right angles to each other







Conditions for equilibrium for two or three coplanar forces acting at a point. Appreciation of the meaning of equilibrium in the context of an object at rest or moving with constant velocity.

The conditions for three coplanar forces acting at a point is that they must form a closed triangle, or their individual forces must be resolved into vertical and horizontal components, so that the sums of both equal 0, if not then they are not in equilibrium. To form a closed triangle, in a scale drawing you can draw the individual vectors, and if you end up in the position you started then the object is in equilibrium. The same principle of resolving into horizontal and vertical components can be used for two coplanar forces also.

You can get equilibrium from an object moving at constant velocity, or at rest. The object moving at constant velocity has no resultant force acting on it, as if there were a resultant force acting on it, it would accelerate and thus would not be in equilibrium. An object at rest may have many forces acting on it, however they may cancel each other out therefore the object will stay at rest. If you look at and object at rest, the weight of the object acts downwards, therefore there must be an equal and opposite force acting upwards to make the resultant force 0.

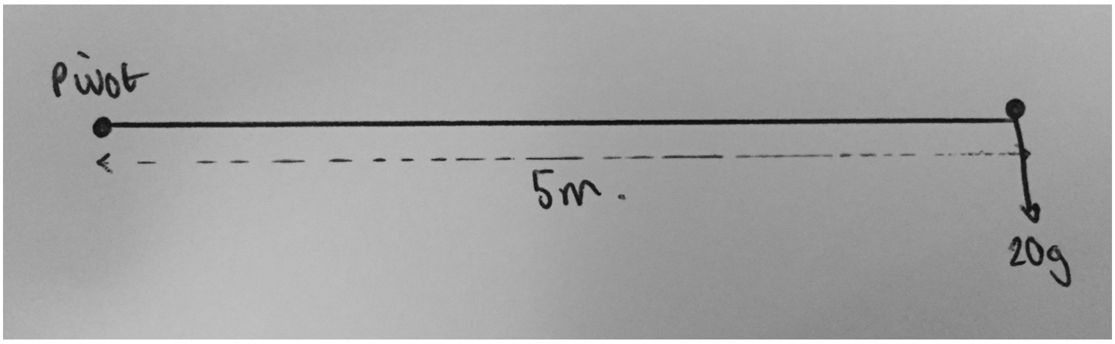
3.4.1.2 Moments

Content

- Moment of a force about a point.
- Moment defined as force x perpendicular distance from the point to the line of action of the force.
- Couple as a pair of equal and opposite coplanar forces.
- Moment of couple defined as force x perpendicular distance between the lines of action of the forces.
- Principle of moments.
- Centre of mass.
- Knowledge that the positon of centre of mass of a uniform regular solid is at its centre.

Moments

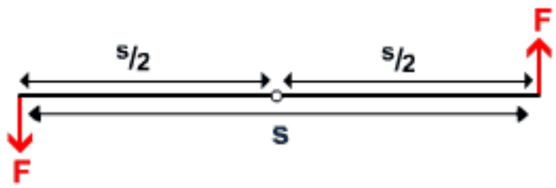
A moment is defined as the force x the perpendicular distance from the point to the line of action of the force. The unit of a moment is the Nm. If you look at the diagram below it gives a simple example of how to calculate the moment of a force about a point. An example of a question is given below.



The moment it is the force x perpendicular distance from the point to the line of action of the force. The force is equal to the mass multiplied by 'g', which is the acceleration due to gravity. So the force is written in shorthand as 20g, therefore the moment = 5 multiplied by 20g, equal to 100g. Leaving it as 100g is much easier than writing out the answer to 100 x 9.81.

Couple as a pair of equal and opposite coplanar forces and that the Moment of couple defined as force x perpendicular distance between the lines of action of the forces.

A couple is a pair of equal and opposite coplanar forces, oppositely directed, and the moment of a couple is called torque. An example is given below, where the moments of both forces are equal, in opposite directions. Therefore the object would keep spinning however there would be no acceleration. The moment would be F x s/2 upwards and downwards, so the resultant moment would be Fs. So in short, the moment of the couple shown below would be the force (F) multiplied by the perpendicular distance between the lines of actions of the forces, (s), so instead of taking of taking individual moments, you can do this and get the final moment equal to Fs.



Principle of Moments

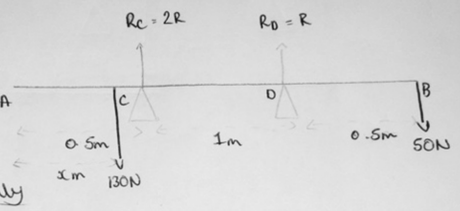
The principle of moments is that, **in equilibrium,** the sum of TOTAL anticlockwise moments is equal to the sum of TOTAL clockwise moments. If asked to state the principle of moments, do not be fooled into giving the equation, as that is the definition of moments, not the principle.

Centre of Mass

Centre of mass is essentially the distribution of mass in an object, and where that centre of mass acts. In uniform objects it always acts down the middle of the object, i.e. a uniform plank of wood 2m long. The centre of mass will act down the middle, as if it did not then if you were to put a pivot in the middle of the plank, there would be a resultant moment on one side as a larger force would be acting on that side. If you are given a non uniform object in a moment question, you may be asked to find where the centre of mass acts. If the system is in equilibrium, then you do as follows in the diagram below.

In the diagram below, the plank is non uniform therefore its centre of mass does not act directly down the middle. Given that the plank is in equilibrium, it means the forces should all be equal in the vertical plane, and the moments should be equal both in the clockwise and anticlockwise direction. Therefore you can get to the fact that 2R + R = 130 + 50, as since there is no vertical movement, the reactions of the supports should be the same value as the forces acting downwards. After you have done this, you can find the value of R and use then when taking clockwise and anticlockwise moments. So the forces that are attempting to move A upwards, in an anticlockwise direction are the supports, and the weight of the plank and object are attempting to make the plank move clockwise, relative to A. Once you put all the information in you can get to the final answer.

Plank is in equilibrium, has a weight of 130 N. Reaction at C is double that of D. Object of weight 50N at B. Find the distance of the centre of mass from A.



Resolve vertically

$2R + R = 130 + 50$

$R = 60N$

Taking moments about A

$(2R \times 0.5) + (R \times 1.5) = (130 \times x) + (50 \times 2)$

$R + 1.5R = 130x + 100$

$150 = 130x + 100$

$50 = 130x$

$0.38m = x$

$R + 1.5R = 150N$

as $R = 60N$

3.4.1.3 Motion along a straight line

Content

- Displacement, speed, velocity, acceleration.
- $v = \Delta s/\Delta t$
- $a = \Delta v/\Delta t$
- Calculations may include average and instantaneous speeds and velocities.
- Representation by graphical methods of uniform and non-uniform acceleration.
- Significance of areas of velocity-time and acceleration-time graphs and gradients of displacement-time and velocity-time graphs for uniform and non-uniform acceleration eg graphs for motion of bouncing ball.
- Equations for uniform acceleration:

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t \qquad v^2 = u^2 + 2as$$

- Acceleration due to gravity, g.

Opportunities for Skills Development

- Distinguish between instantaneous velocity and average velocity.
- Measurements and calculations from displacement-time, velocity-time and acceleration-time graphs.
- Calculations involving motions in a straight line.

Displacement, speed, velocity, acceleration.

Motion along a straight line can be evaluated using these four terms. Displacement is the essentially how far an object is relative to its origin. If a car travels 10km North, then 5km South, its overall displacement from the origin is 5m, however the distance it has travelled it clearly 15km. Speed is how fast something is going, calculated by the distance travelled divided by the time taken to travel the distance. Velocity is a scalar quantity, and is the rate of change of the displacement. Finally, acceleration is the rate of change of velocity.

Calculations may include average and instantaneous speeds and velocities.

To find the average velocity you would divide the displacement by the time taken. For example, if someone move 12 metres in 24 seconds, their average speed would be $0.5ms^{-1}$. However if you were to say they moved 6 metres forward then 6 metres back to their origin, then their displacement is 0m hence an average velocity of $0ms^{-1}$. Instantaneous speed/velocity are the speed/velocity at a given time. One way to calculate these, if you are given a distance time graph or displacement time graph, is to draw a tangent at a given time then work out the gradient to find the instantaneous speed/velocity.

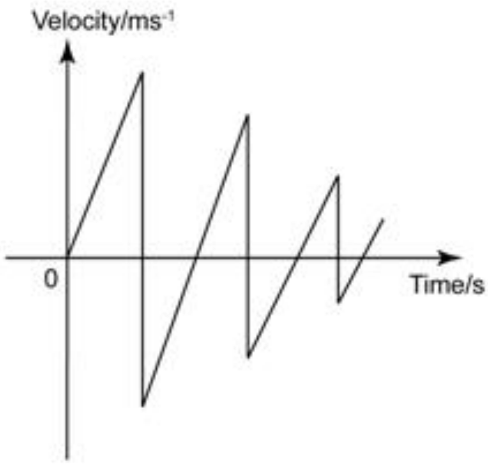
Representation by graphical methods of uniform and non-uniform acceleration.

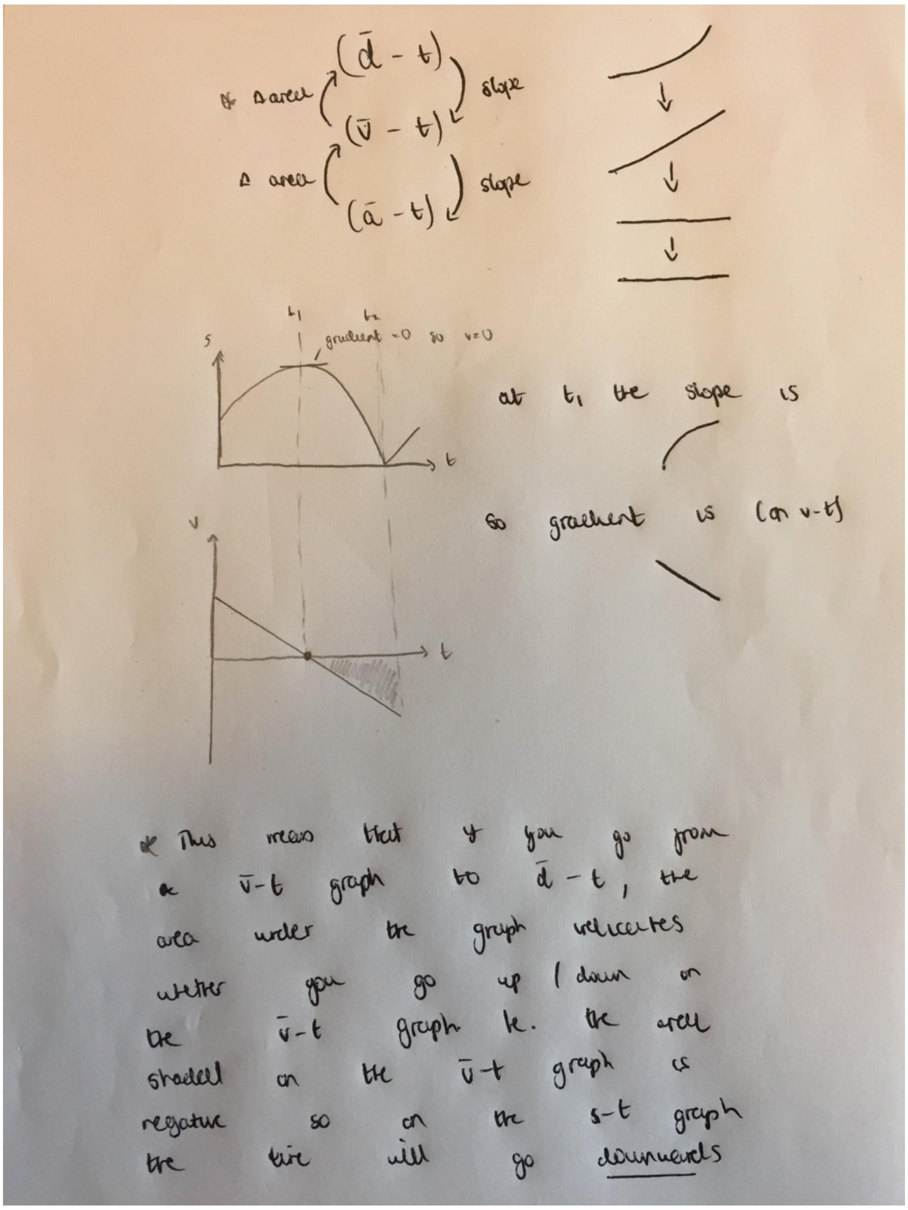
On a velocity time graph, a straight line indicates uniform acceleration (not changing), however if the line curves then there is non uniform acceleration.

Significance of areas of velocity-time and acceleration-time graphs and gradients of displacement-time and velocity-time graphs for uniform and non-uniform acceleration eg graphs for motion of bouncing ball.

Area of velocity-time graph = distance travelled.
Gradient of velocity-time graph = acceleration.
Area of acceleration-time graph = change in velocity.
Gradient of displacement-time graph = velocity.

The graph to the below illustrates a ball bouncing. Initially the ball is at rest raised off the ground. Since velocity is a vector quantity, when the ball changes direction as it hits the ground there will be a change in velocity as it goes from negative to positive.
So if you were to unpick the graph, the gradient should always be constant, as the acceleration/deceleration is always constant, at $9.8ms^{-2}$. This is because gravity is the only force acting on the ball.
When the ball is dropped, since we have picked downwards as the positive direction, it increases to a certain speed Vms^{-1}. As it bounces up its direction has changed so the velocity instantaneously reverts to a negative velocity, falling to 0 as it reaches its maximum height. This maximum height is shown by the point where the velocity is at 0, as once it begins to fall from its maximum height the velocity increases up until a maximum velocity again, before it bounces and repeats the process. The reason that the maximum velocity keeps decreasing is because when the ball hits the ground it loses energy, so there is less energy that can be converted to kinetic energy and therefore gravitational potential energy, thus the maximum speed will keep decreasing.





In the above picture, the slope

Equations for uniform acceleration

The equations for uniform acceleration are shown below, where v = final velocity, u = initial velocity, a = acceleration and t = time.

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t \qquad v^2 = u^2 + 2as$$

Acceleration due to gravity, g.

Acceleration due to gravity, otherwise notated as ‘g’ is equal to $9.81ms^{-2}$, or just $9.8ms^{-2}$ if you are told to use this value of g. It is the rate at which objects accelerate towards earth, or the rate at which objects decelerate if they are travelling in the opposite direction to gravity providing no other forces are acting upon them.

3.4.1.4 Projectile Motion

Content

- Independent effect of motion in horizontal and vertical directions of a uniform gravitational field. Problems will be solvable using the equations of uniform acceleration.
- Qualitative treatment of friction.
- Distinctions between static and dynamic friction will not be tested.
- Qualitative treatment of lift and drag forces.
- Terminal speed.
- Knowledge that air resistance increases with speed.
- Qualitative understanding of the effect of air resistance on the trajectory of a projectile and on the factors that affect the maximum speed of a vehicle.

Opportunities for Skills Development

- Investigation of the factors that determine the motion of an object through a fluid.
- Independent effect of motion in horizontal and vertical directions of a uniform gravitational field. Problems will be solvable using the equations of uniform acceleration.

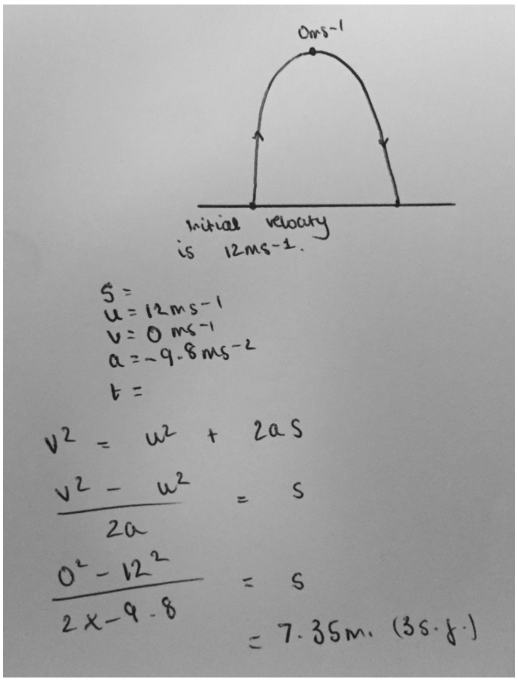
Independent effect of motion in horizontal and vertical directions of a uniform gravitational field. Problems will be solvable using the equations of uniform acceleration.

The effect of motion in the horizontal field is based on an assumption that air resistance is negligible. Thus the acceleration in the horizontal plane will be 0. So if a particle is fired with horizontal velocity of $15ms^{-1}$ then this velocity will still be $15ms^{-1}$ at the instant before the particle hits the ground. In the vertical direction however, if a particle is fired upwards its acceleration due to gravity will be $-9.81ms^{-2}$. This is because gravity acts downwards, so as the particle is moving upwards, gravity is acting against it, causing deceleration at $9.81ms^{-2}$. Therefore, the acceleration due to gravity is $+9.81ms^{-2}$ as the particle is moving in the same direction as gravity is acting, causing an acceleration.

Questions on projectile motion can be solved using the equations for uniform acceleration shown below.

$$v = u + at \qquad s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t \qquad v^2 = u^2 + 2as$$



In the question above, we are trying to find the maximum height reached by an object thrown vertically upwards from the ground. The object is thrown with initial velocity $12ms^{-1}$. So we know the initial velocity, we also know the final velocity, at maximum displacement, will be $0ms^{-1}$. Also since the object is travelling upwards, it is working against gravity so its acceleration is $-9.8ms^{-2}$. Therefore, we know three variables so can work out the displacement (s). It turns out to be 7.35m, using the equation $v^2 = u^2 + 2as$.

Qualitative treatment of friction.

Qualitative refers to looking at somethings quality as opposed to quantity. Thus friction can be defined at the resistance on an object attempting to move over another object. Whilst friction is usually looked at as being a hindrance, it is actually necessary for an object to move. Without friction, objects like cars would not be able to move forward on the road, as the wheels would keep turning but there would be no friction to allow the car to move, like a car moving on ice. Also, objects in motion would never rest until acted on by another force. To understand this fully we must apply Newton's Third Law of motion that states, "If a body A exerts a force on body B, then said body B will exert an equal and opposite force on body A. These forces will be of the same type, acting on two different bodies. So if we apply this to walking, saying body A is your foot and body B if the ground. If your foot exerts a force X in order to move forward, there will be a frictional force X that will act in the opposite direction. Your foot exerts a force backwards to move you forward, thus the frictional force acts forwards on your foot. This frictional force drives the movement. However on a surface with a lower coefficient of friction, clearly it will be easier to walk, but too small of a frictional force e.g. on ice, and you will fall.

3.4.1.5 Newton's Laws of Motion

Content

- Knowledge and application of the three laws of motion in appropriate situations.
- F=ma for situations where mass is constant.

Opportunities for Skills Development

- Students can verify Newton's second law of motion.
- Students can use free-body diagrams.

Knowledge and application of the three laws of motion in appropriate situations.

Newton's First Law: This law is sometimes referred to as inertia, and states that an object will stay in constant motion, or rest, unless acted on by a resultant force.

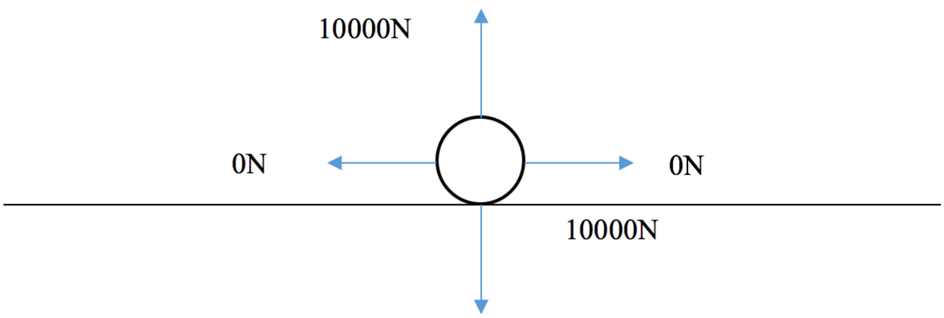
Newton's Second Law: It states that the force applied, F, is equal to the mass multiplied by acceleration, F=ma. This is derived from the fact that Force is proportional to the change in momentum divided by the time, $F \propto \frac{Change\ in\ Momentum}{time}$. We know that change in momentum is equal to the impulse, which is calculated by using m(v-u), otherwise written as Δmv. So substituting in m(v-u) to the equation gives $\frac{m(v-u)}{t}$ where $\frac{(v-u)}{t}$ is equal to acceleration, giving $F \propto ma$. This can be written as F=kma, where k is the constant of proportionality which conveniently is equal to 1, so can be written as F=ma.

Newton's Third Law: This appears to be the simplest law, but in practice can be quite difficult. It states that if a body A exerts a force on body B, then body B will exert an equal and opposite force on body A. These forces, as well as being equal, will be opposite, and act on the two different bodies.

Application of the Laws

Newton's First Law:

1. Consider a car at rest, where we will look at the horizontal and vertical forces acting on the car. If we look at the car as a uniform particle, it gives a clearer representation



The object is taken to have a mass 1000kg, and so an approximate weight of 10000N. This object is at rest, and so if we resolve the forces horizontally we find there is no resultant force. Also if we resolve the forces horizontally we find that 1000-1000 also equals 0. The force acting downwards is the force exerted on the earth by the object. The arrow acting upwards is actually the reaction of this force, and so should really be below the arrow of the weight acting downwards on the diagram. Although this is usually the easiest notation to visualise.

2. If you were to have a particle moving with constant velocity, then clearly the resultant force acting must be 0, otherwise you would get an acceleration. So if there was 2000N acting east and 2000N acting west, this provides a resultant force of 0. So F=ma would be 0 as F = 0, this proving there would be no acceleration.
3. If we were to replace the horizontal forces with 500N acting east, and 200N acting west, you would find a resultant force of 200N. Thus providing the horizontal forces stay balanced, there will be no horizontal movement, and movement east with a force of 300N. This force can then be used to work out the acceleration of the object using F=ma. So for this car/particle of 1000kg, its acceleration would be $0.3ms^{-2}$. This links to Newton's Second Law.

F=ma for situations where mass is constant.

F=ma only works where mass is constant, as where an object approaches the speed of light, its mass changes significantly. This is where Einstein's equation $E=mc^2$ comes in useful, however knowledge of this is not required for this unit.

3.4.1.6 Momentum

Content

- Momentum = mass x velocity
- Conservation of linear momentum.
- Principle applied quantitatively to problems in one dimension.
- Force as the rate of change of momentum, $F = \Delta (mv)/ \Delta t$
- Impulse = Change in momentum.
- $F\Delta t = \Delta(mv)$, where F is constant.
- Significance of area under a force-time graph.
- Quantitative questions may be set on forces that vary with time. Impact forces are related to contact time (eg kicking a football, crumple zones, packaging).
- Elastic and inelastic collisions.
- Appreciation of momentum conservation issues in the context of ethical transport design.

Opportunities for Skills Development

- Students can apply conservation of momentum and rate of change of momentum to a range of examples.

Momentum = mass x velocity

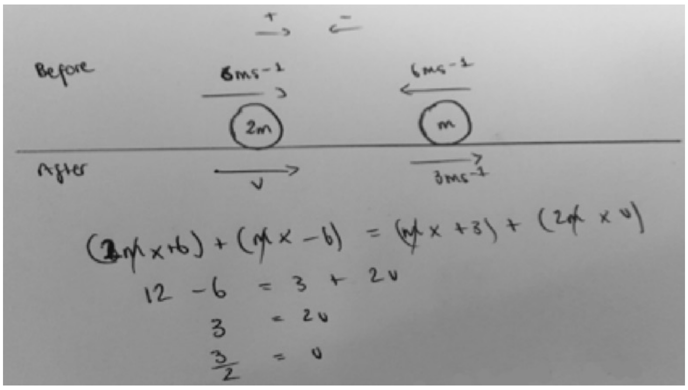
Momentum is a quantity associated will all objects that are moving. It is simply calculated by the mass multiplied by the velocity, but remember that it is a vector. This means that depending on which direction of motion you pick to be positive, the other direction will be negative. For example two particles A and B moving along a horizontal plane, before they collide they both have a velocity of $4ms^{-1}$ and mass 2kg and 4kg respectively. This means that before the collision particle A had momentum $8kgms^{-1}$, and particle B had momentum $16kgms^{-1}$. If we say that particle A was moving in the negative direction, then its momentum would be $-8kgms^{-1}$, and so the momentum of particle B would be positive.

Conservation of linear momentum

This states that provided no external forces act on the objects, total momentum before the collision is equal to total momentum after the collision. An external force could be friction for example.

Principle applied quantitatively to problems in one dimension.

To apply this to real situations, we must first understand that $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$. However this is a complete over complication of the principle, and if you ensure you understand momentum and its properties this equation is unnecessary. I have shown this in the question below. By considering total momentum before and total momentum after, you can perform the equation above without remembering it, as you are not given it in the equation sheet, essentially it is trivial.



The question above shows a simple conservation of linear momentum problem. It asks you to find the final velocity of the particle with mass 2m. Since the momentum before must be equal to the momentum after, you can form the equation and solve to find v, after cancelling the common 'm' in the equation. It turns out to be 1.5ms^{-1} in the positive direction.

Since velocity is a vector quantity, remember the velocity can be negative or positive, so you need to pick a positive and negative direction for the velocity. The left direction in the question above is the negative direction, so that is why the initial momentum of the particle mass m was -6 multiplied by m.

Force as the rate of change of momentum, $F = \Delta(mv)/\Delta t$

Simply, the force on an object i.e. if you kick a football, is equal to the change in momentum divided by the change in time. I will provide examples of this principle later on in this section.

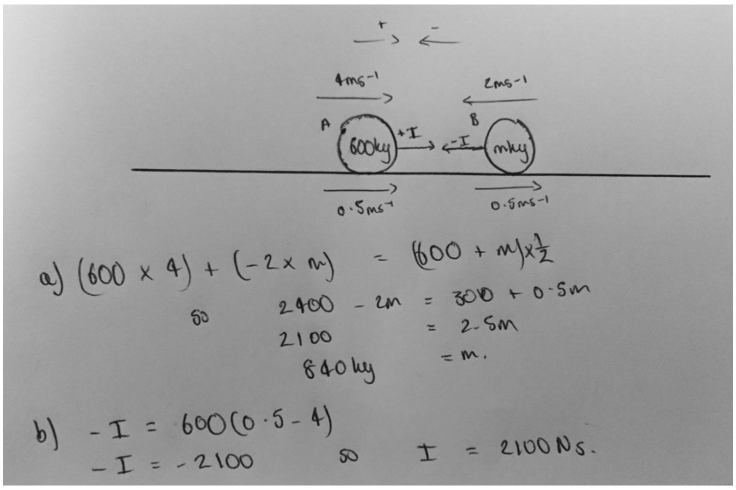
Impulse = Change in momentum.

Impulse is equal to the change in momentum, and so what is the change in momentum? As I have mentioned previously, this is equal to I = m(v-u), or mv-mu. However remember the vector nature of velocity, since mass is a scalar and velocity is a vector, a scalar multiplied by a vector equals a vector. So in summary be careful with the signs used, i.e. the direction of the impulse, where it is negative or positive, and the direction of the velocity. The example below shows this. However, if you are only asked for the 'magnitude' of the impulse then the impulse will not have a positive or negative value, but the velocities you use will still require positive and negative directions. If you are studying M1 then momentum and impulse will become trivial, if you are not then it may be worthwhile to attempt some past paper questions on momentum from M1 to further your knowledge.

Practice Question

Two trucks *A* and *B*, moving in opposite directions on the same horizontal railway track, collide. The mass of *A* is 600 kg. The mass of *B* is *m* kg. Immediately before the collision, th speed of *A* is 4 ms^{-1} and the speed of *B* is 2 ms^{-1} Immediately after the collision, the trucks are joined together and move with the same speed 0.5 ms^{-1}. The direction of motion of *A* is unchanged by the collision. Find

(a) The value of m,
(b) The magnitude of the impulse exerted on A in the collision.



The first answer is another simple conservation of linear momentum question. Again, be careful about the signs you use as velocity is a vector quantity.

However part (b) in an impulse question. Impulse is defined as the rate of change of momentum, so I = mv-mu or m(v-u). For this question it asks to find the impulse exerted on A, which is given by B. The direction of the impulse of B is going in the left direction, which is a negative as we picked left as the negative direction. So the impulse is equal to –I = 600(0.5-4) so I = 2100Ns after you rearrange the equation.

FΔt = Δ(mv), where F is constant.

Using F = Δmv/Δt, we can rearrange to get FΔt = Δmv. This means that the force multiplied by the time is equal to the change in momentum.

Significance of area under a force-time graph.

If you draw a force-time graph, you can find the change in momentum by calculating the area under the graph.

Quantitative questions may be set on forces that vary with time. Impact forces are related to contact time (eg kicking a football, crumple zones, packaging).

If we are to look at forces that vary with time, the best example is kicking a football. Using the equation for calculating the force, we can see that if the change in momentum is constant, however the time is decreased then the impact force increases. Thus kicking a football to produce the same change in momentum, but in a quicker time will increase the force on the ball. This principle is also extremely useful in cars. Crumple zones aim to increase the contact time in a crash, thus reducing the force on both cars, and possibly saving lives. The crumple zones will do exactly as they sound, and essentially fold in on themselves. This can also be applied to packaging of an expensive object i.e. a watch. The packaging will again reduce the force on the object by increasing contact time if for example the watch were to be dropped on the floor in transit.

Elastic and inelastic collisions.

Quite simply, in an elastic collision kinetic energy is conserved, likewise momentum is too and it essentially an idealised situation. However, in an inelastic these quantities are not conserved, as kinetic energy may be lost as thermal energy, or energy from the sound created. In practice, most collisions are inelastic.

Appreciation of momentum conservation issues in the context of ethical transport design.

AQA Jan 2010 Unit 4 Section A Q2

Question:

Water of density 1000 kg **m^{-3}** flows out of a garden hose of cross-sectional area 7.2 x 10^{-4} m^2 at a rate of 2.0 x 10^{-4} $\underline{m^3}$ per second. How much momentum is carried by the water leaving the hose per second?

Answer:

Momentum = Mass x Velocity

Mass = Density x Volume
= 1000 x (2 x 10^{-4}) because 2 x 10^{-4} m^3 are flowing each second (asks for momentum per second)
So mass = 0.2kg

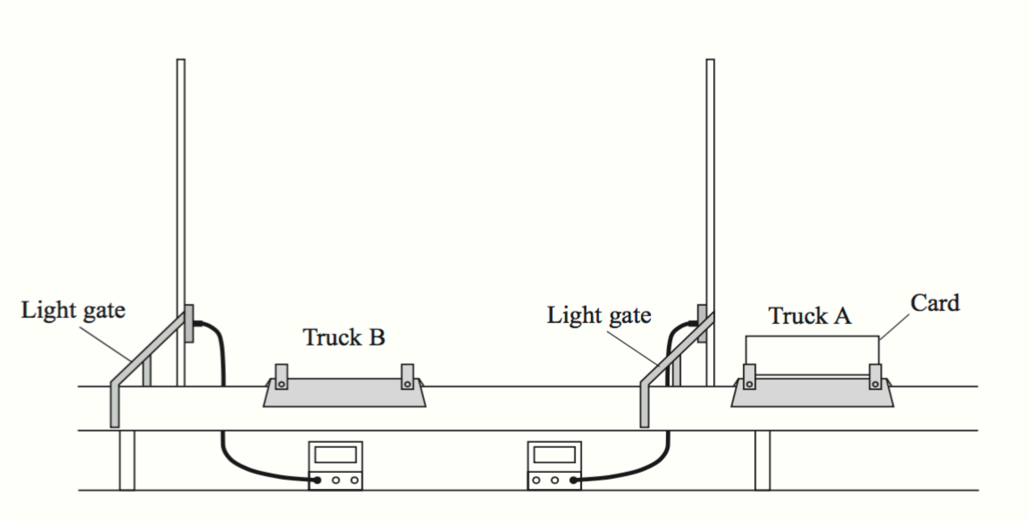
Velocity. If you divide 2 x 10^{-4} m^3s^{-1} by 7.2 x 10^{-4} m^2, you will get 0.277…ms^{-1}, thus the units are units of velocity.

So Momentum = 0.2 x 0.277.. = 0.05555… ≅ 5.6 x 10^{-2} Ns.

Edexcel Jan 2011 Unit 6 Q2a

Question:

A student has an air track which has two trucks, A and B, supported by a cushion of air. He does an experiment to see whether momentum is conserved when the two trucks collide.



Using an air track reduces friction on the trucks. State why this is important in a momentum conservation experiment.

Answer:

Conservation of momentum only applies in there are no (resultant) external forces

Question:

The student uses two light gates as shown in the diagram. Truck A carries a card of negligible mass and length l. A light gate records the time t taken by the card to pass through it.

Explain how you would show that the air track is horizontal before starting the experiment.

Answer:

Truck A given a (gentle) push (1)
The times shown on both timers are similar/calculated velocities are similar (1)
OR
Put truck on track, check if it remains stationary (1) Check in more than one position/use both trucks (1)
OR
Use spirit level (1) Check bubble in middle/check in more than one position (1)
OR
Check height of track above bench with rule and set square (1)
At both ends (1)

Question:

Truck B carries no card and is placed so that it is stationary between the light gates. Truck A is set off towards truck B. As the card passes through the first gate it records a time t_1. Truck A then collides with truck B. They stick together and move through the second gate which records the time t_2

Both trucks have the same mass. Explain why $t_2 = 2t_1$ if momentum is conserved.

Answer:

Mass doubles
(So) velocity halves

time = (card) length/velocity

(so time is doubled)

allow mathematical proof which hides *v* ratio e.g. $mu = 2mv$ then we know that $u = l/t_1$, and $v = l/t_2$ so substitute these values in and solve for $t_2 = 2t_1$

Question:

The student records the following data for 5 separate collisions:

t_1/s	0.34	0.15	0.21	0.28	0.24
t_2/s	0.70	0.35	0.39	0.55	0.52
t_2/t_1	2.1	2.3	1.9	2.0	2.2

Use this data to discuss whether momentum can be considered to be conserved in this experiment.

Answer:
(We know that since $t_2 = 2t_1$ for conservation of momentum, $t_2/t_1 = 2$)

1. Mean ratio $t_2/t_1 = 2.1$ (1)
2. Uncertainty is + 0.2 (1)
3. Uncertainty range includes 2.0 (1)
4. (Hence, yes, momentum is conserved)

Alternatives for last 2 marks

1. Calculates % difference as 5% (1)
2. 5% is less than the experimental uncertainty of 9.5% (1)
3. (Hence, yes, momentum is conserved)

3.4.1.7 Work, Energy and Power

Content

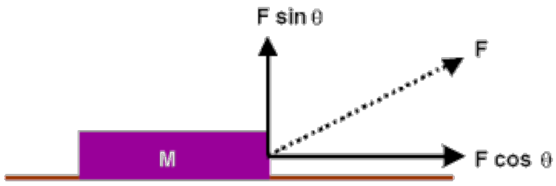
- Energy transferred, $W = Fs\cos\theta$
- Rate of doing work = rate of energy transfer, $P = \Delta W/\Delta t = Fv$
- Quantitative questions may be set on variable forces.
- Significance of the area under a force-displacement graph.
- Efficiency = useful power output/ input power.
- Efficiency can be expressed as a percentage.

Opportunities for Skills Development

- Investigate the efficiency of an electric motor being used to raise a mass through a measured height. Students should be able to identify random and systematic errors in the experiment and suggest ways to remove them.

Energy transferred, $W = Fs\cos\theta$

For an object to move, work needs to be done on said object as a result of action from a force. Work done is defined as the force multiplied by the distance moved in the direction of the force. This therefore means that one joule of energy is equal to the energy required to move 1N through a vertical height of 1m. To further the equation 'W=Fd", the equation "W=Fscosθ" provides you with the work done when the force may be acting at an angle. So you can calculate the horizontal component of the force and multiply this by the displacement to find the work done. If θ were to equal 90 then the work done would be 0, as cos(90) is equal to 0.



Rate of doing work = rate of energy transfer, $P = \Delta W/\Delta t = Fv$

The rate of doing work is called the power output, so the work done over the time taken. Since the work done is equal to the force x distance moved in direction of force, the equation P = Fv can also be derived. F = Force and v = velocity.

Quantitative questions may be set on variable forces.

Significance of the area under a force-displacement graph.

If you are given a force-displacement graph, then the work done will be the area underneath the graph. If the line is not a straight line then you may be required to use the trapezium rule to provide an estimate for the value.

Efficiency = useful power output/ input power.

Simply the efficiency is the useful power output divided by the input power. So if the input power were 40kW and the useful power output was 20kW, then 20/40 = 0.5

Efficiency can be expressed as a percentage.

The efficiency can be expressed as a percentage, so the fraction above could be multiplied by 100 to get 50% efficiency.

3.4.1.8 Conservation of Energy

Content

- Principle of conversation of energy.
- $\Delta E_p = mg\Delta h$ and $E_k = 1/2mv^2$
- Quantitative and qualitative application of energy conservation to examples involving gravitational potential energy, kinetic energy, and work done against resistive forces.

Opportunities for Skills Development

- Estimate the energy that can be derived from food consumption.

Principle of conversation of energy.

Energy cannot be destroyed or created, so for example in a collision, the total energy before the collision will be equal to the total energy after the collision if you ensure to include energy loss to things like heat or sound.

$\Delta E_p = mg\Delta h$ and $E_k = 1/2mv^2$

The gravitational potential energy is equal to the mass multiplied by the gravitational field strength (9.81), multiplied by the change in height. Whereas kinetic energy is equal to $1/2mv^2$ where m equals the mass, and v being velocity. These are quantities always possessed by objects, however if an object is 0m from the surface of the earth it will therefore have 0J of gravitational potential energy. For a football mass 1kg and velocity $10ms^{-1}$, its kinetic energy would be 50J.

Quantitative and qualitative application of energy conservation to examples involving gravitational potential energy, kinetic energy, and work done against resistive forces.

3.4.2 Materials

3.4.2.1 Bulk Properties of Solids

Content

- Density, $\rho = m/v$.
- Hooke's Law, elastic limit,
- $F = k\Delta L$, k as stiffness and spring constant.
- Tensile strain and tensile stress.
- Elastic strain energy, breaking stress.
- Energy stored = $1/2F\Delta L$ = area under force-extension graph.
- Description of plastic behaviour, fracture and brittle behaviour linked to force-extension graphs.
- Quantitative and qualitative application of energy conservation to examples involving elastic strain energy and energy to deform.
- Spring energy transformed to kinetic and gravitational potential energy.
- Interpretation of simple stress-strain curves.
- Appreciation of energy conservation issues in the context of ethical transport design.

Opportunities for Skills Development

- Students can compare the use of analogue and digital metres.
- Estimate the volume of an object leading to an estimate of its density.

Density, $\rho = m/v$

Density is equal to the mass per unit volume, so has the units kgm^{-3}.

Hooke's Law, elastic limit

Hooke's Law states that the extension of a spring/wire etc. is proportional to the force applied. Once a spring reaches its elastic limit, it will not return to its original length and is permanently deformed (plastic deformation).

$F = k\Delta L$, k as stiffness and spring constant.

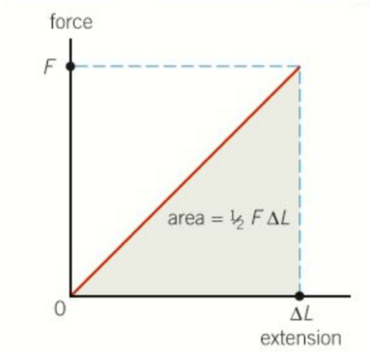
The equation $F = k\Delta L$ gives us the equation to find the force applied (F), the extension (L) or the spring constant (k). k is essentially the stiffness of a spring, which is referred to as the spring constant. The greater the value of k, the stiffer the spring.

Tensile strain and tensile stress.

Tensile strain is the force per unit area, so F/A, so has the units Nm^{-2}. Tensile strain is the extension divided by the length, so $\Delta L/L$, thus has no units as is a ratio.

Elastic strain energy, breaking stress

Elastic potential energy is stored in a stretched spring, and so when released, this potential energy will become kinetic energy. However the work done in stretching a spring by an extension ΔL, is given by $\frac{1}{2}$ FΔL. The change in extension is from the unstretched length, and F is the force required to extend it by ΔL. This work done in stretching the spring is elastic potential energy stored, and is what the area under a graph represents in a force – extension graph, as you can see below.

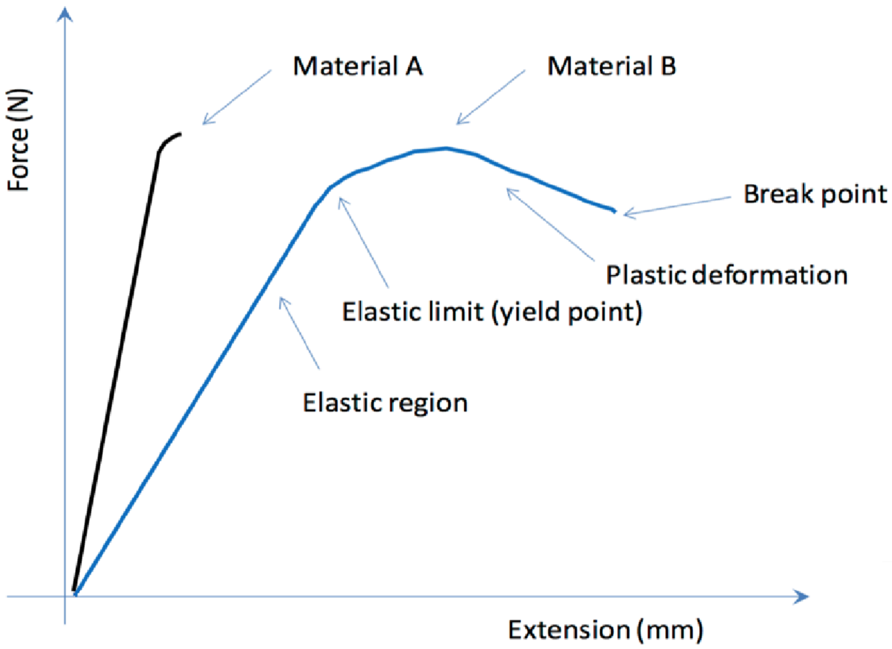


Breaking stress is the maximum tensile stress that an object can take before failing by breaking.

Energy stored = 1/2FΔL = area under force-extension graph.

Description of plastic behaviour, fracture and brittle behaviour linked to force-extension graphs.

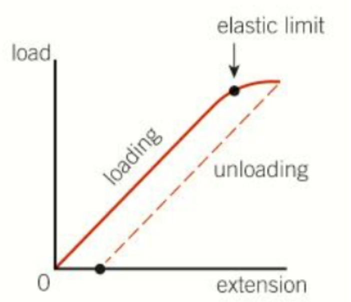
Plastic behaviour is permanent deformation or change in shape of a solid body under the action of a sustained force, without the object breaking. A graph below shows a brittle material (A), and a ductile material showing plastic behaviour (B).



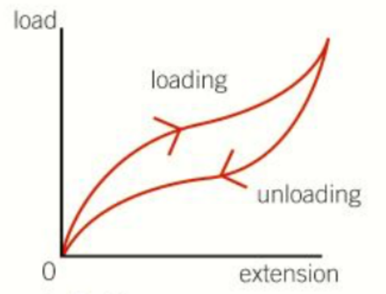
A brittle material (A) under load breaks without undergoing a significant amount of prior deformation, so is usually quite sudden. It will absorb relatively little energy prior to its fracture, shown by the small area underneath its line. So brittle materials can usually withstand a large force, showing little plastic deformation. Examples are glass or concrete. If we look at material B, it can be described as ductile. This is because it can be easily stretched without breaking or decreasing in strength, so can also withstand large forces. They show areas of elastic deformation and also plastic deformation, as seen on the graph above. Therefore these types of materials can be drawn into long thin wires without breaking, like copper or steel.

Loading and Unloading Graphs

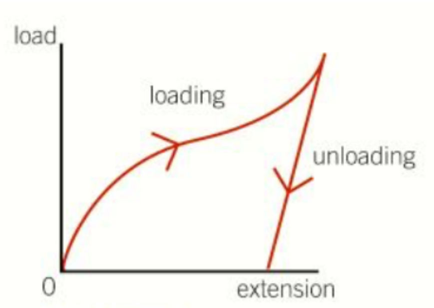
The graph below shows what you would expect from a metal wire. The wire will return to its original length when unloaded provided the elastic limit is not reached. Although beyond this limit, the unloading wire will return parallel to the loading wire. It also means that the wire will be permanently longer than its original length.



This graph shows what you expect to see from a rubber band. The rubber band will return to its original length, with the unloading curve lower than the loading curve apart from when there is maximum extension, and when they are at 0. The rubber band stays elastic as it regains its initial length, but it has a low limit of proportionality so curves quickly. Another thing is that the extension when unloading is greater than that when loading for given tensions.



For a polythene strip, the graph will look as is shown below. The unloading extension will also be greater than that of the loading extension. Also the strip will not return to its original length, and will follow a straight line almost directly downwards. The strip suffers from plastic deformation and has a very low limit of proportionality, as shown by the graph.



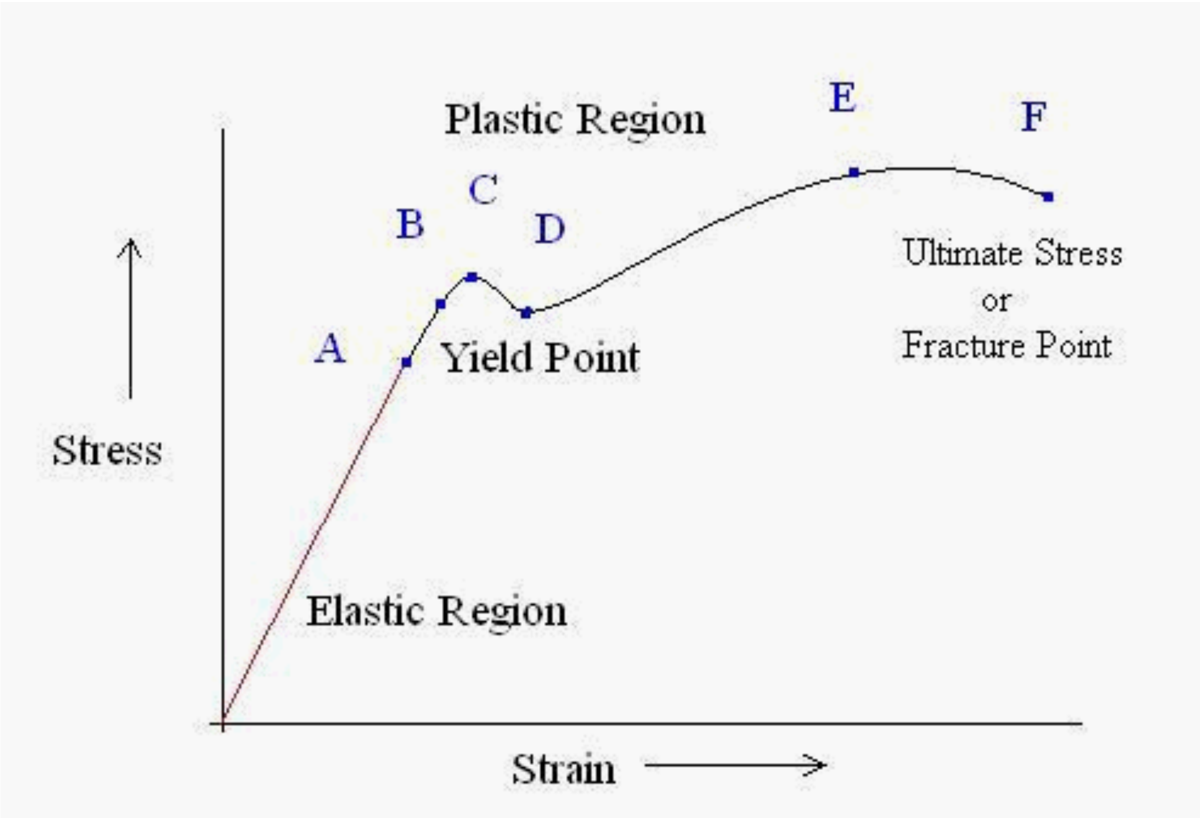
Quantitative and qualitative application of energy conservation to examples involving elastic strain energy and energy to deform.

If you were to take a rubber band, work is done in stretching the rubber band, so work is done to deform it. This work is converted into elastic strain energy within the rubber, and if this applied stretching force is then slowly reduced, the rubber band will use this energy to "pull" back. If the force is removed then the band will retract quickly, and this strain energy would be converted into kinetic energy and sound energy. If you were to bend a metal bar, and then lay it on the ground, the bar will not nearly recover its original position. This is because it has undergone permanent deformation, and the elastic potential energy that was stored was then converted into heat energy, resulting in a slight temperature rise of the bar. In any material **undergoing deformation, at least some of the supplied energy will be converted into heat**. However looking at an ideal elastic material, we assume that all energy supplied is converted into strain energy, and when the material returns to its original position there is no energy loss.

Spring energy transformed to kinetic and gravitational potential energy.

As mentioned in the previous point, spring energy can be transformed to kinetic energy. The same is also true for gravitational potential energy, so if a spring is pulled downward, its loss in potential energy will be transformed to elastic potential energy.

Interpretation of simple stress-strain curves.



O to A: The limit of proportionality is shown by A, and from the origin to point A the material obeys Hooke's Law. After A the proportionality is lost and the graph begins to curve. The straight line can be referred to as the elastic region, as the material can regain its original shape after the load has been removed.

A to B: Between A and B the strain increases faster than the stress, and point B is where any continuous stress will result in permanent, plastic deformation. B is known as the elastic limit.

B to C: Beyond B, the material goes to the plastic stage. After C, the material starts decreasing in its cross-sectional area, so the stress decreases down to point D. This point is called yield point 1.

C to D: This is the second yield point, and is where a small increase in the tensile stress causes a large increase in tensile strain, as the material undergoes plastic flow.

D to E: This point is the point of **ultimate tensile stress** (UTS), and beyond this point the wire loses its strength, extends, and becomes narrower approaching its weakest point. This is also shown by a decreased cross section as well, and this continues until the wire breaks (breaking stress).

E to F: The point of fracture.

The strength of a material is therefore its point of maximum tensile stress (UTS).

Appreciation of energy conservation issues in the context of ethical transport design.

OCR (A) A Level Specimen 1 Q18bc

Question:

State the meaning of *elastic* and *plastic* behaviour.

Answer:

Elastic: material returns to original dimensions when load is **B1** removed.
Plastic: material has permanent change of shape when load is removed

Question:

Repeatedly stretching and releasing rubber warms it up.

Fig. 18.1 shows a force-extension graph for rubber.

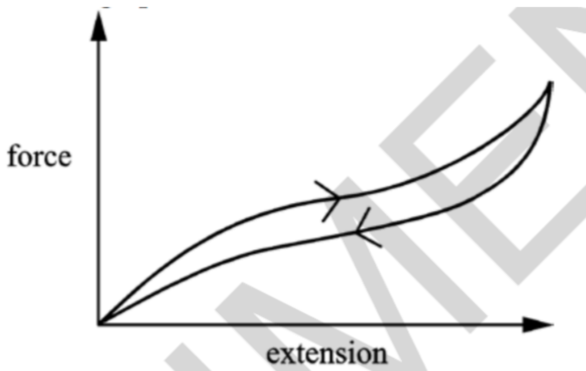


Fig. 18.1

Rubber is an ideal material for aeroplane tyres. Using the information provided, discuss the behaviour and properties of rubber and how its properties minimise the risks when aeroplanes land

Answer:

- The material is elastic because the removal of force returns the **B1** rubber to its original length.
- The area under force-extension graph is work done.
- Repeated stretching and releasing the rubber warms up the rubber because not all the strain energy is returned back. The area enclosed represents the amount of thermal energy. During landing, some of the aeroplane's kinetic energy is transferred to thermal energy and therefore the aeroplane does not "bounce" during landing; hence this minimises the risk to passengers.

3.4.2.2 The Young Modulus

Content

- Young Modulus = Tensile Stress/ Tensile Strain = FL / AΔL
- Use of stress-strain graphs to find Young Modulus.
- (One simple method of measurement is required.)

Young Modulus = Tensile Stress/ Tensile Strain = FL / AΔL

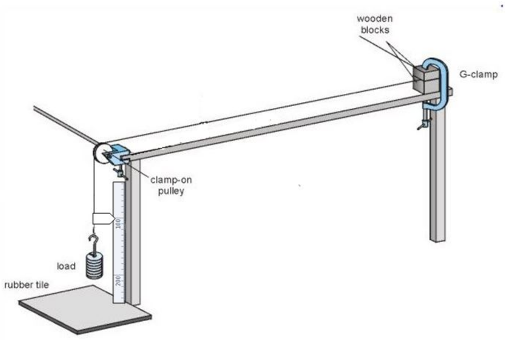
Young modulus is equal to the tensile stress divided by the tensile strain, as given in the equation above. It is essentially a measure of elasticity.

Use of stress-strain graphs to find Young Modulus.

The gradient of the stress-strain graph is the value of young's modulus.

One simple method of measurement is required.

You might be given a 6 marker on how to find the young's modulus of a material. Here is one method you could use. Initially you may like to draw a diagram, showing a workable arrangement of suitable apparatus. An example of an arrangement is shown in the picture below. So if you were finding the Young's modulus of a wire, you would need to find its cross sectional area, so would need to measure its diameter. Its important you say you are measuring the diameter, not the radius, as you are going to calculate the radius. To measure the diameter a micrometer can be used, various measurements along the wire should be taken to obtain a mean diameter. Then you can half this value for the radius. After this you can proceed to outline your arrangement of apparatus. You will need to make sure that the wire is taut, so you could add an initial mass to ensure this, and then measure the original length of the wire. After this you will be adding a range of masses to the wire, ensuring you use at least seven different masses, and repeat measurements for the same wire. You will then need to measure the extension i.e. using a metre rule alongside a pointer touching scale or set square (to reduce parallax error). The weight (force) can be calculated using the mass multiplied by g. Once you have all of your measurements of L, ΔL, F and A, you can plot a graph of stress against strain and calculate the gradient to find the Young's Modulus.



These points are a variety of different points that you can use, and you may decide to choose a different method. However irrespective of this, it would be ideal to include over 5 points, alongside points on how you can make your results more accurate i.e. repeat readings.

Edexcel June 2009 Unit 3B Q6d

(not added yet)

OCR (A) A Level Specimen 1 Q18a

Question:

A group of scientists have designed an alloy which is less dense than copper but may have similar mechanical properties. A researcher is given the task to determine the Young modulus of this alloy in the form of a wire.

Write a plan of how the researcher could do this in a laboratory to obtain accurate results. Include the equipment used and any safety precautions necessary.

Answer:

Equipment used safety

- Wire fixed at one end with load added to wire
- Suitable scale with suitable marker on wire.
- Micrometer screw-gauge or digital/vernier callipers for measuring diameter of wire
- Reference to safety concerning wire snapping

Measurements

- Original length from fixed end to marker on wire
- Diameter of wire.
- Measure load.
- New length of wire when load increased.

Calculation of Young modulus

- Find extension (for each load) OR strain
- Determine cross-sectional area of wire or stress
- Plot stress-strain graph OR graph of load-extension
- Young modulus = gradient OR Young modulus equals gradient x original length/area
- Calculate Young modulus from single set of measurements of load, extension, area and length

Reliability of results

- Measure diameter in 3 or more places and take average.
- Put on initial load to tension wire and take up ‘slack’ before measuring original length.
- Take measurement of extension while unloading to check elastic limit has not been exceeded
- Use long wire (to give measureable extension and reduce percentage uncertainty)

Scale or ruler parallel to wire. Readings read off parallel to scale (reduce parallax error)