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Keep going...

A few more...



## 3.6 Further mechanics and thermal physics

### 3.6.1 Periodic motion

#### 3.6.1.1 Circular motion

##### Content

- **Motion in a circular path at constant speed implies there is an acceleration and requires a centripetal force.**
- Magnitude of angular speed,  $\omega = \frac{v}{r} = 2\pi f$ .
- Radian as a measure of angle.
- Direction of angular velocity will not be considered.
- Centripetal acceleration,  $a = v^2/r = \omega^2 r$ .
- The derivation of the centripetal acceleration formula will not be examined.
- Centripetal force,  $F = mv^2/r = m\omega^2 r$ .

*Motion in a circular path at constant speed implies there is an acceleration and requires a centripetal force.*

Acceleration is defined as the rate of change of velocity. The speed is the magnitude of the velocity. If an object is travelling at a constant speed in a circle, then its direction is constantly changing. Therefore, its velocity is constantly changing, and so must its acceleration too. For a given acceleration and mass, there must be a resultant force according to  $F = ma$ . This resultant force is referred to as the centripetal force, which acts towards the centre of the circle that the object is travelling in.

*Magnitude of angular speed,  $\omega = \frac{v}{r} = 2\pi f$ .*

Angular speed is defined as the angular displacement per second, represented by  $2\pi/T$ , and since  $T = 1/f$ , it is also equal to  $2\pi f$ . It is also equal to  $v/r$ , where  $r$  is the radius of the circle being travelled in.

*Radian as a measure of angle.*

The radian is the standard unit of measurement of an angle.

The webpage below gives a good explanation of radians as a measure of angle.

<http://www.themathpage.com/atrig/radian-measure.htm>

*Direction of angular velocity will not be considered.*

*Centripetal acceleration,  $a = v^2/r = \omega^2 r$ .*

Centripetal acceleration is calculated using the formulae given above. Since  $\omega = v/r$ ,  $a$  can be written as  $\omega^2 r$  instead. The centripetal acceleration can be looked at as the rate of change of the tangential velocity.

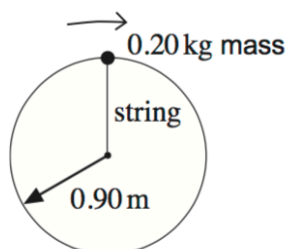
*The derivation of the centripetal acceleration formula will not be examined.*

*Centripetal force,  $F = mv^2/r = m\omega^2r$ .*

The centripetal force is calculated by using the equations above. It is the force that keeps an object moving in a uniform circular path. Since  $\omega = v/r$ , then the equation can be written as  $m\omega^2r$ . Some practice questions are given below to demonstrate use of this equation.

## AQA June 2014 Unit 4 Section A Q8

- 6 A 0.20 kg mass is whirled round in a vertical circle on the end of a light string of length 0.90 m.



At the top point of the circle the speed of the mass is  $8.2 \text{ m s}^{-1}$ . What is the tension in the string at this point?

- A 10 N
- B 13 N
- C 17 N
- D 20 N

The resultant (centripetal) force is equal to  $mv^2/r$ , and when the mass is at the top of the swing, both  $T$  and  $mg$  act downwards so  $T + mg = mv^2/r$ , thus  $T = mv^2/r - mg$ .

$T$  must always be its lowest value at the top of the swing because the weight of the object is working with  $T$ , as opposed to working against it at the bottom, so  $T$  does not need to be as large.

The answer is 13 N so B after you sub in the values to  $T = mv^2/r - mg$ .



## AQA Jan 2010 Unit 4 Section A Q10

### Question:

A planet of mass  $M$  and radius  $R$  rotates so rapidly that loose material at the equator only just remains on the surface. What is the period of rotation of the planet?

### Answer:

If the material only just remains on the surface, then it can be said to feel weightless. This occurs when the centripetal force is equal to the weight of the object ie  $m\omega^2 R = mg$ .

It can then be proven that  $T^2 = 4\pi^2 \sqrt{R/g}$

We also know  $g = GM/R^2$ , which we can substitute in then solve for  $T$  to get  $T = 2\pi \sqrt{R/GM}$   
**It should say  $R^3$  but the equation will not allow it.**

### 3.6.1.2 Simple harmonic motion (SHM)

#### Content

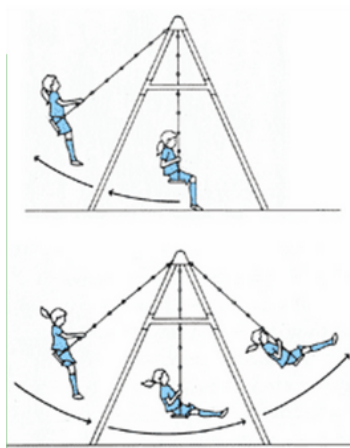
- Analysis of characteristics of simple harmonic motion (SHM)
- Condition for SHM:  $a \propto x$
- Defining equation:  $a = -\omega^2 x$
- $x = A \cos \omega t$  and  $v = \pm \omega \sqrt{A^2 - x^2}$
- Graphical representations linking the variations of  $x$ ,  $v$  and  $a$  with time.
- Appreciation that the  $v$ - $t$  graph is derived from the gradient of the  $x$ - $t$  graph and that the  $a$ - $t$  graph is derived from the gradient of the  $v$ - $t$  graph
- Maximum speed =  $\omega A$
- Maximum acceleration =  $\omega^2 A$

#### Opportunities for skills development

- Data loggers can be used to produce  $s$ - $t$ ,  $v$ - $t$  and  $a$ - $t$  graphs for SHM
- Sketch relationships between  $x$ ,  $v$ ,  $a$  and  $t$  for simple harmonic oscillators.

#### Analysis of characteristics of simple harmonic motion (SHM)

For a pendulum swinging from side to side, it can be said (under certain conditions) to oscillate under simple harmonic motion (SHM). When the pendulum is at its lowest position, this can be said to be its equilibrium position, as this will be the final position of rest. An example of this is a person on a swing, the picture below demonstrates this.



Some of the key characteristics of SHM are displacement, amplitude, time period and frequency. To look at displacement, you can take the above diagram as guidance. The object's displacement will change continuously, and in one full cycle as the person on the swing starts from a position other than equilibrium will undergo the following changes.

The displacement will decrease as it moves back to its equilibrium position, and then will reverse and increase as it moves away from the equilibrium position but in the opposite direction. Then it will decrease again and then finally increase, moving towards its starting position from its equilibrium position.

Amplitude is the maximum displacement from the equilibrium position. You will get something called free vibrations if there is no frictional force and the amplitude is constant. The time period is the time taken to complete one full oscillation, denoted by the symbol  $T$ . Frequency,  $f$ , is the number of waves passing a certain point per second, and is related to time period by  $f=1/T$ . For an oscillating system you will also get an angular frequency,  $\omega$ .

If you were to have two pendulums side by side with the same time period, you may be able to observe a phase difference between them. This is the difference in the fraction of the cycle that has elapsed between the two oscillating systems. This difference will be constant and is equal to the difference in time divided by the time period.

For any oscillating system, it is said that there is a restoring force that restores the object towards its equilibrium position. This resultant force must be proportional to displacement, as from equilibrium position, so that the acceleration will then be proportional to displacement.

Restoring force calculation:

## AQA June 2006 Unit 4 Q1bii

### Question:

The magnitude of the restoring force that acts on the bob when at its maximum displacement.

We already know the mass of the bob is  $1.2 \times 10^{-2}$  kg, the amplitude is 51mm, and that it takes 46.5s to complete 25 oscillations: so  $T = 1.86$ .

### Answer:

At maximum displacement, acceleration = max, so  $F = ma_{\max}$ .

$$a_{\max} \{ = (-)(2\pi f)^2 A \} = (2\pi \times 0.538)^2 \times 51 \times 10^{-3} \quad \checkmark (= 0.583 \text{ m s}^{-2})$$

(allow C.E. for value of  $f$  from (i))

$$F_{\max} (= ma_{\max}) = 1.2 \times 10^{-2} \times 0.583 \quad \checkmark$$

$$= 7.0 \times 10^{-3} \text{ N} \quad \checkmark (6.99 \times 10^{-3} \text{ N})$$

### Condition for SHM: $a \propto -x$

The condition for SHM is that the acceleration is proportional to the displacement, but acting in the opposite direction.

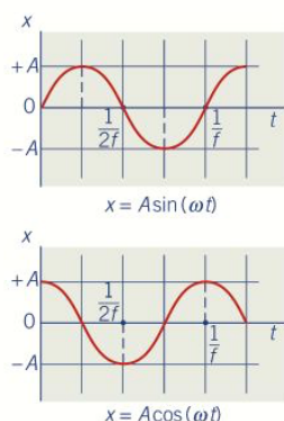
### Defining equation: $a = -\omega^2 x$

Since  $a \propto -x$ , then  $a = -kx$ , where  $k$  is a constant. It turns out that the constant is the angular frequency squared, denoted by the  $\omega^2$ , where  $\omega = 2\pi f = v/r$ .

$$x = A\cos(\omega t) \text{ and } v = \pm \omega\sqrt{(A^2 - x^2)}$$

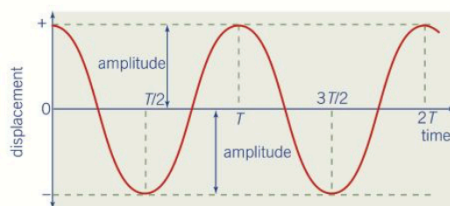
The equations above will find you the displacement and velocity at any point in an oscillation.  $x$  = displacement,  $A$  = amplitude,  $\omega$  = angular frequency,  $v$  = velocity.

The displacement of a system oscillating with SHM can be modelled by using a sinusoidal curve, and by convention when  $t = 0$ ,  $x = +A$  therefore the equation  $A\cos(\omega t)$  is used to model this situation. However if the starting point of the oscillation was chosen to be at equilibrium position moving out in a positive direction, the equation  $A\sin(\omega t)$  would apply.

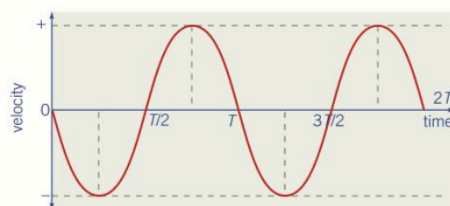


The second equation (for velocity) can be derived by differentiating the first and then applying trigonometric identities. However, the derivation is not required for the course, just the application of the equation.

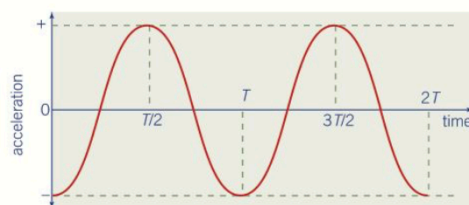
## Graphical representations linking the variations of $x$ , $v$ and $a$ with time



▲ Figure 2(i) Displacement against time



▲ Figure 2(ii) Velocity against time



▲ Figure 2(iii) Acceleration against time

*Appreciation that the  $v$ - $t$  graph is derived from the gradient of the  $x$ - $t$  graph and that the  $a$ - $t$  graph is derived from the gradient of the  $v$ - $t$  graph*

The gradient of the displacement-time graph gives the velocity-time graph, and the gradient of the velocity-time graph derives the acceleration-time graph. If studying mathematics, knowing that differentiation can help you draw the graphs. You know that  $x = A\cos(\omega t)$ , and so if you were to differentiate this with respect to time, you would get  $v = -A\omega\sin(\omega t)$ . The negative sign indicates that you need to draw a sine graph, but reflect it in the line  $y = 0$ . To differentiate this again with respect to time, you would get  $a = -A\omega^2\cos(\omega t)$ . Thus you would draw a cosine curve reflected in the line  $y = 0$ .

**Maximum speed =  $\omega A$**

This equation gives you the maximum speed of an oscillating system, where  $\omega$  = angular frequency and  $A$  = amplitude

**Maximum acceleration =  $\omega^2 A$**

This equation gives you the maximum acceleration of an oscillating system, where  $\omega$  = angular frequency and  $A$  = amplitude

## AQA June 2006 Unit 4 Q1bii

Question:

The magnitude of the restoring force that acts on the bob when at its maximum displacement.

We already know the mass of the bob is  $1.2 \times 10^{-2}$  kg, the amplitude is 51mm, and that it takes 46.5s to complete 25 oscillations: so  $T = 1.86$ .

Answer:

At maximum displacement, acceleration = max, so  $F = ma_{\max}$ .

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(allow C.E. for value of  $f$  from (i))

$$F_{\max} (= ma_{\max}) = 1.2 \times 10^{-2} \times 0.583 \quad \checkmark$$

$$= 7.0 \times 10^{-3} \text{ N} \quad \checkmark \quad (6.99 \times 10^{-3} \text{ N})$$

## AQA Jan 2010 Unit 4 Section A Q6

### Question:

Which one of the following gives the phase difference between the particle velocity and the particle displacement in simple harmonic motion?

### Answer:

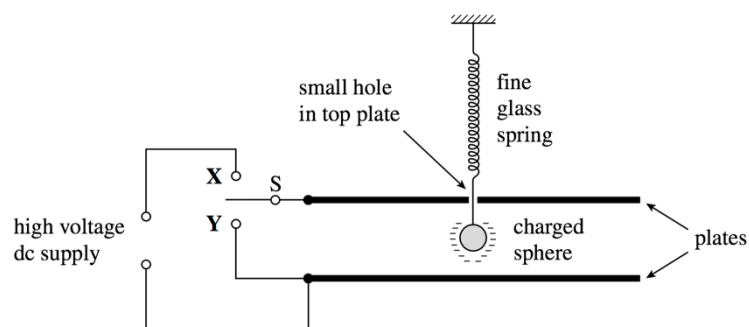
$$\pi/2$$

## AQA June 2010 Unit 4 Section B Q2bii

### Question:

A small negatively charged sphere is suspended from a fine glass spring between parallel horizontal metal plates, as shown in **Figure 1**.

**Figure 1**



‘Switch S is now moved to position Y (in a previous question it was at X).

‘With reference to the forces acting on the sphere, explain why it starts to move with simple harmonic motion’

### Answer:

- Net downward force on sphere (when E becomes zero) [or gravitational force acts on sphere, or force is weight]
- This force extends spring
- Force (or acceleration) is proportional to (change in) extension of spring and acceleration is in opposite direction to displacement (or towards equilibrium)
- for shm, acceleration  $\propto (-)$  displacement [or for shm, force  $\propto (-)$  displacement]

## AQA Jan 2010 Unit 4 Q1a

### Question:

Describe the energy changes that take place as the bob of a simple pendulum makes one complete oscillation, starting at its maximum displacement.

### Answer:

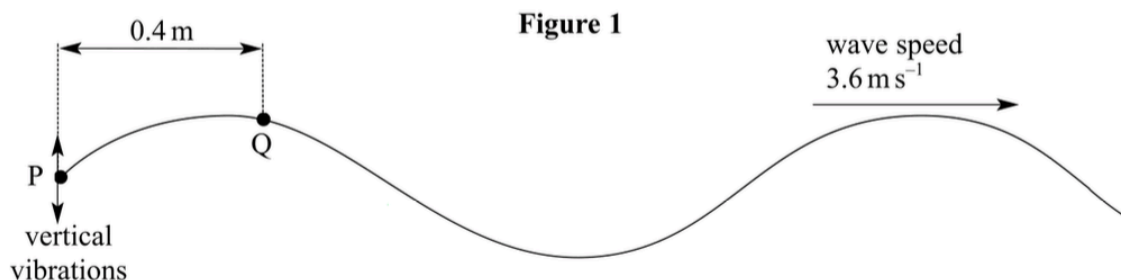
- (grav) potential energy  $\rightarrow$  kinetic energy  $\rightarrow$  (grav) potential energy  $\rightarrow$  kinetic energy  $\rightarrow$  gravitational potential energy
- energy lost to surroundings in overcoming air resistance

## AQA June 2006 Unit 4 Q2

### Question:

- 2 Progressive waves are generated on a rope by vibrating vertically the end, P, in simple harmonic motion of amplitude 90 mm, as shown in **Figure 1**. The wavelength of the waves is 1.2 m and they travel along the rope at a speed of  $3.6 \text{ m s}^{-1}$ . Assume that the wave motion is not damped.

**Q2 Jun 2006**



- (a) Point Q is 0.4 m along the rope from P. Describe the motion of Q in as much detail as you can and state how it differs from the motion of P. Where possible, give quantitative values in your answer.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

### Answer:

Question 2	Q2 Jun 2006	
(a)	vibrates or oscillates or moves in shm ✓ vibration/oscillation is vertical/perpendicular to wave propagation direction ✓ frequency ( $=c/\lambda$ ) = 3.0 (Hz) ✓ (or same as P) amplitude = 90 (mm) ✓ (or same as P) Q has a phase lag on P ✓ (or vice versa) phase difference of $\left(\frac{0.4}{1.2} \times 2\pi\right) = \frac{2\pi}{3}$ (rad) or $120^\circ$ ✓	<b>max 5</b>



## OCR (A) A Level Specimen 1 Q21c)d)

### Question:

A stabilising mechanism for electrical equipment on board a high-speed train is modelled using a 5.0g mass and two springs, as shown in Fig. 21.1. For testing purposes, the springs are horizontal and attached to two fixed supports in a laboratory.

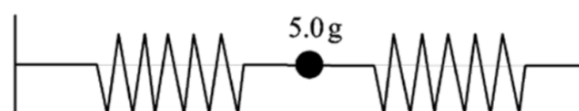


Fig. 21.1

Explain why the mass oscillates with simple harmonic motion when displaced horizontally.

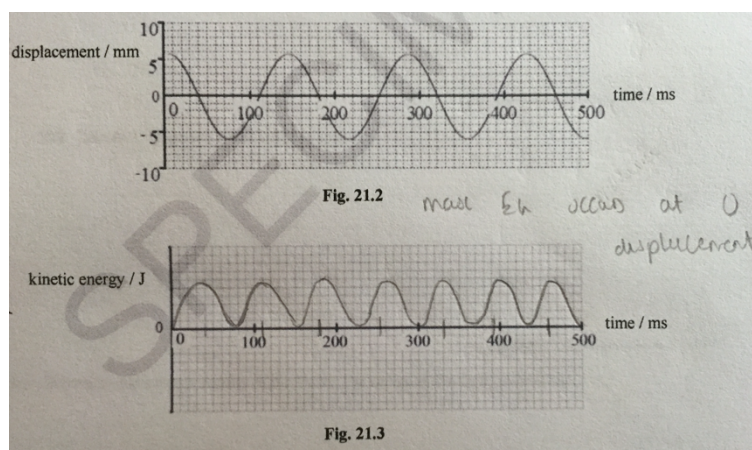
### Answer:

- Resultant force from springs is proportional to displacement from **B1** centre **or** acceleration (of mass) is proportional to displacement from centre.
- Directed to centre or fixed point.

### Question:

Fig. 21.2 shows the graph of displacement against time for the oscillating mass.

### Answer:



### Question:

Plan how you can obtain experimentally the displacement against time graph for the oscillating mass in the laboratory. Include any steps taken to ensure the graph is an accurate representation of the motion

### Answer:

## Method Examples

### Stroboscope

- Use of stroboscope of known frequency or period
- Photograph to capture several positions on one picture
- Measure displacement from centre using a scale put behind the mass.

### Motion sensor:

- Motion sensor connected to data logger which sends information on displacement and time to computer.
- Sensor placed close to moving mass to eliminate reflections from other objects.
- Small reflector attached to mass.

## Safeguards to ensure accuracy

### Stroboscope:

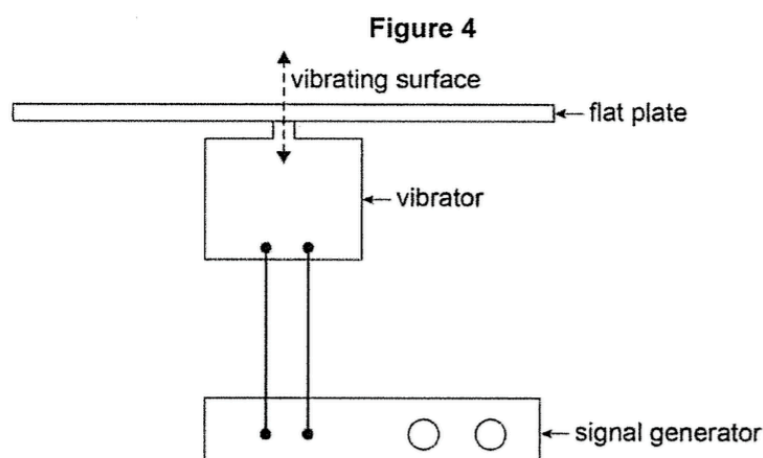
- Use frequency such that positions of mass are close together on photograph.
- Distance scale close to oscillating mass or camera set back from mass to reduce parallax.
- Camera should be directed at equilibrium point or at  $90^\circ$  to oscillation.

### Motion sensor:

- Any attached reflector should not cause damping.
- Motion sensor directed along line of oscillation or motion sensor signal blocked by supports so must be as near to line of oscillation as possible.
- Use thin supports to reduce reflections

## AQA A Level Specimen Paper Q4.5

A rigid flat plate is made to vibrate vertically with simple harmonic motion. The frequency of the vibration is controlled by a signal generator as shown in **Figure 4**.



Question:

A small quantity of fine sand is placed onto the surface of the plate. Initially the sand grains stay in contact with the plate as it vibrates. The amplitude of the vibrating surface remains constant at  $3.5 \times 10^{-4}$  m over the full frequency range of the signal generator. Above a particular frequency the sand grains lose contact with the surface.

Explain how and why this happens.

Answer:

- When the vibrating surface accelerates down with an acceleration less than the acceleration of free fall the sand stays in contact.
- Above a particular frequency, the acceleration is greater than  $g$
- There is no contact force on the sand *or* sand no longer in contact when downwards acceleration of plate is greater than acceleration of sand due to gravity



### 3.6.1.3 Simple harmonic systems

#### Content

- Study of mass-spring system:  $T = 2\pi\sqrt{\frac{m}{k}}$
- Study of simple pendulum:  $T = 2\pi\sqrt{\frac{l}{g}}$
- Questions may involve other harmonic oscillators (eg liquid in U-tube) but full information will be provided in questions where necessary.
- Variation of  $E_K$ ,  $E_P$  and total energy with both displacement and time.
- Effects of damping on oscillations.

#### Opportunities for Skills Development

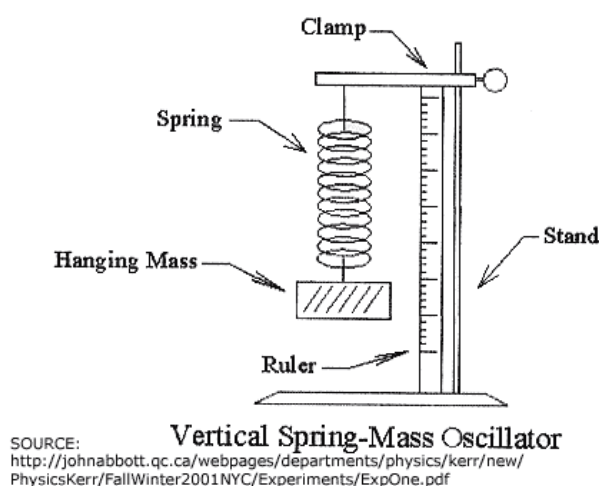
- Students should recognise the use of the small-angle approximation in the derivation of the time period for examples of approximate SHM.

#### Study of mass-spring system $T = 2\pi\sqrt{\frac{m}{k}}$

Describing a mass-spring oscillating system can be done using the equation above, where  $T$  = Time period (s),  $m$  = mass (kg) and  $k$  = spring constant ( $\text{Nm}^{-1}$ ). The diagram below demonstrates an example of a mass-spring system.

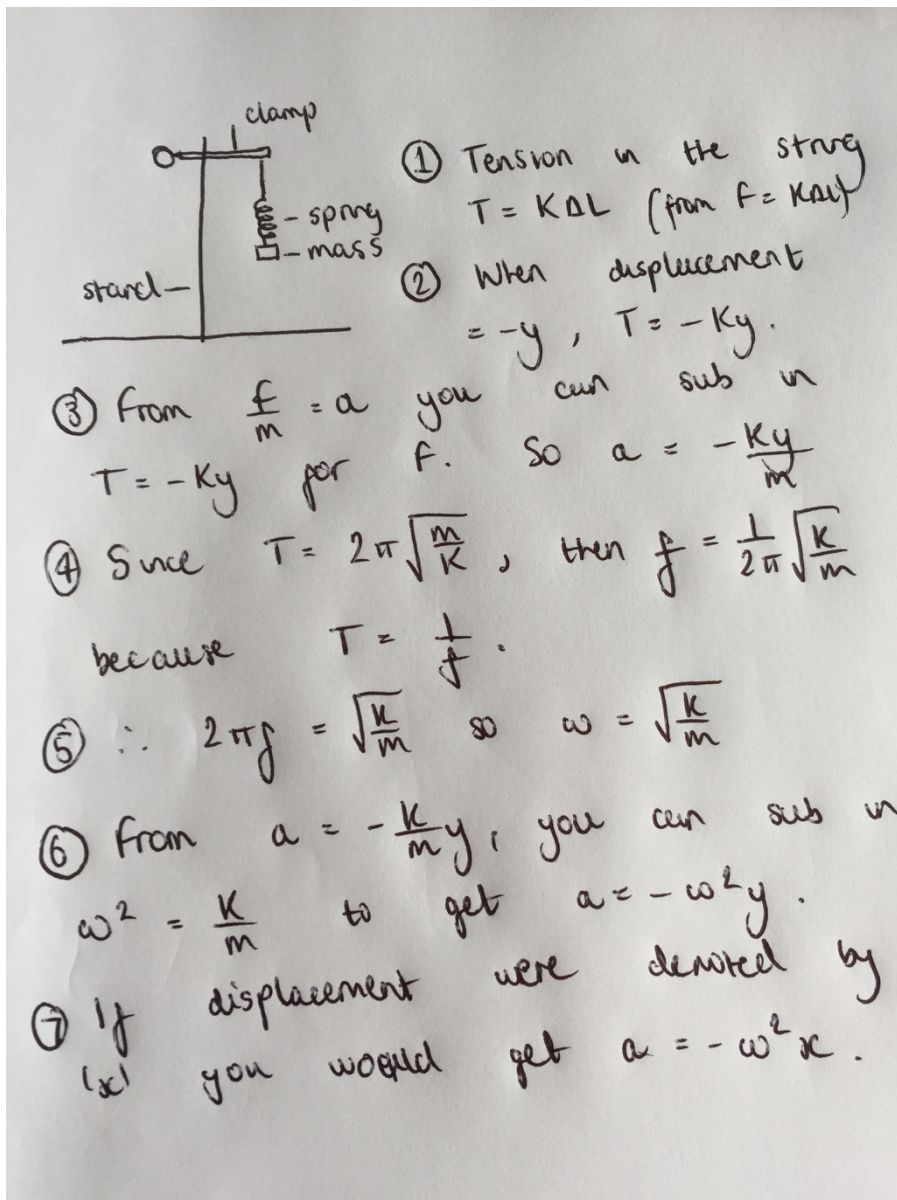
The time period of the oscillation of a mass-spring system, unlike a pendulum is independent of  $g$ , so time periods of mass-spring systems would be the same anywhere in the universe, we think.

Furthermore, maximum tension in a spring occurs when the maximum displacement downwards is reached, and minimum displacement occurs when the spring is at maximum displacement upwards.



It can be said that the tension in the string provides the restoring force, or more specifically, the change in tension. The tension  $T = k\Delta L$ . When the spring is at a displacement (from equilibrium) of  $y$ , the restoring force  $T = -ky$ , where the negative sign is representative of the change in tension always attempting to bring the mass-spring system back to its equilibrium position. This then tells you that the acceleration ( $F/m = a$ ), equals  $-kx/m$ .

If you follow the working out shown below, you can derive the equation  $a = -\omega^2 x$ . This ultimately proves that the mass-spring system oscillates with simple harmonic motion.



The diagram shows a mass-spring system. A horizontal line represents a 'stand'. A vertical line descends from the stand, ending in a 'clamp'. A coiled line representing a 'spring' is attached to the clamp. A small square representing a 'mass' is attached to the bottom of the spring. To the right of the diagram is a list of seven steps:

- ① Tension in the string  
 $T = K\Delta L$  (from  $F = K\Delta L$ )
- ② When displacement  
 $= -y$ ,  $T = -Ky$ .
- ③ From  $\frac{F}{m} = a$  you can sub in  
 $T = -Ky$  for  $F$ . So  $a = -\frac{Ky}{m}$
- ④ Since  $T = 2\pi\sqrt{\frac{m}{K}}$ , then  $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$   
because  $T = \frac{1}{f}$ .
- ⑤  $\therefore 2\pi f = \sqrt{\frac{K}{m}}$  so  $\omega = \sqrt{\frac{K}{m}}$
- ⑥ From  $a = -\frac{K}{m}y$ , you can sub in  
 $\omega^2 = \frac{K}{m}$  to get  $a = -\omega^2 y$ .
- ⑦ If displacement were denoted by  
 $(x)$  you would get  $a = -\omega^2 x$ .

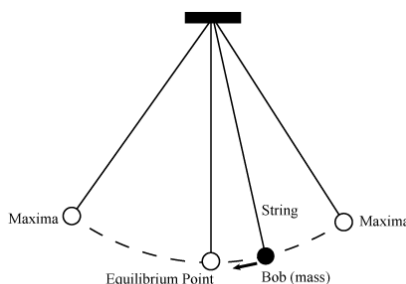
From this equation describing the mass-spring system, you can determine how frequency of the oscillation is affected, since  $T = 1/f$ . You are able to reduce the frequency of the oscillation if you either add extra mass, or use a spring of smaller spring constant (a spring that is more easily stretched).



**Study of simple pendulum:**  $T = 2\pi\sqrt{\frac{l}{g}}$

The equation approximating the time period of a pendulum is the same as that for the mass-spring system, but contains  $l$  instead of  $m$ , and  $g$  instead of  $k$ . The  $l$  = length of the string (m), and the  $g$  is the gravitational field strength (taken to be 9.81 unless stated otherwise). Therefore, the gravitational field strength affects the time period of a pendulum, so on the moon a pendulum would swing slower than an identical set up on earth.

This equation only applies for small amplitude oscillations.



Likewise, in a simple pendulum system you are able to derive the equation  $a = -\omega^2 x$ , this working is shown below.

① Simple Pendulum System

②

From ① it can be said that  $\sin\theta = \frac{s}{L}$ , although this only applies if the angle does not exceed around  $10^\circ$ .

The restoring force  $F = -mg\sin\theta$

$a = \frac{F}{m} = -\frac{mg\sin\theta}{m} = -g\sin\theta$

We know that  $\sin\theta = \frac{s}{L}$  so

$a = -\frac{g}{L}s$

Also, from the same reasoning as in the mass-spring system, we know that  $\omega^2 = \frac{g}{L}$ , so we get  $a = -\omega^2 s$

Again, this  $s$  could be replaced with the more familiar  $x$ .

*Questions may involve other harmonic oscillators (eg liquid in U-tube) but full information will be provided in questions where necessary.*

The main oscillating systems that are dealt with are mass-spring systems and simple pendulums. However other examples are used like liquid in a U-tube, but the necessary information will be provided in questions.

<https://www.youtube.com/watch?v=prZ1qbAL960> - this video provides a good explanation of SHM in a liquid U-tube.

### *Variation of $E_K$ , $E_P$ and total energy with both displacement and time.*

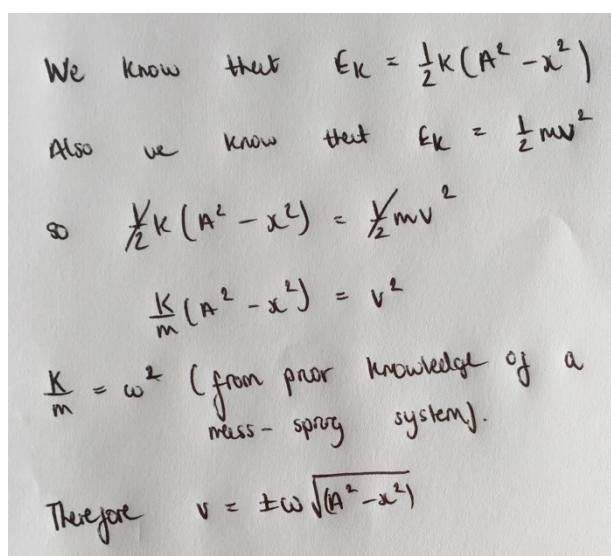
As long as there is no resistance to the direction of motion or travel of say, a mass spring system, then the total energy of the system is constant, and equal to the maximum potential energy of the mass-spring system.

To consider the total energy of for example, a mass-spring system, you must first consider the variation of the kinetic energy, and potential energy of the system. The elastic potential energy  $E_P$ , is given by  $E_P = 1/2k\Delta L^2$ . This is more commonly written as  $E_P = 1/2kx^2$ , where  $x$  is the displacement (from equilibrium).

We know that the total energy of the system is equal to the maximum potential energy of the system. This means that at maximum displacement, where  $x = A$  (amplitude), the potential energy is equal to  $1/2kA^2$ .

We also know that the total energy  $E = E_K + E_P$ , where  $E_K$  is the kinetic energy. We can then calculate the kinetic energy as we know the total energy of the system and the potential energy. This calculation gives  $E_K = 1/2k(A^2 - x^2)$ .

We can then use this knowledge to find the velocity at any point of the oscillation, although this equation is given in the equation sheet.



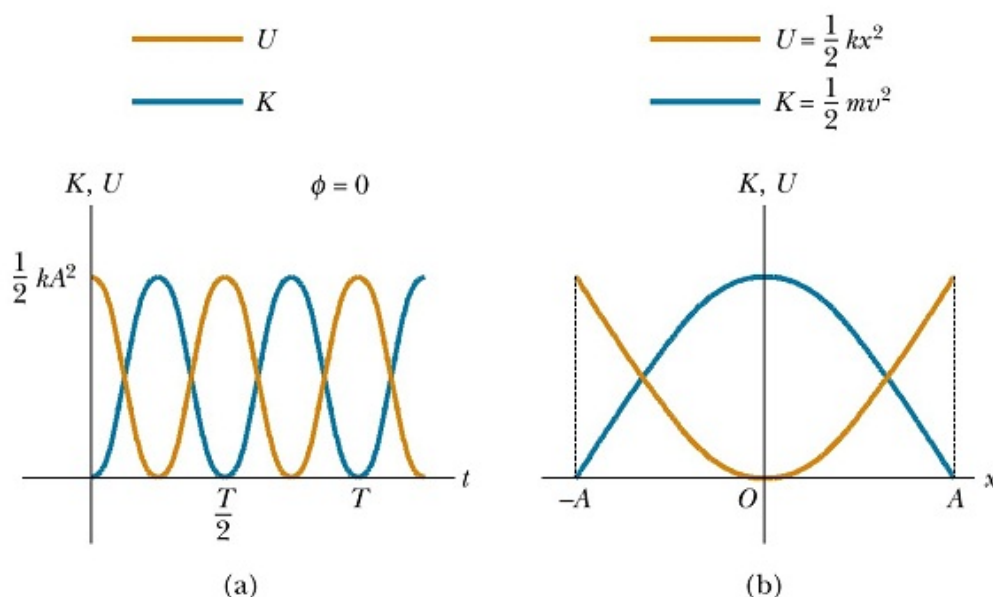
Handwritten derivation showing the steps to find velocity  $v$  from energy conservation:

$$\begin{aligned} \text{We know that } E_K &= \frac{1}{2}k(A^2 - x^2) \\ \text{Also we know that } E_K &= \frac{1}{2}mv^2 \\ \text{So } \frac{1}{2}k(A^2 - x^2) &= \frac{1}{2}mv^2 \\ \frac{k}{m}(A^2 - x^2) &= v^2 \\ \frac{k}{m} &= \omega^2 \quad (\text{from prior knowledge of a mass-spring system}) \\ \text{Therefore } v &= \pm \omega \sqrt{A^2 - x^2} \end{aligned}$$

We also know that the maximum speed occurs when  $x = 0$ , so maximum speed occurs when  $v = \omega A$



The variation of kinetic and potential energy can be modelled on a graph of energy vs displacement. An example of this graph is shown below:



For the graphs of energy against time, to remember which line is which, you could use your knowledge that  $E_p = \frac{1}{2} kx^2$ . If  $k$  were equal to 2, then you would have  $E_p = x^2$ , which could be written in the form  $y = x^2$  to make it easier to see the relationship between the parabolic curve and the equation. Hence, you get the shape of the graph shown.

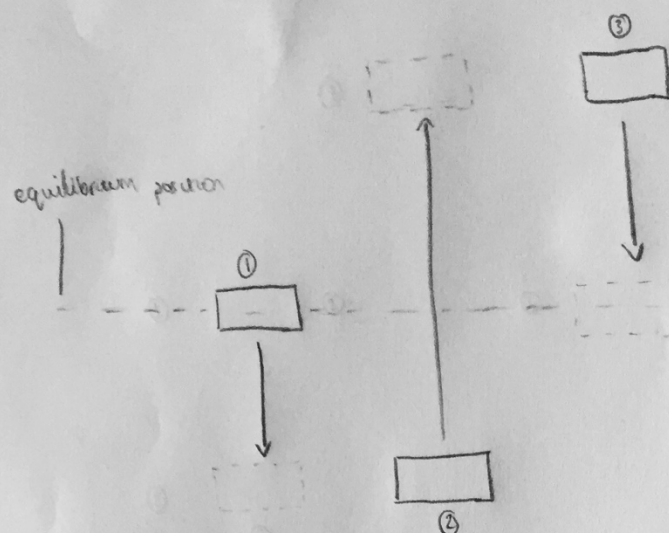
On the contrary, the kinetic energy is represented by an inverted parabolic curve. This comes from the equation  $E_k = \frac{1}{2} k(A^2 - x^2)$ . If you were to make  $k = 2$ , you would be left with  $E_k = A^2 - x^2$ . For simplicity, you can look at this as  $y = -x^2$ , thus an inverted parabolic curve.

The image below gives a description of the energy changes that take place during one complete oscillation of a vertical mass-spring system, as well as the velocity/acceleration at each point.

Some of the key points are that:

- At minimum displacement ( $x=0$ ), the velocity is at its maximum value ( $\omega A$ ), thus acceleration is at its minimum value (0).
- At maximum displacement ( $x=A$ ), velocity is at its minimum value (0), and so acceleration is at its maximum value ( $\omega^2 A$ )

From these points you could draw a displacement-time, velocity-time and acceleration-time graph, as well as determine the variation of kinetic and potential energy in the system.



Energy changes in one complete oscillation (mass-spring system).

- ① Kinetic Energy
- ② Elastic PE
- ③ Gravitational PE
- ④ Kinetic Energy
- ⑤ Elastic PE

At ① the velocity is at its maximum because  $x = 0$ . This also means that acceleration must be at its minimum value.

If velocity = max then  $E_k$  must be its max value because  $E_k = \frac{1}{2}mv^2$ .

At this point the potential energy stored in the spring is at a minimum value because  $x = 0$ .

At ② the velocity takes its minimum value this acceleration is at its maximum value.

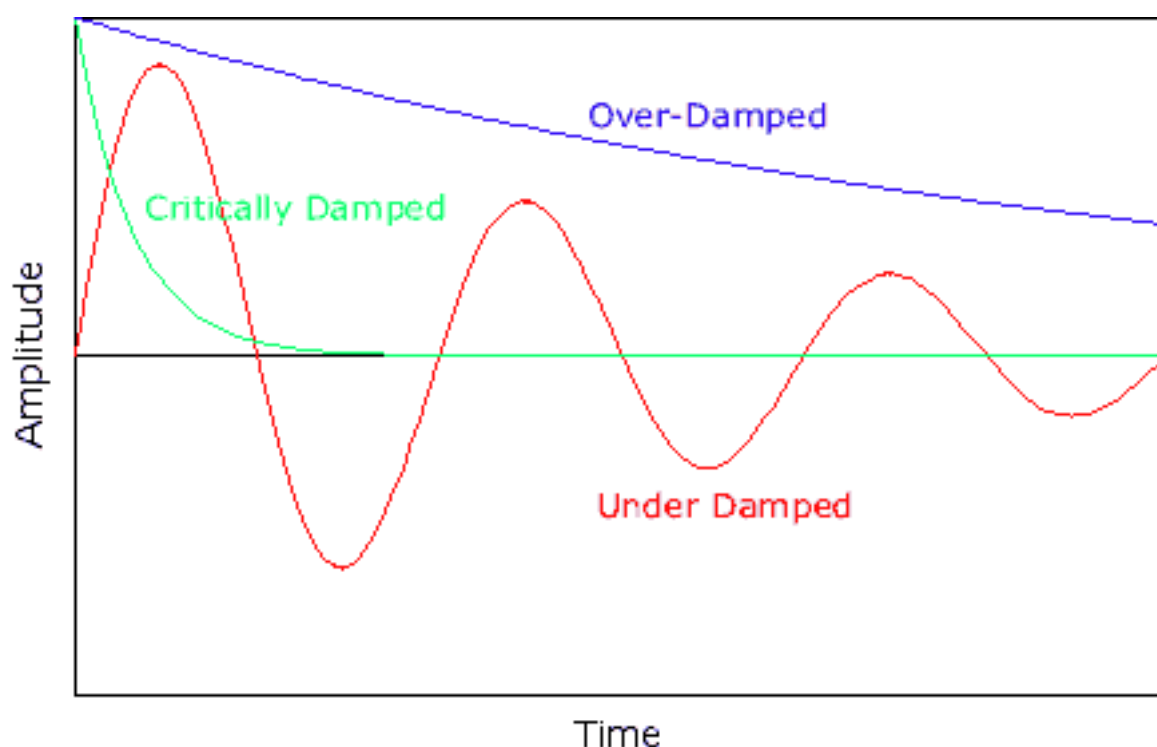
This means that  $E_k = \min$  and  $E_p = \max$ .

At ③ the same principles as before can be applied, but you should also remember that the system will gain gravitational potential energy as it moves to this position.

### *Effects of damping on oscillations.*

Damping creates a force which acts in the opposite direction to the direction of travel of the object. This means that harmonic oscillators, in real life, will lose energy as they oscillate. This reduces the total mechanical energy of the system, thus reducing the amplitude of the oscillations. The reason for this energy loss is due to air resistance, or frictional forces. These forces disperse energy to the surrounding environment as thermal energy.

There are three types of damping: light damping, critical damping and heavy damping. The effects of each type of damping are shown in the picture below.



Source: [http://www.splung.com/kinematics/images/damped\\_oscillations/damped\\_oscillations.gif](http://www.splung.com/kinematics/images/damped_oscillations/damped_oscillations.gif)

**Light damping:** This occurs when the time period does not change, but the amplitude decreases gradually with time. The amount that the amplitude decreases is the same fraction for each cycle.

**Critical damping:** This is where the damping causes the oscillating system to return back to its equilibrium position almost immediately. This type of damping is used in a car's suspension, as you would not want the car to oscillate after it went over a bump, so critical damping returns the motion of the car back to its equilibrium state.

**Over damping:** This is sometimes called heavy damping. It refers to the situation in which damping is so strong that when the object is released from a non equilibrium position (ie maximum displacement), the object will return to equilibrium far slower than if it were critically damped.

## AQA June 2006 Unit 4 Q1b

### Question:

State the conditions under which this equation applies.

The equation it is referring to is the one shown to the right,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

### Answer:

Oscillations must be of small amplitude



### 3.6.1.4 Forced vibrations and resonance

#### Content

- Qualitative treatment of free and forced vibrations.
- Resonance and the effects of damping on the sharpness of resonance.
- Examples of these effects in mechanical systems and situations involving stationary waves.

#### Opportunities for Skills Development

- Investigation of the factors that determine the resonant frequency of a driven system.

#### *Qualitative treatment of free and forced vibrations.*

**Free vibrations** are oscillations in which the total mechanical energy of the system remains the same over time. The consequence of this is that the amplitude will remain constant throughout the oscillations. However, this idea is purely theoretical as in real life systems, energy is always dissipated to the surroundings over time, so amplitude will fall to 0. Essentially, free vibrations can be looked at as being undamped (no damping present).

In damped oscillations, clearly the amplitude will fall to 0 over time. However, in addition to the damping, if we add energy into the system to keep it oscillating, then we could say that we are providing a **periodic force**. A periodic force is, as it sounds, a force applied at unvarying intervals.

In forced vibrations, as the applied frequency increased from 0, the amplitude of the oscillations would increase until a maximum, and then the amplitude would decrease again. When the phase difference between the displacement and applied periodic force is  $\pi/2$ , the amplitude will be at its maximum. This will then increase to  $\pi$  as the frequency is increased, and amplitude decreases.

#### *Resonance and the effects of damping on the sharpness of resonance.*

When a system oscillates with resonance, it is oscillating at maximum amplitude, as the phase difference between the driving force and oscillator is  $\pi/2$ . Essentially, the velocity and periodic force are in phase. Resonant frequency is the name given to the frequency at which maximum amplitude occurs.

The natural frequency of an oscillating system is the frequency at which the system will oscillate when not exposed to a constant or recurrent external force. The natural frequency can only be reached when little/no damping is present. **This also means that for an oscillating system at resonance, the applied frequency is equal to the natural frequency.**

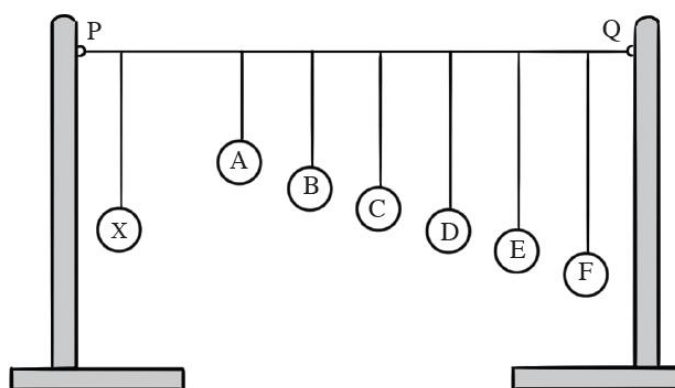
If a system is lightly damped, then it will have a larger maximum amplitude at resonance. Also, more damping means that the resonant frequency produced will be further from the

natural frequency of the system. This means that the resonance curve is sharper when damping is lighter.

The amplitude of oscillations will decrease drastically as the applied frequency becomes larger than the resonant frequency.

*Examples of these effects in mechanical systems and situations involving stationary waves.*

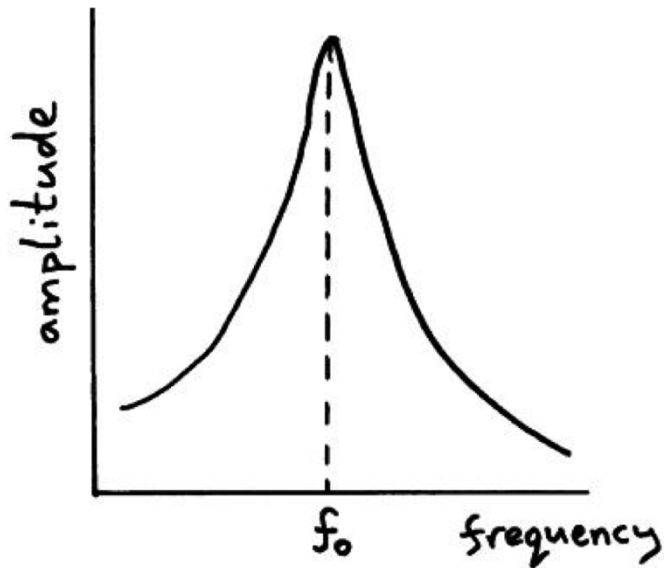
Barton's pendulum is an example of a resonance in a mechanical system. The diagram below shows a single driver pendulum (X), and six other pendulums (ABCDEF). Clearly, X has the same length as D.



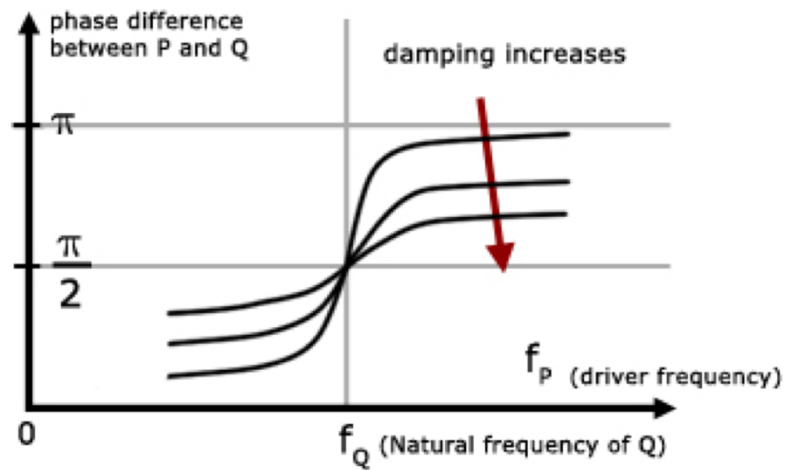
When the driver pendulum is displaced, it will cause the other pendulums to oscillate. However, pendulum D will respond more than any of the other pendulums as its length is very similar to that of D. The consequence of their similar length is that the two pendulums will have the same time period, and so their natural frequencies will be the same. This means D oscillates in resonance with X, and so has the largest amplitude. The other pendulums will respond according to their length, and how far they are from X.

Oscillations can also occur in bridges, and if they are not damped, can result in bridges oscillating at resonance if the correct periodic force is applied. This is very dangerous and can be caused by crosswinds, or people walking in step with each other.

## Amplitude versus frequency



## Phase difference between driver and oscillator versus frequency



**Coupled oscillations could also be studied**

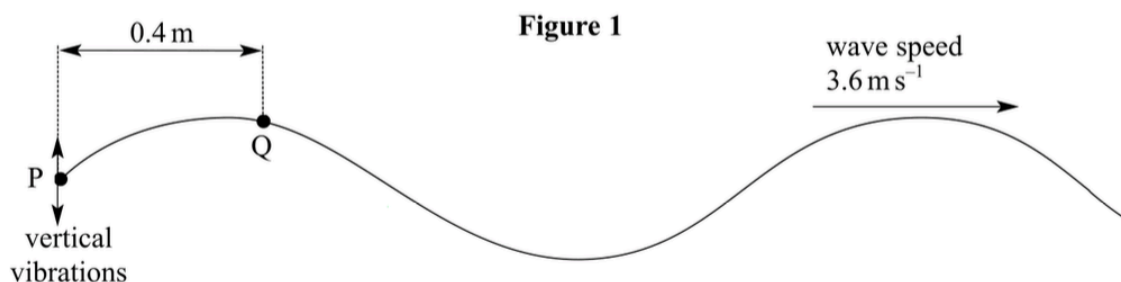


## AQA June 2006 Unit 4 Q2

### Question:

- 2 Progressive waves are generated on a rope by vibrating vertically the end, P, in simple harmonic motion of amplitude 90 mm, as shown in **Figure 1**. The wavelength of the waves is 1.2 m and they travel along the rope at a speed of  $3.6 \text{ m s}^{-1}$ . Assume that the wave motion is not damped.

**Q2 Jun 2006**



- (a) Point Q is 0.4 m along the rope from P. Describe the motion of Q in as much detail as you can and state how it differs from the motion of P. Where possible, give quantitative values in your answer.

You may be awarded additional marks to those shown in brackets for the quality of written communication in your answer.

### Answer:

Question 2	Q2 Jun 2006	
(a)	vibrates or oscillates or moves in shm ✓ vibration/oscillation is vertical/perpendicular to wave propagation direction ✓ frequency ( $=c/\lambda$ ) = 3.0 (Hz) ✓ (or same as P) amplitude = 90 (mm) ✓ (or same as P) Q has a phase lag on P ✓ (or vice versa) phase difference of $\left(\frac{0.4}{1.2} \times 2\pi\right) = \frac{2\pi}{3}$ (rad) or $120^\circ$ ✓	<b>max 5</b>

## AQA Jan 2011 Q5c

### Question:

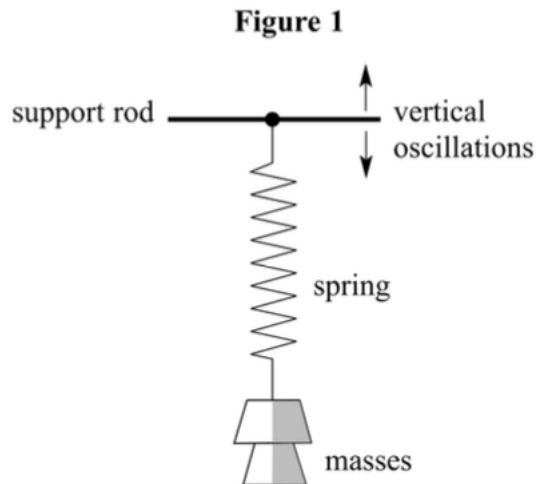
‘The length of **PQ** is 0.40 m. When the wire is vibrating, transverse waves are propagated along the wire at a speed of  $64 \text{ ms}^{-1}$ . Explain why the wire is set into large amplitude vibration when the frequency of the a.c. Supply is 80 Hz.’

### Answer:

- Wavelength ( $\lambda$ ) of waves  $64/80 = 0.80\text{m}$
- Length of wire is  $\lambda/2$  causing fundamental vibration because natural frequency of wire  $64/80 = 80\text{Hz}$
- Wire resonates (at frequency of ac supply), or a statement that fundamental frequency (or a natural frequency) of the wire is the same as applied frequency.

## AQA Jan 2006 Unit 4 Q1bi)ii)iii)

With both masses still in place, the spring is now suspended from a horizontal support rod that can be made to oscillate vertically, as shown in Figure 1, with amplitude 30mm at several different frequencies.



Describe fully, with reference to amplitude, frequency and phase, the motion of the masses suspended from the spring in each of the following cases.

(From the previous question we work out that the oscillation frequency of the system is approximately 1.5Hz)

### Question:

The support rod oscillates at a frequency of 0.2Hz.

### Answer:

- Forced vibrations (at 0.2Hz)
- Amplitude less than resonance
- Almost in phase with driver

### Question:

The support rod oscillates at a frequency of 1.5Hz

### Answer:

Resonance (for oscillations at 1.5Hz)  
Amplitude very large (over 30mm)  
Oscillations may appear violent  
Phase difference is  $90^\circ$

**Question:**

The support rod oscillates at a frequency of 10Hz

**Answer:**

Forced vibrations (at 10Hz)

Small amplitude

Out of phase with driver (ie antiphase,  $\pi$  out of phase)

## 3.6.2 Thermal physics

### 3.6.2.1 Thermal energy transfer

#### Content

- Internal energy is the sum of the randomly distributed kinetic energies and potential energies of the particles in a body.
- The internal energy of a system is increased when energy is transferred to it by heating or when work is done on it (and vice versa), eg a qualitative treatment of the first law of thermodynamics.
- Appreciation that during a change of state the potential energies of the particle ensemble are changing but not the kinetic energies. Calculations involving transfer of energy.
- For a change of temperature:  $Q = mc\Delta\theta$  where  $c$  is specific heat capacity
- Calculations including continuous flow
- For a change of state  $Q = ml$  where  $l$  is the specific latent heat.

#### Opportunities for Skills Development

- Investigate the factors that affect the change in temperature of a substance using an electrical method or the method of mixtures.
- Students should be able to identify random and systematic errors in the experiment and suggest ways to
- Investigate, with a data logger and temperature sensor, the change in temperature with time of a substance undergoing a phase change when energy is supplied at a constant rate.

*Internal energy is the sum of the randomly distributed kinetic energies and potential energies of the particles in a body.*

The internal energy of a system is the sum of the randomly distributed kinetic energies and potential energies of the particles in a body. The internal energy of a system can be thought of as the thermal energy, although some of an objects internal energy may be as a result of other factors.

*The internal energy of a system is increased when energy is transferred to it by heating or when work is done on it (and vice versa), eg a qualitative treatment of the first law of thermodynamics.*

There are two ways to increase the internal energy, the first of which is to do work on an object, the second of which is to transfer energy to the object by heating it. The internal energy of a system can be decreased when work is done on another object.

An example of energy transfer is in a filament lamp. Work is done by the electricity supply on the filament as electrons are pushed through. This causes the internal energy of the filament to increase, until it eventually emanates light and thermal energy to its surroundings.

Thermodynamics is a branch of physics that deals with the energy and work of a system.

**The first law of thermodynamics** states that the change of internal energy of an object is equal to the total energy transfer due to work done and heating.

*Appreciation that during a change of state the potential energies of the particle ensemble are changing but not the kinetic energies. Calculations involving transfer of energy.*

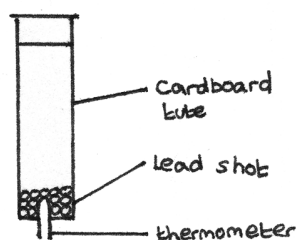
When a solid is given enough energy, the atoms will vibrate so rigorously that they may have enough energy to break free from each other. This will change the state of the solid into a liquid. The energy given to the solid raises the potential energy of the molecules as they break free from each other, although the kinetic energy does not change. The forces between molecules of a liquid are of insufficient strength to hold the molecules in fixed positions, as in a solid. On the contrary, molecules in a gas pretty much have no forces of attraction between them.

*For a change of temperature:  $Q = mc\Delta\theta$  where  $c$  is specific heat capacity*

The specific heat capacity,  $c$ , of a substance is defined as the energy required to raise the temperature of 1kg of substance by  $1^{\circ}\text{C}/1\text{K}$ .  $Q$  = energy (J),  $m$  = mass (kg),  $\theta$  = temperature ( $1^{\circ}\text{C}/1\text{K}$ ) and the unit of specific heat capacity is either  $\text{Jkg}^{-1}\text{K}^{-1}$  or  $\text{Jkg}^{-1}\text{C}^{-1}$ , depending on the unit of temperature used. It does not matter whether  $\Delta\theta$  is in Kelvin or Celsius because the difference between each unit on either scale is the same, ie a rise in temperature of 1K is of the same magnitude as a rise in temperature of  $1^{\circ}\text{C}$ . Although certain questions may require answers in Kelvin/Celsius.

Experimental methods to verify the specific heat capacity of a material.

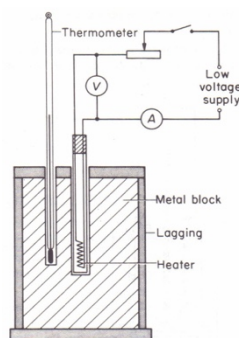
1. The inversion tube experiment can be used, knowing only the length of the tube, the temperature increase of the lead, and the number of inversions.
  - a. An object ie a lead shot, is dropped from the top of the tube and falls to hit the lead shots at the bottom of the tube. The tube has a bung at the top to prevent the lead shot escaping the tube. Each time the shot hits the bottom, the tube must be flipped vertically, then once the lead shot reaches the bung it must be flipped again. This must be repeated a number of times ie 50 times, and the temperature rise noted.
  - b. The total loss in gravitational potential energy should be equal to the gain of internal energy within the lead shots at the bottom of the tube.
  - c. So for example, 50 inversions, the loss of gravitational potential energy is  $50mgL$ , where  $L$  is the length of the tube.
  - d. The gain in internal energy will be  $mc\Delta T$
  - e. So  $mc\Delta T = 50mgL$ , so  $c = 50gL/\Delta T$ .



<https://revise.im/physics/unit-5/thermal-energy>

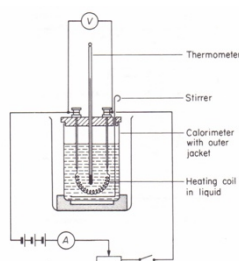
These final two methods are electrical calorimeters used to determine the specific heat capacity of a solid and a liquid respectively.

2. The electrical energy supplied by the heater, assuming no thermal energy is lost to the surroundings, will all be transferred to the metal block. The thermometer can be used to measure the temperature difference, so it can be said that  $IVt = mc\Delta T$ , so  $c = IVt/m\Delta T$ . It is assumed that the mass of the block is known and of value  $m$ . The electrical energy supplied can be calculated using a stopwatch, ammeter and voltmeter.
  - a. To improve the thermal conductivity of the thermometer and metal block, you can put water/oil in the hole you place the thermometer inside.



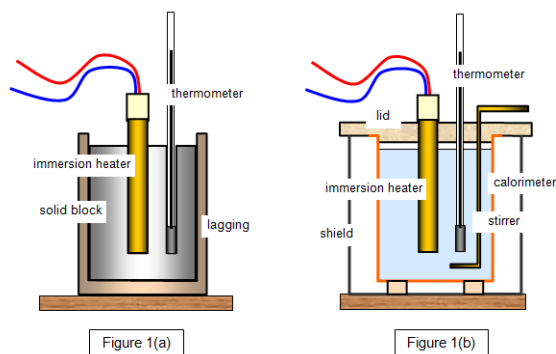
<http://physicsmax.com/measure-specific-heat-capacity-metal-solid-block-method-8008>

3. Measurements to determine the specific capacity of a liquid can be made in a similar procedure to that above. If you take into account the energy required to heat the calorimeter, then you are left with  $IVt = m_1c_1\Delta T + m_2c_2\Delta T$ , where  $c_1$  is the specific heat capacity of the liquid, and all other variables are known.
  - a. It is important to stir the water at regular intervals.



<http://physicsmax.com/measure-specific-heat-capacity-metal-solid-block-method-8008>

The picture below gives a comparison of the two electrical methods of determination.

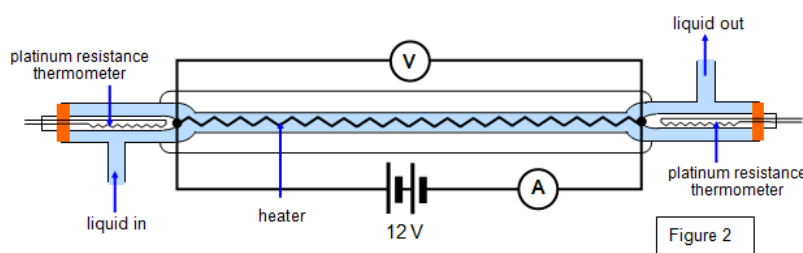


[http://www.schoolphysics.co.uk/age16-19/Thermal%20physics/Heat%20energy/text/Specific\\_heat\\_capacity\\_measurement/index.html](http://www.schoolphysics.co.uk/age16-19/Thermal%20physics/Heat%20energy/text/Specific_heat_capacity_measurement/index.html)

### Calculations including continuous flow

An electric shower is an example of a continuous flow hot water system, where hot water is delivered continuously at a constant temperature and with no requirement for storage. Continuous flow only uses energy when the heat is necessitated ie when the shower is turned on.

The picture below gives a good explanation of a continuous flow heater.



[http://www.schoolphysics.co.uk/age16-19/Thermal%20physics/Heat%20energy/text/Specific\\_heat\\_capacity\\_measurement/index.html](http://www.schoolphysics.co.uk/age16-19/Thermal%20physics/Heat%20energy/text/Specific_heat_capacity_measurement/index.html)

The energy supplied to the liquid is equal to  $IVt = mc\Delta T$ . This process is also used in solar heating panels.

*For a change of state  $Q = ml$  where  $l$  is the specific latent heat.*

The equation above describes the energy changes when a solid/liquid/gas undergoes a change of state.  $Q$  is the energy required to change state,  $m$  is the mass of the substance and  $l$  is either the specific latent heat of **fusion** or **vaporisation** (depending on which change of state is taking place).

When a solid reaches its melting point, it requires a certain amount of energy to melt it into a liquid. This energy required is called the latent heat of **fusion**, as a solid 'fuses' into a liquid when it melts. On the reverse, when a liquid cools to become a solid, latent heat is released. At the melting point, the molecules in the liquid are moving slow enough for bonds to form and keep the molecules together.



The word ‘latent’ means hidden, because when for example a solid reaches its melting point, the energy required to cause the change of state does not cause an increase in temperature of the solid.

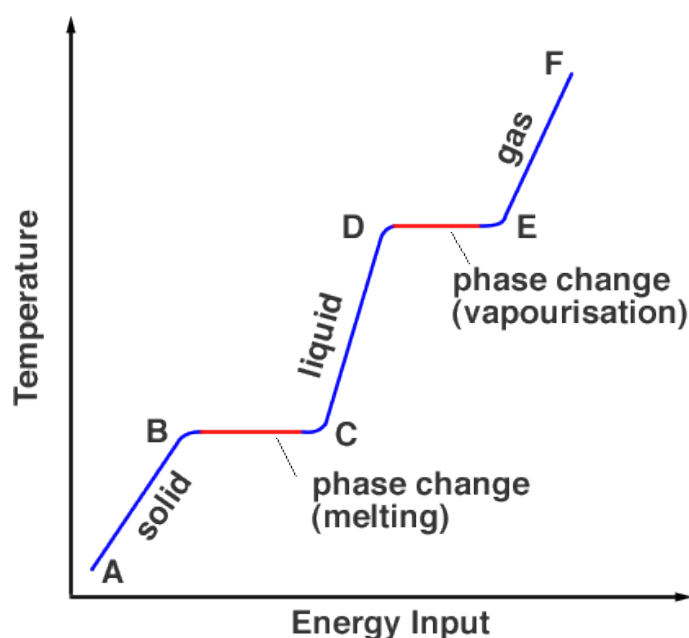
For a liquid heated at its boiling point, the energy required to vaporise the liquid is referred to as the latent heat of **vaporisation**. This will occur once the molecules have been given enough kinetic energy to break free of the bonds holding them in their liquid form. On the contrary, latent heat will be released as for example, water vapour, condenses into its liquid form. As a rule of thumb, it requires more energy to vaporise a substance than melt it. Water vapour (steam) given off from a kettle is extremely hot, compared with ice that has just melted into a liquid.

There is also a phenomenon called **sublimation** which occurs in some solids. In these solids, instead of changing state to a liquid, when heated they are vaporised directly instead.

By definition, ‘the specific latent heat of **fusion**,  $l$ , of a substance is the heat required to change a unit mass of a substance from a solid, at its melting point, into a liquid, whilst remaining at the same temperature.’

By definition, ‘the specific latent heat of **vaporisation**,  $l$ , of a substance is the heat required to change a unit mass of a substance from a liquid at its boiling point, into a vapour, whilst remaining at the same temperature.’

A graph of temperature vs energy input is shown below for the changes of state from a solid to a gas.



<http://www.splung.com/content/sid/6/page/latentheat>

Let the substance in the graph shown be called X. At A, X is a solid and is heated towards its melting point at B. During this stage, and until C, the solid will receive latent heat of fusion, which will go towards changing the state of the solid. During this time, it is clear from the graph that the temperature does not change. Substance X is now a liquid at C, and is heated towards its boiling point at D. From D to E the liquid is provided with latent heat of

vaporisation, and the liquid will change state towards becoming a gas. The temperature can be increased further towards F, but no more changes of states will occur unless the substance is contained and cooled.

### 3.6.2.2 Ideal gases

#### Content

- Gas laws as experimental relationships between  $p$ ,  $V$ ,  $T$  and the mass of the gas.
- Concept of absolute zero of temperature.
- Ideal gas equation:  $pV = nRT$  for  $n$  moles and  $pV = NkT$  for  $N$  molecules.
- $Work\ done = p\Delta V$
- Avogadro constant  $N_A$ , molar gas constant  $R$ , Boltzmann constant  $k$
- Molar mass and molecular mass.

#### *Gas laws as experimental relationships between $p$ , $V$ , $T$ and the mass of the gas.*

The pressure within a gas is the force per unit area that the gas exerts on a surface. This force acts perpendicularly to the surface in which the gas is in contact with. The units of pressure,  $p$  are pascals (Pa), which can be written as  $\text{Nm}^{-2}$ . The pressure within a gas is caused by the molecules hitting the surfaces of a container, each individual collision is insignificant, but due to the vast number of impacts occurring every second the pressure is detectable. Another thing worth noting is that the gas molecules bounce off the surfaces, or each other, without losing speed.

It can be said that the pressure of a gas depends on the volume, the temperature and the mass of the gas (in a given container). Moreover, the values of each of these properties determines the state of the gas.

The relationships between  $p$ ,  $V$ ,  $T$  and the mass of the gas was first established by Boyle and Charles.

**Boyles** Law states that, for a gas of constant mass and temperature,  $pV = k$ , where  $k$  is a constant. Any gas that obeys Boyle's law is referred to as an ideal gas.

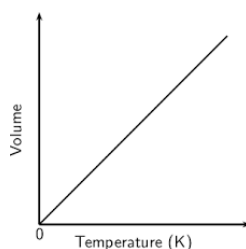
**Charles** Law states that for a given mass, at constant pressure, volume is proportional to temperature. This can be written as  $V/T = k_2$  where  $k_2$  is a constant.

The **pressure** law states that for a given mass at constant volume, pressure is proportional to temperature. This can be written as  $p/T = k_3$  where  $k_3$  is a constant.

Any changes that occur at a constant temperature are called isothermal changes. On the contrary, an isobaric change is one which takes place at constant pressure.

#### *Concept of absolute zero of temperature.*

Absolute zero is the lowest possible theoretical temperature. This temperature is equal to 0K on the Kelvin scale, which is equivalent to  $-273.15^\circ\text{C}$ . A graph of volume against temperature will yield a straight line (Charles law states they are proportional under certain circumstances). This graph relates to the temperature of 'absolute zero' because when volume is 0, temperature in Kelvin will be 0 ie at the origin. Although this is only followed by *ideal gases*.



This means that no matter how much gas is present, as long as it is an ideal gas, at absolute zero its volume will be 0.

***Ideal gas equation:  $pV = nRT$  for  $n$  moles and  $pV = NkT$  for  $N$  molecules.***

The three experimental laws covered previously can be combined to give the ideal gas equation  $pV/T = \text{constant}$ , which is only true for a fixed mass of an ideal gas.  $p$  is the pressure,  $V$  is the volume and  $T$  is the absolute temperature. The constant is equal to  $nR$ , where  $n$  is the number of moles and  $R$  is the molar gas constant. Thus the equation can be rewritten as  $pV = nRT$ .

Something worth noting is that any ideal gases of the same volume, temperature and pressure will have the same number of moles. Rearranging the  $pV = nRT$  to  $pV/RT = n$  represents this.

For  $n=1$ , ie for 1 mol of any ideal gas in a container,  $pV/T = R$ . This value of  $R$  can be calculated to be equal to  $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$  (given in the formula sheet). This value is called the molar gas constant, and a graph of  $pV$  vs  $T$  yields a straight line through absolute zero with a gradient  $= nR$ .

It can be said that the number of moles,  $n = N/N_A$ , where  $N$  is the number of molecules and  $N_A$  is Avogadro's constant (covered in the next section). If you substitute this value of  $n$  into  $pV = nRT$ , you get  $pV = NRT/N_A$ , and it can then be said that  $pV = NkT$ , because  $k = R/N_A$ , where  $k$  is the Boltzmann constant, and  $N$  is the number of molecules.

***Work done  $= p\Delta V$***

For a gas, the work done is the product of the pressure and volume during a change of volume.

Work done is defined as the product of the force acting through a distance  $s$ , where  $W = Fs$ . The units can be written in SI form as  $\text{Nm}$ .

The unit of pressure can be written as  $\text{Nm}^{-2}$ , and the unit of volume as  $\text{m}^3$ . The combined unit gives you  $\text{Nm}$ , so the unit of work.

***Avogadro constant  $N_A$ , molar gas constant  $R$ , Boltzmann constant  $k$***

The Avogadro constant,  $N_A$  is the number of atoms in 12g of  $^{12}\text{C}$  equal to  $6.023 \times 10^{23}$ . From this you can work out the mass of one atom of carbon 12 to be  $12/6.023 \times 10^{23} = 1.993 \times 10^{-23} \text{ g}$ . We will also use, in a later topic, something called the atomic mass unit, denoted by the

symbol u. This atomic mass unit is  $1/12^{\text{th}}$  the mass of carbon-12. The value of  $u = 1.661 \times 10^{-27}$  kg.

The molar gas constant was covered previously, to recap:

For  $n=1$ , ie for 1 mol of any ideal gas in a container,  $pV/T = R$ . This value of  $R$  can be calculated to be equal to  $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$  (given in the formula sheet). This value is called the molar gas constant, and a graph of  $pV$  vs  $T$  yields a straight line through absolute zero with a gradient  $= nR$ .

The Boltzmann constant ( $k = 1.38 \times 10^{-23}$ ) relates the average kinetic energy of particles in a gas with the temperature of a gas. The Boltzmann constant  $k = R/N_A$ , so the constant is a combination of two other constants.

### *Molar mass and molecular mass.*

One mole of a substance is defined as the mass of a substance containing the same number of fundamental units as there are atoms in exactly 12g of  $^{12}\text{C}$ . These fundamental units could be atoms or molecule depending on the question. This means that one mole of a substance contains  $6.023 \times 10^{23}$  atoms/molecules. ‘Molarity’ is the number of moles in a certain amount of substance, with the unit ‘mol’.

The mass of one mol of a substance is called its molar mass, units  $\text{kg mol}^{-1}$ . The molar mass of sulfur is roughly  $0.032 \text{ kg mol}^{-1}$ , so 0.032kg of sulfur contains  $6.023 \times 10^{23}$  particles.

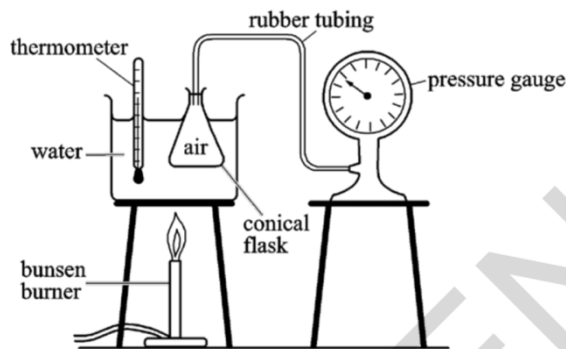
To find the molarity of a substance, you must divide the mass of the substance ( $m_s$ ) by the molar mass of the substance ( $M$ ). ie  $n = m_s/M$ . You can find the number of molecules in a given mass by multiplying  $m_s/M$  by Avogadro’s number, so  $N = N_A m_s/M$ .

On the contrary, molecular mass is simply the mass of a molecule. It is found by summing the constituent masses of the atoms within a molecule.

## OCR (A) A Level Specimen 1

### Question:

The apparatus shown in **Fig. 20.1** is used to investigate the variation of the product  $PV$  with temperature in the range  $20\text{ }^{\circ}\text{C}$  to  $100\text{ }^{\circ}\text{C}$ . The pressure exerted by the air is  $P$  and the volume of air inside the flask is  $V$ .



**Fig. 20.1**

Describe how this apparatus can be set up and used to ensure accurate results.

### Answer:

- Ensure largest possible proportion of flask is immersed.
- Make volume of tubing small compared to volume of flask.
- Remove heat source and stir water to ensure water at uniform temperature throughout.
- Allow time for heat energy to conduct through glass to air before reading temperature



### 3.6.2.3 Molecular kinetic theory model

#### Content

- Brownian motion as evidence for existence of atoms.
- Explanation of relationships between  $p$ ,  $V$  and  $T$  in terms of a simple molecular model.
- Students should understand that the gas laws are empirical in nature whereas the kinetic theory model arises from theory.
- Assumptions leading to  $pV = \frac{1}{3}Nm(c_{rms})^2$  including derivation of the equation and calculations.
- A simple algebraic approach involving conservation of momentum is required.
- Appreciation that for an ideal gas internal energy is kinetic energy of the atoms.
- Use of *average molecular kinetic energy*  $= \frac{1}{3}Nm(c_{rms})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$
- Appreciation of how knowledge and understanding of the behaviour of a gas has changed over time.

#### *Brownian motion as evidence for existence of atoms.*

The existence of atoms and subsequently molecules was first established in 1827, by Robert Brown. His observation of pollen grains in water showed the irregular random motion of particles in a fluid to be as a result of continuous bombardment of molecules in the surrounding medium.

#### *Explanation of relationships between $p$ , $V$ and $T$ in terms of a simple molecular model.*

The pressure of a gas at constant temperature can be increased by decreasing the volume in which the gas is contained. This is because the molecules within the container need to travel a smaller distance between impacts on the surface of the container, so there are more impacts per unit time, and so an increased pressure. This explains Boyle's Law.

Increasing the temperature of a gas increases the average speed, so the impacts within the container are more frequent and of larger significance. This increases pressure and relates to the pressure law.

#### *Students should understand that the gas laws are empirical in nature whereas the kinetic theory model arises from theory.*

The gas laws are empirical in a nature; ie they have been found experimentally. Whereas, the kinetic theory model arises from mathematics and theories.

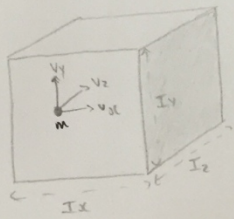
#### *Assumptions leading to $pV = \frac{1}{3}Nm(c_{rms})^2$ including derivation of the equation and calculations.*

In deriving this equation shown above, you must first make assumptions about the molecules within a gas. These assumptions are as follows:



- You can treat molecules as point objects, with a negligible volume relative to the volume of the gas
- There are no attractive forces between the molecules within the gas. If there were attractive forces the impact forces on the surfaces of the container would be reduced
- Their motion is continuous and completely unpredictable
- All collisions, either between molecules or molecules and the container surface, are elastic. This means there is no loss in kinetic energy after collisions
- Finally, the duration of contact between molecules and the container surface is negligible compared to the time between impacts.

A simple derivation of the equation  $pV = \frac{1}{3}Nm(\text{crms})^2$  is given below. In this derivation, the velocity in the x direction is represented by  $v_x$ , in the y direction by  $v_y$  and z direction by  $v_z$ . Pythagoras' theorem, as noted in the derivation, can be applied to three dimensions. Finding the resultant velocity in 2 dimensions can be done by doing  $v^2 = (v_x)^2 + (v_y)^2$ , and this can be extended in three dimensions so  $v^2 = (v_x)^2 + (v_y)^2 + (v_z)^2$  as the individual velocities are all perpendicular to each other. Other than that the derivation should be trivial.



• If we consider one particle of mass  $m$ , the speed  $c = \sqrt{(v_x)^2 + (v_y)^2 + (v_z)^2}$  (Pythagoras in 3 dimensions).  

$$c^2 = (v_x)^2 + (v_y)^2 + (v_z)^2$$

• We also know that when this molecule of mass  $m$  collides with the shaded wall, its velocity in the x-direction will reverse.

• This means that the change in momentum  $\Delta mv = -2mv_x$ .

• The time taken for this particle to complete successive impacts on this wall is  $t = \frac{2l_x}{v_x}$ .

• We know that  $f = \frac{\Delta mv}{t} = \frac{-2mv_x}{2l_x/v_x} = -\frac{mv_x^2}{l_x}$ .

• According to Newton's Third Law, the force of impact is equal and opposite to the force on the molecule, so force on the molecule  $= +\frac{m(v_x)^2}{l_x}$ .

- We also know that pressure =  $\frac{\text{force}}{\text{area}}$
- So 
$$p_1 = \frac{\frac{m(v_{x1})^2}{Lx}}{LyLz} = \frac{m(v_{x1})^2}{LxLyLz} \quad LxLyLz = \text{volume}$$
$$= \frac{m(v_{x1})^2}{V}$$
- So overall pressure is sum of all pressures i.e.  $p_1 + p_2 + p_3 + \dots + p_n$
- $$p = \frac{m(v_{x1})^2}{V} + \frac{m(v_{x2})^2}{V} + \frac{m(v_{x3})^2}{V} + \dots$$
$$= \frac{m}{V} (v_{x1}^2 + v_{x2}^2 + v_{x3}^2 + \dots)$$

each represents a different particle.
- If we say that  $(\bar{v}_x)^2 = \frac{v_{x1}^2 + v_{x2}^2 + \dots}{N}$   
where  $N$  is the number of molecules.
- Then we can write  $p = \frac{Nm(\bar{v}_x)^2}{V}$
- The same equation applies in all directions i.e.  $p = \frac{Nm(\bar{v}_y)^2}{V}$  or  $p = \frac{Nm(\bar{v}_z)^2}{V}$
- So 
$$p = \frac{Nm}{3V} ((\bar{v}_x)^2 + (\bar{v}_y)^2 + (\bar{v}_z)^2)$$
- $C_{rms}$  (the root mean square of the speed)  
 $= (\bar{v}_x)^2 + (\bar{v}_y)^2 + (\bar{v}_z)^2$
- So 
$$p = \frac{1}{3} Nm (C_{rms})^2$$

*A simple algebraic approach involving conservation of momentum is required.*

In the derivation above, momentum is treated algebraically. This is because the actual speed and mass of the particle is unknown, thus are denoted by  $m$  and  $v$  respectively. This means that change in momentum =  $-mv - (mv) = -2mv$ .

*Appreciation that for an ideal gas internal energy is kinetic energy of the atoms.*

For an ideal gas, the internal energy should be due only to the kinetic energy of the atoms. As the temperature of a gas increases, the average kinetic energy of the molecules within a container will increase.

$$\text{Use of average molecular kinetic energy} = \frac{1}{3} Nm(c_{rms}) = \frac{3}{2} kT = \frac{3RT}{2N_A}$$

The average molecular kinetic energy is given by  $\frac{1}{3} Nm(c_{rms})$  which is also equal to  $\frac{3}{2} kT$ , equal also to  $\frac{3RT}{2N_A}$ . The derivation of these equations is not required, only their application. Each symbol has the same associated unit and name as mentioned already.

**REMEMBER THIS IS ONLY FOR 1  
GAS MOLECULE!!**

## AQA June 2011 Unit 5

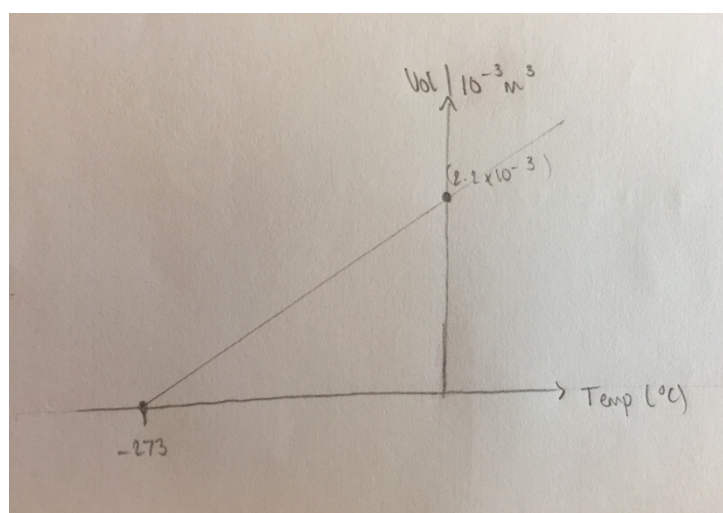
### Question:

A fixed mass of ideal gas at a low temperature is trapped in a container at constant pressure. The gas is then heated and the volume of the container changes so that the pressure stays at  $1.00 \times 10^5 \text{ Pa}$ .

When the gas reaches a temperature of  $0^\circ\text{C}$  the volume is  $2.20 \times 10^{-3}$ .

Draw a graph on the axes below to show how the volume of the gas varies with temperature in  $^\circ\text{C}$ .

### Answer:



We know that at absolute 0 ( $-273.15^\circ\text{C}$ ), the volume of an ideal gas will be 0. Thus the graph must go through this point. We also know that  $V = nRT/P$ , where  $nR/P$  is constant, so  $V$  is proportional to  $T$ . Thus a straight line must be drawn from absolute zero, to the point ( $2.2 \times 10^{-3} \text{ m}^3$ ) that we are given in the question.

### Question:

Calculate the number of moles of gas present in the container.

### Answer:

The gradient must be equal to  $nR/P$ . So if we calculate the gradient using our two known points, we get approximately  $8.05 \times 10^{-6}$ .

So  $8.05 \times 10^{-6} = nR/P$ , and if we solve for  $n$  we get 0.970 moles (3sf).

**Question:**

Calculate the average kinetic energy of a molecule when this gas is at a temperature of 50.0°C. Give your answer to an appropriate number of significant figures.

**Answer:**

$3kT/2$  = average kinetic energy of a molecule. Remember that we are given T in Celsius, so add 273 to convert it to Kelvin.

- (use of mean kinetic energy =  $3/2 k T$ ) =  $3/2 \times 1.38 \times 10^{-23} \times 323$
- =  $6.69 \times 10^{-21} \text{ J}$  (3sf)

**Question:**

Calculate the total internal energy of the gas at a temperature of 50.0°C.

**Answer:**

We already know the number of moles present, and since in 1mol there are  $6.02 \times 10^{23}$  molecules, in 0.097 moles there are  $0.097 \times 6.02 \times 10^{23}$  molecules present. We also know the average kinetic energy of a molecule, so we simply multiply the number of molecules present by the average kinetic energy of 1 molecule.

$$\text{Total internal energy} = 6.69 \times 10^{-21} \times 0.0970 \times 6.02 \times 10^{23} = 390 \text{ J}$$

**Question:**

By considering the motion of the molecules explain how a gas exerts a pressure and why the volume of the container must change if the pressure is to remain constant as the temperature increases.

The quality of your written communication will be assessed in this question.

**Answer:**

- Molecules are in rapid random motion/many molecules are involved molecules change their momentum or accelerate on collision with the walls
- Reference to newton's 2<sup>nd</sup> law either  $F = ma$  or  $F = \text{rate of change of momentum}$
- Reference to newton's 3<sup>rd</sup> law between molecule and wall
- Relate pressure to force  $P = F/A$
- Mean square speed of molecules is proportional to temperature
- As temperature increases so does change of momentum or change in velocity
- Compensated for by longer time between collisions as the temperature increases
- As the volume increases the surface area increases which reduces the pressure

**EXAM TIP:**

**REMEMBER TO CONVERT TO KELVIN FROM CELCIUS!**



## 3.7 Fields and their consequences

### 3.7.1 Fields

#### Content

- Concept of a force field as a region in which a body experiences a non-contact force.
- Students should recognise that a force field can be represented as a vector, the direction of which must be determined by inspection.
- Force fields arise from the interaction of mass, of static charge, and between moving charges.
- Similarities and differences between gravitational and electrostatic forces:
- Similarities: Both have inverse-square force laws that have many characteristics in common, eg use of field lines, use of potential concept, equipotential surfaces etc.
- Differences: masses always attract, but charges may attract or repel.

*Concept of a force field as a region in which a body experiences a non-contact force.*

The definition of a force field is a region in which a body experiences a non-contact force, and an example of a field is the gravitational field.

*Students should recognise that a force field can be represented as a vector, the direction of which must be determined by inspection.*

A force field can be represented as a vector, i.e. the gravitational field lines will point towards the earth as this is the direction that they act. If you have another field i.e. an electric field interacting with another electric field, you may have to determine the resultant direction and magnitude of the electric field.

*Force fields arise from the interaction of mass, of static charge, and between moving charges.*

The interaction of mass is determined by the gravitational field, the electric field determines the interaction of static charge, and the electromagnetic for moving charges.

*Similarities and differences between gravitational and electrostatic forces:*

The similarities are that they both have inverse-square force laws that have many characteristics in common, eg the use of field lines, use of the 'potential' concept, equipotential surfaces etc.

Their differences are that masses will always attract; however, charges may attract or repel depending on whether they are like charges or not.



## 3.7.2 Gravitational Fields

### 3.7.2.1 Newton's law

#### Content

- Gravity as a universal attractive force acting between all matter.
- Magnitude of force between point masses:  $F = \frac{Gm_1m_2}{r^2}$  where  $G$  is the gravitational constant.

#### *Gravity as a universal attractive force acting between all matter.*

Gravity is a universal attractive force that acts between all matter. Our first real descriptions of gravity came from Sir Isaac Newton, and then these were furthered by Albert Einstein in the early 1900s. Gravity acts towards the centre of the Earth, and causes objects to be attracted downwards, to the surface of the Earth.

#### *Magnitude of force between point masses: $F = \frac{Gm_1m_2}{r^2}$ where $G$ is the gravitational constant.*

The equation  $F = Gm_1m_2/r^2$  is the equation that enables you to calculate the magnitude of the force between point masses. The value of  $G$  is given in the equation sheet, and is  $6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$ . The 'r' is the distance between the centre of mass of the two objects, so for a uniform particle on the surface of the Earth, r will not be 0, but approximately  $6.4 \times 10^6 \text{ m}$ , as this is the radius of the Earth. Usually the equation will be written using 'mM' where the 'm' will be the larger mass making the noticeable attraction with the other mass i.e. the Earth.



## AQA June 2010 Unit 4 Section B Q1a

**Question:**

State Newton's law of gravitation

**Answer:**

- Force of attraction between two point masses
- Proportional to product of masses
- Inversely proportional to square of distance between them

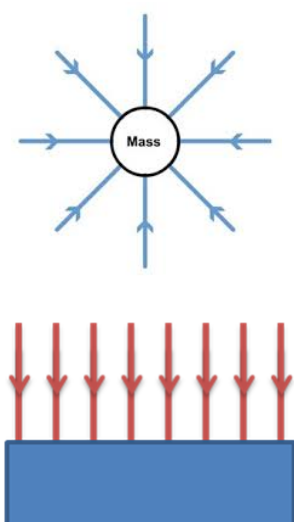
### 3.7.2.2 Gravitational field strength

#### Content:

- Representation of a gravitational field by gravitational field lines.
- $g$  as force per unit mass as defined by  $g = F/m$ .
- Magnitude of  $g$  in a radial field given by  $g = GM/r^2$ .

#### *Representation of a gravitational field by gravitational field lines.*

The diagram below shows a representation of a gravitational field. The field lines go radially outwards from the mass. However, if you were to look at a small cross section of the gravitational field lines at the surface of the Earth, they would appear to radiate uniformly. This can be seen in the second picture.



#### *$g$ as force per unit mass as defined by $g = F/m$ .*

$g$  is defined as the force per unit mass, so  $F/m$ . As you move vertically upwards from the surface of the Earth, the attraction  $F$  between you and the Earth will decrease, illustrated by the equation  $F = GmM/r^2$ . However, for small distances above the surface of the Earth  $F$  will not change by a huge amount, so the force per unit mass ( $g$ ), will also not change very much. This explains why  $g$  is a relative constant at the surface of the Earth (compared to how much it fluctuates as you move distances of thousands of kilometres from the surface). This can also be explained by fields, as the field is pretty much uniform at the surface of the Earth, so  $F$  will not change by large amounts.

#### *Magnitude of $g$ in a radial field given by $g = GM/r^2$ .*

Since the gravitational field is not a uniform field on a large scale, and it is a radial field,  $g$  will vary. This means that we need an equation to calculate  $g$  at different points in the radial field, which is done by using  $GM/r^2$ .

### 3.7.2.3 Gravitational potential

#### Content

- Understanding of definition of gravitational potential, including zero value at infinity
- Understanding of gravitational potential difference.
- Work done in moving a mass  $m$  given by  $\Delta W = m\Delta V$ .
- Equipotential surfaces.
- Idea that no work is done when moving along an equipotential surface.
- $V$  in a radial field given by  $V = -GM/r$ .
- Significance of negative sign.
- Graphical representations of variations of  $g$  and  $V$  with  $r$ .
- $V$  related to  $g$  by:  $g = -\Delta V/\Delta r$ .
- $\Delta V$  from area under graph of  $g$  against  $r$ .

#### Opportunities for Skills Development

- Students use graphical representations to investigate relationships between  $v$ ,  $r$  and  $g$ .

#### *Understanding of definition of gravitational potential, including zero value at infinity.*

Gravitational potential is the work done per unit mass to move a test mass from infinity to that point. Therefore, the scale starts at infinity, which can also be taken as 0. For this reason, the scale will increase up to 0, as work is done to move a unit mass towards this point (infinity). This could be modelled in the equation  $\Delta V = \Delta W/m$ , so therefore the unit for gravitational potential is  $\text{Jkg}^{-1}$ .

#### *Understanding of gravitational potential difference.*

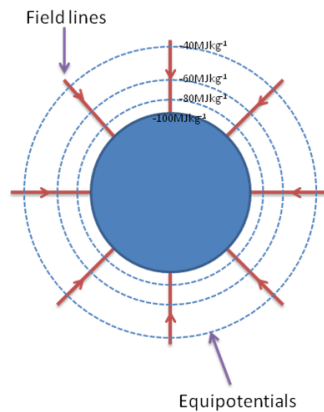
The gravitational potential difference is the difference in potential between two points in a gravitational field. The gravitational potential on the surface of the Earth is around  $-63\text{MJkg}^{-1}$  and at another point 1000km above the surface of the Earth is approximately  $-54\text{MJkg}^{-1}$ . Therefore, the gravitational potential difference is  $-54 - (-63) = 9\text{MJkg}^{-1}$ .

#### *Work done in moving a mass $m$ given by $\Delta W = m\Delta V$ .*

Since the gravitational potential is the work done per unit mass, the change in gravitational potential energy is found by multiplying the change in the gravitational potential by the mass of the object. Therefore the equation is  $\Delta W = m\Delta V$ , where  $W$  = the work done (change in gravitational potential energy),  $m$  = mass and  $V$  = gravitational potential.

#### *Equipotential surfaces.*

These are surfaces in which no work is done when moved along. This is because the gravitational potential at these equipotential surfaces is the same the whole way along, so in the equation  $\Delta W = m\Delta V$ , the change in gravitational potential ( $\Delta V$ ) is 0, so work done is 0. The diagram below shows equipotential surfaces.



*Idea that no work is done when moving along an equipotential surface.*

This point is covered above.

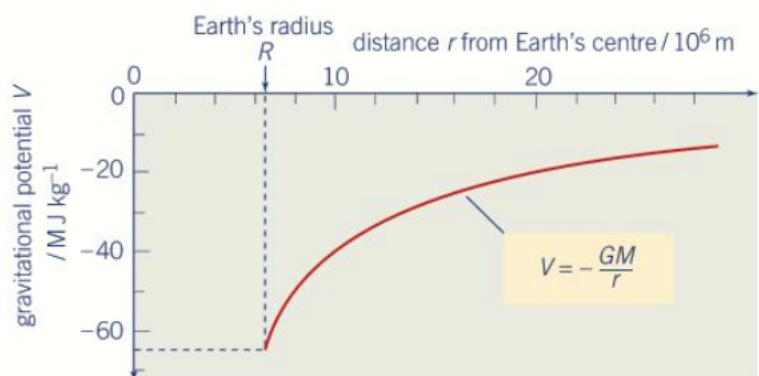
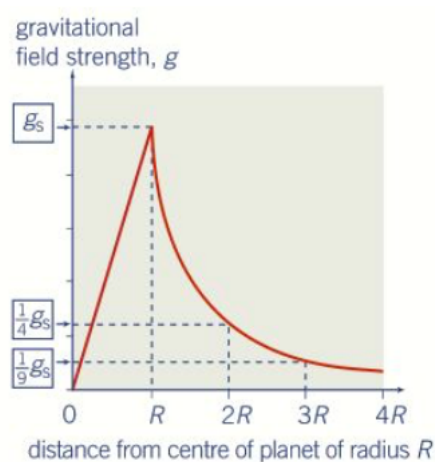
*$V$  in a radial field given by  $V = -GM/r$ .*

To calculate the gravitational potential in a radial field you use the equation given above.  $G$  = gravitational constant given in the equation sheet,  $M$  is mass, for example mass of the Earth.  $r$  is the distance from the centre of the object. The  $M$  in the equation will always be the mass of the object responsible for the gravitational attraction.

*Significance of negative sign.*

The negative sign indicates that gravity is an attractive force, pulling objects together.

*Graphical representations of variations of  $g$  and  $V$  with  $r$ .*



Since  $g = GM/r^2$ , it can be said that  $g \propto 1/r^2$ . This gives the shape of the graph that you can see. However, when moving towards the centre of a planet, the gravitational field strength reduces linearly from  $g$  to 0. On the other contrary,  $V = -GM/r$ , so it can be said that  $V \propto 1/r$ , to give the shape of the graph shown above.

*V related to g by:  $g = - \Delta V / \Delta r$ .*

The gravitational field strength can also be calculated from the potential gradient, where  $g = - \Delta V / \Delta r$ . It can also be looked at as the change in potential per metre at a point, near the surface of the Earth a 1kg mass gains 9.81J of energy per metre it is raised, but this becomes less the further from the surface of the Earth that you move.

*$\Delta V$  from area under graph of g against r.*

The change in potential can be found by calculating the area under a graph of g against r.

## AQA A Level Specimen Q4.1

### Question:

Explain why gravitational potential is always negative

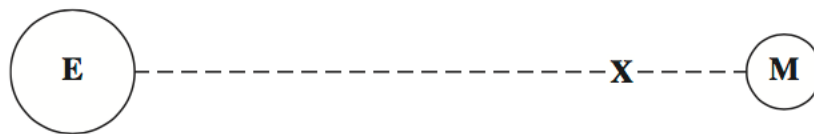
### Answer:

- Gravitational potential is defined as zero at  $\infty$
- (Forces attractive) so work must be done (on a mass) to reach  $\infty$  (hence negative)

## AQA June 2012 Unit 4 Q4b)iii)c)

**Figure 5**, which is not drawn to scale, shows the region between the Earth (**E**) and the Moon (**M**).

**Figure 5**



**Question:**

In the vicinity of the Earth's orbit the gravitational potential due to the Sun's mass is  $-885 \text{ MJ kg}^{-1}$ . With reference to the variation in gravitational potential with distance, explain why the gravitational potential due to the Sun's mass need not be considered when carrying out the calculation in part (b)(i).

**Answer:**

- Distance from Earth to Sun  $\gg$  distance from Earth to Moon
- Change in  $V_{\text{sun}}$  (or in  $g_{\text{sun}}$ ) over Earth to Moon distance is negligible
- Value of  $V_{\text{sun}}$  (or  $g_{\text{sun}}$ ) is not (significantly) changed by relative positions of E+M

**Question:**

The amount of energy required to move a manned spacecraft from the Earth to the Moon is much greater than that required to return it to the Earth. By reference to the forces involved, to gravitational field strength and gravitational potential, and to the point **X**, explain why this is so.

The quality of your written communication will be assessed in your answer.

**Answer:**

**For 6 marks:**

*The candidate discusses the forces of attraction due to the Earth and due to the Moon, appreciates that they act in opposite directions, and that the former is generally much greater than the latter.*

*The candidate discusses the resultant gravitational field between E and M, understands that there is a 'neutral' point at which the resultant field strength is zero and that this point is much closer to M than E. It is recognised that this point has to be passed for the journey to be completed in either direction.*

*There is a discussion of gravitational potential, in which it is pointed out that the resultant potential rises to a maximum at the neutral point. There is a reference to the much greater amount of work that has to be done on the spacecraft to reach this point from E than from M.*

### **Gravitational forces**

- The spacecraft experiences gravitational attractions to both the Earth and the Moon during its journey.
- These forces pull in opposite directions on the spacecraft.
- Because E is much more massive than M, for most of the outward journey the force towards E is greater than that towards M.
- Only in the later stages of the outward journey is the resultant force directed towards M.
- On the return journey the resultant force is predominantly towards E.

### **Gravitational field strength**

- During the outward journey E's gravitational field becomes weaker and M's becomes stronger.
- The resultant field is the vector sum of those due to E and M separately.
- A point (X) is reached at which these two component fields are equal and opposite, giving zero resultant.
- X is much closer to M than to E.
- Once X has been passed, the spacecraft will be attracted to M by M's gravitational field.
- On the return journey the spacecraft will 'fall' to E once it is beyond X.

### **Gravitational potential**

- The gravitational potential due E increases (i.e. becomes less negative) as the spacecraft moves away from E.
- The resultant gravitational potential is the (scalar) sum of those due to E and M separately.
- At X the gravitational potential reaches a maximum value before decreasing as M is approached.
- In order to reach M on the outward journey, the spacecraft has to be given at least enough energy to reach X, and vice-versa for the return.
- Much more work is needed to move the spacecraft from E to X than from M to X, since a larger force has to be overcome over a larger distance



### 3.7.2.4 Orbits of planets and satellites

#### Content

- Orbital period and speed related to radius of orbit; derivation of  $T^2 \propto R^3$
- Energy considerations for an orbiting satellite.
- Total energy of an orbiting satellite.
- Escape velocity.
- Synchronous orbits.
- Use of satellites in low orbits and geostationary orbits, to include plane and radius of geostationary orbit.

#### Opportunities for Skills Development

- Estimate various parameters of planetary orbits, eg kinetic energy of a planet in orbit.
- Use logarithmic plots to show relationships between  $T$  and  $r$  for given data.
- 

### *Orbital period and speed related to radius of circular orbit; derivation of $T^2 \propto R^3$*

Kepler's Third Law states that the orbital period is related to the radius of the circular orbit where  $T^2 \propto R^3$ . The picture below shows the derivation of Kepler's Law.

The image shows a handwritten derivation of Kepler's Third Law. The steps are as follows:

$$F = \frac{GMm}{r^2} \quad F = \frac{mv^2}{r}$$

since  $\frac{v}{r} = \omega$

$$F = \frac{GMm}{r^2} \quad F = m\omega^2 r$$

since  $\omega = 2\pi f$

$$F = \frac{GMm}{r^2} \quad F = m(2\pi f)^2 r$$

Equate two equations

$$\frac{GMm}{r^2} = m(4\pi^2 f^2) r$$

since  $f = \frac{1}{T}$

$$\frac{GM}{r^2} = \frac{4\pi^2}{T^2} r$$
$$T^2 = \frac{4\pi^2}{Gm} r^3$$

since  $\frac{4\pi^2}{Gm}$  is constant = k

$$T^2 \propto r^3$$
$$T^2 = kr^3$$

### *Energy considerations for an orbiting satellite.*

An orbiting satellite will possess both kinetic energy and gravitational potential energy. Its kinetic energy will be as a result of its velocity and mass, and its gravitational potential energy will be as a result of its position in the gravitational field as well as its mass. If a satellite moved closer to the Earth, it would lose gravitational potential energy, so this energy must be converted to kinetic energy. Thus the orbital time of the satellite would decrease as its kinetic energy would increase thus its velocity also.

### *Total energy of an orbiting satellite.*

In order to calculate the kinetic energy of satellite we use  $\frac{1}{2}mv^2$ , so we need to calculate the velocity of the satellite. In order to do this, we need an equation with  $v$  in, so we can use  $F = mv^2/r$ . However if we do not have all of the information that we need to solve  $v$ , you can equate the equations  $F = mv^2/r$  and  $F = GmM/r^2$ . The result of this when you solve for the velocity is  $v^2 = \frac{GM}{r}$ . So if we substitute this into  $\frac{1}{2}mv^2$  we end up with  $GmM/2r$ . Now we have an equation for the kinetic energy we need to find one for the gravitational potential energy. Gravitational potential is equal to  $V = -\frac{GM}{r}$ , so to find its gravitational potential energy at this point we must multiply by  $m$ . This gives us  $-\frac{Gmm}{r}$ , so when we do  $E_k + E_p$  as shown below, we are left with  $-\frac{Gmm}{2r}$ .

The image shows a handwritten derivation of the total energy of an orbiting satellite. The steps are as follows:

$$E_k = \frac{1}{2} m v^2$$
$$F = \frac{mv^2}{r} \quad F = \frac{GmM}{r^2}$$
$$\frac{mv^2}{r} = \frac{GmM}{r^2}$$
$$v^2 = \frac{GM}{r}$$
$$\text{so } \frac{1}{2} m \frac{GM}{r} = \frac{GmM}{2r} = E_k.$$
$$E_p = mV$$
$$V = -\frac{GM}{r}$$
$$E_p = -\frac{Gmm}{r}$$
$$E_k + E_p = \text{Total energy.}$$
$$= \frac{GmM}{2r} + -\frac{Gmm}{r}$$
$$= \frac{GmM}{2r} - \frac{2Gmm}{2r}$$
$$= -\frac{Gmm}{2r} = \text{Total energy.}$$

### *Escape velocity.*

Escape velocity is defined simply as '*The lowest velocity which a body must have in order to escape the gravitational attraction of a particular planet or other object.*' The gravitational potential at the surface of the Earth is roughly -63MJ. So if you were attempting to find the escape velocity of a 1kg mass at the surface of the earth, you are to consider the energy transformations of kinetic and potential energies.

For a 1kg mass at the surface of the Earth, you must give roughly 63MJ of energy in order for potential energy to equal 0, thus it can be said that there is no gravitational attraction from the Earth on the mass as it has escaped its arena of influence. Now, if you wanted to find the escape velocity, you must equate the equation for kinetic energy ( $mv^2/2$ ), to the equation for work done on a mass  $GMm/r$ , and solve for  $v = (2GM/r)^{1/2}$ . This is the escape velocity. You can see that the escape velocity is independent of mass of the object, as the masses cancel.

**For an object, such as a space rocket, to escape from the gravitational attraction of the Earth it must be given an amount of energy equal to the gravitational potential energy that it has on the Earth's surface. The minimum initial vertical velocity at the surface of the Earth that it requires to achieve this is known as the escape velocity.**

### *Synchronous orbits.*

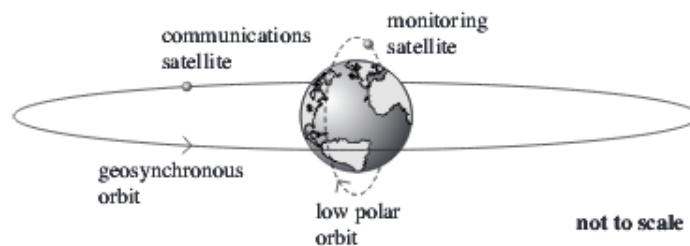
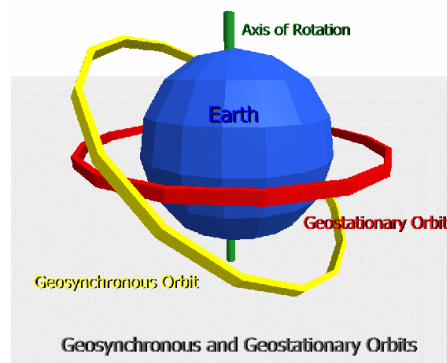
A synchronous orbit is defined as "an orbit in which an orbiting body (usually a satellite) has a period equal to the average rotational period of the body being orbited (usually a planet), and in the same direction of rotation as that body." This type of orbit is also referred to as geosynchronous. This type of orbit can have any inclination and eccentricity, that is, it can be tilted with respect to the poles of the earth and it may not be a circular orbit.

### *Use of satellites in low orbits and geostationary orbits, to include plane and radius of geostationary orbit.*

A synchronous orbit is defined in the previous topic. There is however, a special type of synchronous orbit, called a geostationary orbit. A geostationary orbit is a geosynchronous orbit that occurs around the equator of the Earth. It orbits at 35,786 km above the surface of the Earth, thus the radius of its orbit is roughly  $4.2 \times 10^7$  m (this is 35786 km added to the radius of the Earth: 6400 km). This special case of geosynchronous orbit is unique as an observer from Earth would appear to see a satellite in geostationary orbit in the same position, all day, every day. It can be said that every geostationary orbit is also geosynchronous, but not the other way around. The diagram below illustrates the special case of geosynchronous orbit, and also shows a different type of geosynchronous orbit.

Low polar orbits are also shown below; these orbits are not geosynchronous as they do not have the same orbital time period as the Earth. Since their radius is smaller, and  $T^2 \propto R^3$ , then the time period must also be much shorter, so they are useful as satellites to monitor

changing conditions on Earth, whereas the geostationary orbit is more useful for communication.



Geosynchronous orbit above Equator (Geostationary):

- Its orbital period matches the Earth's rotational period exactly (roughly 24 hours).
- A satellite in geosynchronous orbit will maintain the same position relative to the Earth.
- There is only one type of orbital radius possible.
- A satellite will travel west to east above the Equator (the same direction as the Earth's rotation).
- The orbital height is higher than a polar orbit satellite.
- The speed is much less than a satellite in polar orbit.
- It will scan a restricted, and fixed area of the Earth's surface only, due to its orbit.
- Its applications are usually in telecommunications, so cable and satellite TV, radio and other digital information.
- The satellite is in continuous contact with the receiving or transmitting aerial as its position is maintained relative to the Earth, thus aerials can be in fixed positions.
- You do need a higher signal strength than that of a polar satellite as its height above the surface of the earth is much greater.

Low Polar Orbit

- It usually has an orbital period of only a few hours.
- The Earth rotates relative to the orbit of a satellite in low polar orbit,
- You are able to get many orbits with different radii and periods.
- Much lower orbital height than geosynchronous.

- The speed of satellites in low polar orbit is much greater than that of one in a synchronous orbit.
- Its main applications are in surveillance on Earth i.e. mapping, weather observations or environmental monitoring.
- It enables access to every point on the Earth's surface every day.
- You can collect data from places inaccessible to humans.
- The contact with the transmitting or receiving aerial is intermittent as the Earth moves relative to the orbit of a satellite in low polar orbit.
- An aerial would require a tracking facility.
- You would require a lower signal strength than a synchronous satellite.

## AQA A Level Specimen Q4.3

### Question:

A satellite is launched into a geostationary orbit

Describe and explain two features of a geostationary orbit.

### Answer:

- In the plane of the equator
- Always above the same location on the earth
- Having the same period as the earth/24 hours



## 3.7.3 Electric Fields

### 3.7.3.1 Coulomb's law

#### Content

- Force between point charges in a vacuum:  $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$
- Permittivity of free space,  $\epsilon_0$ .
- Appreciation that air can be treated as a vacuum when calculating force between charges.
- For a charged sphere, charge may be considered to be at the centre.
- Comparison of magnitude of gravitational and electrostatic forces between subatomic particles.

*Force between point charges in a vacuum:*  $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$

The force between point charges follows similar rules as the force between two masses, where the force is proportional to the product of the two charges, and inversely proportional to the square of their distance apart.

#### *Permittivity of free space, $\epsilon_0$ .*

The permittivity of free space is given by the symbol  $\epsilon_0$ , and has the value of  $8.85 \times 10^{-12}$ . It is basically the permittivity of a vacuum, and is given to you in the equation sheet. **Another way of looking at it is that it is the resistance to formation of an electric field.**

#### *Appreciation that air can be treated as a vacuum when calculating force between charges.*

Since the force between point charges is for a vacuum, in questions we can assume that air is a vacuum.

#### *For a charged sphere, charge may be considered to be at the centre.*

#### *Comparison of magnitude of gravitational and electrostatic forces between subatomic particles.*

The electrostatic force between subatomic particles is far greater than the gravitational attraction between them. For example, if you hold two electrons 1m apart from each other, the gravitational attraction between them will be  $F = G \times (9.11 \times 10^{-31})^2 / 1^2$ , which equals around  $5.5 \times 10^{-71}$  N. The electrostatic force of repulsion will be equal to  $1/4\pi\epsilon_0 \times (-1.6 \times 10^{-19})^2 / 1^2$ , which equals around  $2.3 \times 10^{-28}$  N. Therefore, the electrostatic force divided by the gravitational force equals  $4 \times 10^{42}$ . This means the electrostatic force is  $4 \times 10^{42}$  times bigger, so the gravitational force is pretty much negligible when dealing with subatomic particles.



### 3.7.3.2 Electric Field strength

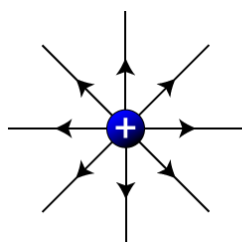
#### Content

- Representation of electric fields by electric field lines.
- Electric field strength.
- $E$  as force per unit charge defined by  $E = F/Q$
- Magnitude of  $E$  in a uniform field given by  $E = V/d$
- Derivation from work done moving charge between plates:  $Fd = Q\Delta V$ .
- Trajectory of moving charged particle entering a uniform electric field initially at right angles.
- Magnitude of  $E$  in a radial field given by  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

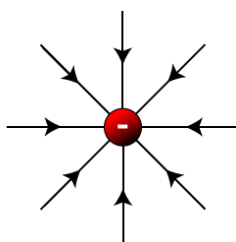
#### *Representation of electric fields by electric field lines.*

Since charges can be positive and negative, you will get different electric field lines depending on this factor.

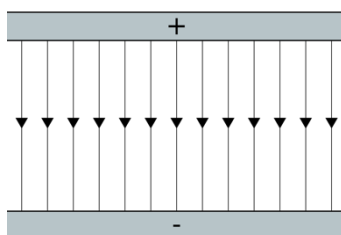
For a positive charge its electric field lines will look as follows:



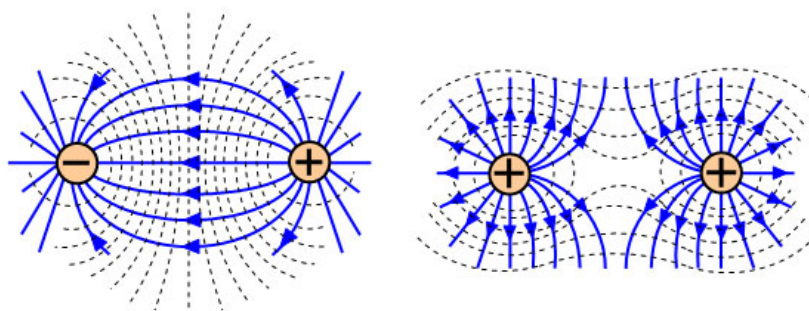
For a negative charge its electric field lines will look as follows:



These fields are radial fields, just like the radial field you will get in gravitational fields. It is also possible to get a uniform field. In a uniform field i.e. between two plates, the field will look as follows.



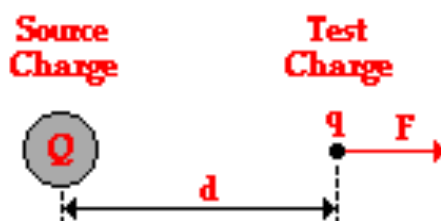
You will also see different electric field line when two charges interact with each other.



The electric field between a positive and negative charge is shown on the left, and the electric field between two positive charges on the right. If the diagram on the right were to be two positive charges, it would be exactly the same except the arrows would point towards the charge.

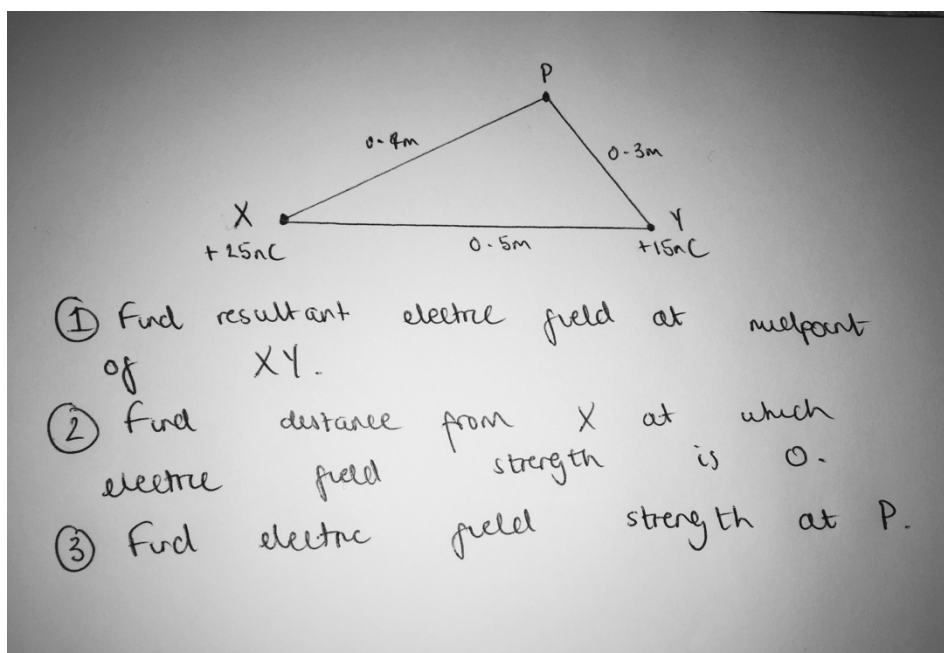
### *Electric field strength. $E$ as force per unit charge defined by $E = F/Q$*

Electric field strength is the force per unit charge acting on a positive test charge placed at that point. As an equation it is:  $E = F/Q$ , and has the unit  $\text{NC}^{-1}$  or  $\text{Vm}^{-1}$ . Electric field strength is a vector quantity, so if two charges are interacting, you may be asked to find the resultant electric field. By convention, the electric field points in the direction that a positive charge placed at that point in the field would feel a force.



The electric field strength at a distance  $d$  from the source charge  $Q$ , would be  $E = F/q$ . Since it is the force per unit charge acting on a positive test charge placed at that point, the charge used is therefore the charge on the test charge ( $q$ ), not the source charge ( $Q$ ).

The electric field strength is not actually dependent upon the quantity of the charge on the test charge. The electric field strength at any given location around the source charge  $Q$  will be measured to be the same regardless of what test charge is used. This is because if you increase the charge on the test charge by a factor of say, 2, then according to Coulomb's Law, this increase in charge will be accompanied by an increase in the force. The increase in force would be by the same factor, so the denominator and numerator in  $E = F/Q$  would always increase by the same factor, thus electric field strength at any given point would be constant regardless of the charges.



- To find the resultant electric field from X and Y, we must first find the magnitude of the electric field strength from each of them. Electric field strength,  $E$ , can be calculated by using  $E = Q/4\pi\epsilon_0 r^2$ . The picture below shows how to approach this question. You must first work out the electric field from X ( $E_X$ ) and then from Y ( $E_Y$ ). Since electric field strength is a vector, and these two fields are working in opposite directions, the magnitude of the resultant is  $3600 - 2200 = 1400 \text{ NC}^{-1}$ . Therefore, this is the electric field strength at the mid-point of XY.

$$E_X = \frac{25 \times 10^{-9}}{4\pi\epsilon_0 (0.25)^2} \approx 3600 \text{ NC}^{-1}$$

$$E_Y = \frac{15 \times 10^{-9}}{4\pi\epsilon_0 (0.25)^2} \approx 2200 \text{ NC}^{-1}$$

$$\begin{array}{c} 3600 \quad \leftarrow \quad 2200 \\ \hline 3600 - 2200 = 1400 \text{ NC}^{-1} \end{array}$$

- We know that at a distance we can call  $d$ , the electric field strength is 0. Since the remaining distance must be  $(0.5 - d)$ , so we can place this onto the diagram.

put where electric field strength = 0.

$$E_X = \frac{25 \times 10^{-9}}{4\pi\epsilon_0 d^2}$$

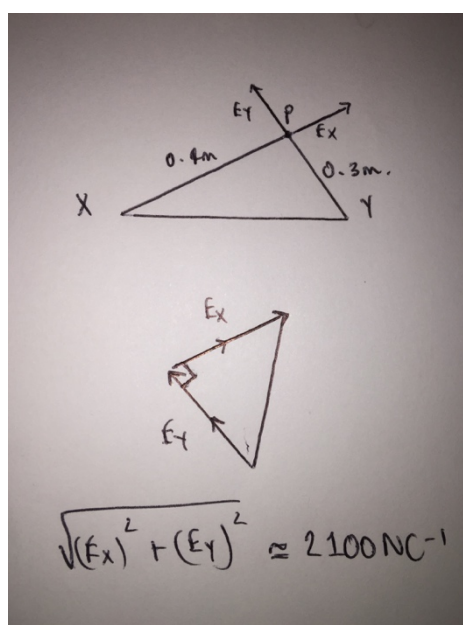
$$E_Y = \frac{15 \times 10^{-9}}{4\pi\epsilon_0 (0.5-d)^2}$$

$$\text{so } \frac{25 \times 10^{-9}}{4\pi\epsilon_0 d^2} = \frac{15 \times 10^{-9}}{4\pi\epsilon_0 (0.5-d)^2} = \frac{25}{d^2} = \frac{15}{(0.5-d)^2}$$

solve for  $d = 0.28 \text{ m}$

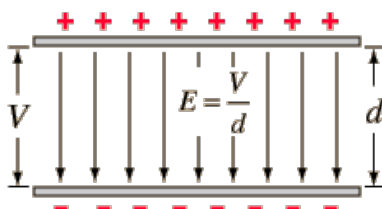
Then if we make the electric field strength at that distance from X equal to the electric field strength at that distance from Y, and solve to find  $d$ , we get 0.28m from X.

- First we must calculate the electric field at P due to X and then due to Y. These electric fields are not acting in the same direction (their directions are shown on the diagram), and since they are vectors we can use the head to tail method for vectors to find the resultant electric field. Also to link back to the definition of an electric field, the direction of these vectors is the direction that a positive test charge placed at P would feel a force. Using this method, we construct a right angled triangle and can use Pythagoras to solve for the resultant electric field.



### *Magnitude of $E$ in a uniform field given by $E = V/d$ .*

In a uniform field, the electric field strength is given by the potential difference between two plates, divided by their distance apart. So if the potential difference were 10V and the distance were 1m, the electric field strength would be  $1 \text{ Vm}^{-1}/\text{NC}^{-1}$ .



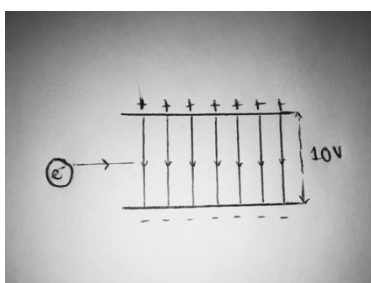
### *Derivation from work done moving charge between plates: $Fd = Q\Delta V$ .*

Since  $E = F/Q$ , then  $F/Q = \Delta V/d$ , therefore  $Fd = Q\Delta V$ . The force multiplied by the distance is equal to the work done, so the charge multiplied by the change in potential difference is equal to the work done.

### *Trajectory of moving charged particle entering a uniform electric field initially at right angles.*

If a positively charged particle enters a uniform electric field initially travelling perpendicular to the field, then it will accelerate uniformly in the direction of the field and move in a parabolic fashion. For a negatively charged particle i.e. an electron, it would move in the opposite direction to the electric field as it is repelled by the negative plate.

In the question below we have an electron entering a uniform field between two plates 1m apart, travelling perpendicular to the direction of the field. The question is to find the vertical distance moved by the electron in  $0.5\mu\text{s}$ , and which direction the electron moves in.



The electron is clearly going to move towards the positive charge, in a parabolic direction upwards. To find the vertical distance moved by the electron we need to find what its acceleration is. For acceleration we can use  $F = ma$ , and also  $F = EQ$ . First of all we need to know the electric field strength within the uniform field, which is given by  $E = V/d$ , so equal to  $10/1 = 10\text{Vm}^{-1}$ . Now we have the electric field strength we can find the force acting on the electron, equal to  $F = EQ$ , so  $10 \times 1.6 \times 10^{-19}$ . This value of  $F$  can be used now to find the acceleration using  $F = ma$ . So this value of  $F$  calculated previously ( $1.6 \times 10^{-18}$ ), can be divided by the mass of the electron ( $9.11 \times 10^{-31}$ ) to give the acceleration of the electron. This acceleration is equal to  $1.75 \times 10^{12} \text{ms}^{-2}$ .

Now we have the acceleration of the electron, we can use the suvat equations to find the vertical distance moved. The information that we have is  $u = 0$  as the electron is initially moving perpendicular to the direction of the field with no vertical velocity. We also now know  $a = 1.75 \times 10^{12} \text{ms}^{-2}$ , and we want to find the distance travelled in  $0.5\mu\text{s}$  ( $0.5 \times 10^{-6}\text{s}$ ). Using  $s = ut + at^2/2$  we can obtain a value of  $s$  equal to  $0.22\text{m}$ .

If asked to find the horizontal distance travelled by the electron we would need to know its initial velocity, and assuming that the acceleration horizontally is 0, it would be possible to obtain a value for the horizontal distance travelled in a given time.

### *Magnitude of E in a radial field given by $E = 1/4\pi\epsilon_0 \times Q/r^2$ .*

The equation above can also be written as  $E = Q/4\pi\epsilon_0 r^2$ . This is the value of  $E$  in a radial field, and can be derived from the fact that  $E = F/q$ , so using Coulomb's Law that  $F =$

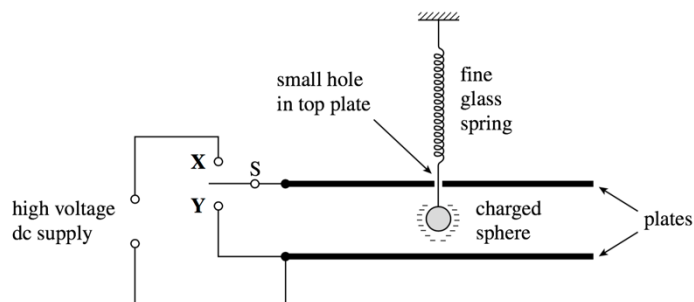
$qQ/4\pi\epsilon_0r^2$ . We can substitute this equation in for  $F$ , and when you divide it by  $q$ , you get  $E = Q/4\pi\epsilon_0r^2$ .

## AQA June 2010 Unit 4 Section B Q2bi

### Question:

A small negatively charged sphere is suspended from a fine glass spring between parallel horizontal metal plates, as shown in **Figure 1**.

**Figure 1**



‘Switch S is now moved to position Y (It has been moved to X in a previous question)’

State and explain the effect of this on the electric field between the plates.’

### Answer:

- Electric field becomes zero (or ceases to exist)
- There is a flow of charge (or electrons) from one plate to the other [or it can be said that the plates discharge]
- (until) pd across plates becomes zero [or no pd across plates or the plates are at the same potential]

### 3.7.3.3 Electric potential

#### Content

- Understanding of definition of absolute electric potential, including zero value at infinity, and of electric potential difference.
- Work done in moving charge  $Q$  given by  $\Delta W = Q\Delta V$ .
- Equipotential surfaces.
- No work done moving charge along an equipotential surface.
- Magnitude of  $V$  in a radial field given by  $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$
- Graphical representations of variations of  $E$  with  $V$  and  $r$ .
- $V$  related to  $E$  by  $E = \Delta V/\Delta r$ .
- $\Delta V$  from area under graph of  $E$  against  $r$ .

#### *Understanding of definition of absolute electric potential, including zero value at infinity, and of electric potential difference.*

Absolute electric potential means that it is not relative, and means the amount of electric potential that a point has is the same irrespective of charge. Electric potential is the work done per unit positive charge on moving a small positive test charge from infinity to that point. **Electric potential is a scalar quantity, so the electric potential at one point due to two different charges will add up.** Thus the electric potential at an equidistant point of 40mm from two different charges will be the sum of their electric potentials.

Because of the existence of repulsion, it is possible to obtain positive and negative potential energy values. If you were to bring a positive test charge from infinity towards another positive source charge, there would be a repulsion between the charges, so we must do work to move the test charge towards the source charge. This work is stored as electric potential energy of the test charge. This concept would also apply to two negative charges. Although in both cases the potential energy of the test charge increases (from 0) as it approaches the source charge. Therefore the electric potential of the test charge is positive. However if you had a test charge and source charge of opposite signs, they will attract each other. This means it now takes work to separate the two charges, which is stored as electric potential energy in the test charge. This energy on the test charge will increase towards 0 as their separation increases, so the test charge must have a negative potential energy.

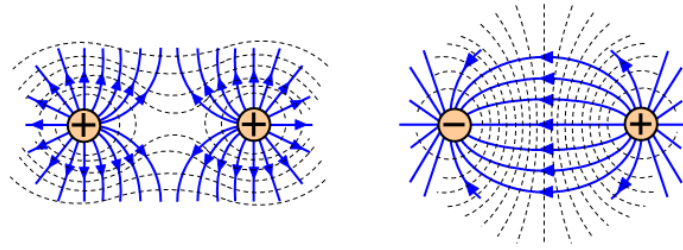
#### *Work done in moving charge $Q$ given by $\Delta W = Q\Delta V$ .*

The work done in moving a charge  $Q$ , is given by multiplying the charge by the change in potential over a given distance. So if a 10C charge is moved across a potential of 10V, the work done would be 100J.

#### *Equipotential surfaces.*

Equipotential surfaces are surfaces whereby the electric potential does not change. These equipotential surfaces are similar to the equipotential surfaces of the gravitational field. The diagram below shows different equipotential lines around like charges and unlike charges.





Since electric potential is a scalar quantity, the potential at a given point will add up. For unlike charges shown, the electric potential at the middle will cancel out, hence the vertical equipotential line separating the two charges.

*No work done moving charge along an equipotential surface.*

In the equation for work done  $\Delta W = Q\Delta V$ , the  $\Delta V$  is significant as this is the change in potential. When moving over an equipotential surface, your electric potential will not change, so  $\Delta V = 0$ , thus  $\Delta W = 0$ .

*Magnitude of  $V$  in a radial field given by*  $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

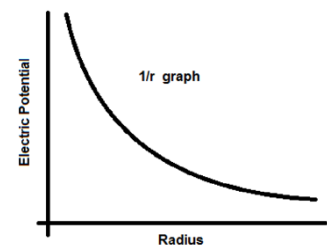
Calculating  $V$  in a radial field is done by using the equation above.

*Graphical representations of variations of  $E$  with  $V$  and  $r$ .*

Since electric field strength is proportional to  $1/r^2$ , and electric potential is proportional to  $1/r$ , the graphs will look similar but have a slight difference.

The graph of electric field strength against  $r$  would be different in that it would decrease at a faster rate since electric field strength would decrease by  $1/r^2$  as opposed to  $1/r$ .

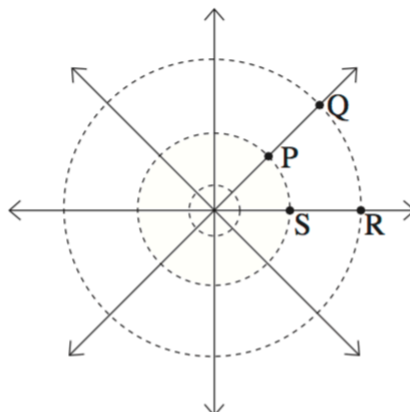
*$V$  related to  $E$  by  $E = \Delta V/\Delta r$ .*



The electric field strength at a point is equal to the change in potential divided by the change in distance between the charges. This is proven by the fact that electric potential equals  $Q/4\pi\epsilon_0 r$ , so if you divided by  $r$  you will get  $Q/4\pi\epsilon_0 r^2$ , which is equal to the electric field strength. Thus if you have an electric field strength you can multiply it by the change in distance to get the electric potential.

## AQA June 2014 Unit 4 Section A Q16

- 16 The diagram below shows the field lines and equipotential lines around an isolated positive point charge.



Which one of the following statements concerning the work done when a small charge is moved in the field is **incorrect**?

- A When it is moved from either P to Q or S to R, the work done is the same in each case.
- B When it is moved from Q to R no work is done.
- C When it is moved around the path PQRS, the overall work done is zero.
- D When it is moved around the path PQRS, the overall work done is equal to twice the work done in moving from P to Q.

A = correct as they are equipotentials so  $W=QV$

B = Equipotential so change in  $V$  is 0 so true

C = When a charge is moved completely around a closed path in an electric field the net work done is zero, so correct

D = Must be incorrect according to previous statement



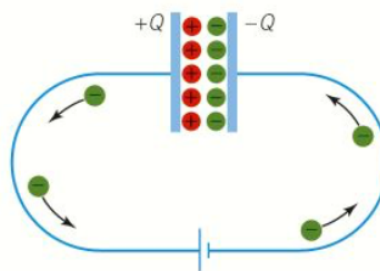
## 3.7.4 Capacitance

### Content

- Definition of capacitance:  $C = Q/V$

### *Definition of capacitance: $C = Q/V$*

Capacitors are designed to store charge, and you can make a capacitor by placing two nearby metal plates parallel to each other. One plate is positive and one negative, with an equal and opposite charge. The equation to calculate capacitance is  $C = Q/V$ , where  $C$  is capacitance units Farads (F),  $Q$  is the charge in Coulombs (C) and  $V$  is potential difference between the plates (V).



The two metal plates are insulated from each other, and in the capacitor electrons move from the positive terminal to the negative terminal. This explains the loss of charge on the positive end being equal to the gain in charge at the negative terminal.

### 3.7.4.3 Parallel Plate Capacitor

#### Content

- Dielectric action in a capacitor  $C = A\epsilon_0\epsilon_r/d$ .
- Relative permittivity and dielectric constant
- Students should be able to describe the action of a simple polar molecule that rotates in the presence of an electric field.

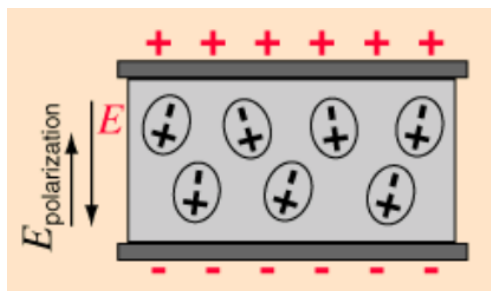
#### Opportunities for Skills Development

- Determine the relative permittivity of a dielectric using a parallel-plate capacitor.
- Investigate the relationship between  $C$  and the dimensions of a parallel-plate capacitor eg using a capacitance meter.

#### *Dielectric action in a capacitor $C = A\epsilon_0\epsilon_r/d$ .*

To store more charge on the plates of a capacitor, an electrically insulating dielectric can be put in between them. The presence of a dielectric will reduce the effective electric field as when placed between two metal plates, it will produce an electric field in the opposing direction to the field of the charges on the plate. The equation for calculating capacitance in a dielectric is  $C = A\epsilon_0\epsilon_r/d$ , where  $C$  is capacitance (F),  $A$  is **cross sectional area** ( $\text{m}^2$ ),  $\epsilon_0$  is permittivity of free space and  $\epsilon_r$  is relative permittivity, and  $d$  is the distance between parallel plates.

A material may contain polar molecules, and if so, they are usually arranged randomly in the absence of an electric field. Applying an electric field will polarise the molecules in material as shown below.



This means that one surface will gain positive charge and one will gain negative charge, i.e. the side of the dielectric that faces the positive plate will gain more negative charge. In dielectric materials that are already polarised, but face in opposite directions, when placed in a parallel plate capacitor the molecules will be attracted towards the positive plate due to the negatively charged electrons.

The resultant effect of the dielectric is that more charge can be stored, where the surface of the dielectric by the positive plate will gain negative charge, and negative plate positive charge. The positive end on the dielectric will draw more electrons onto the negative plate, then the negative side will repel electrons back to the battery from the positive plate.

Since  $Q/V = C$ , then if  $Q$  increases  $C$  will also increase, so in conclusion, the effect of a dielectric in a circuit is to store more charge in a capacitor for a given potential difference, so increase the capacitance.

### *Relative permittivity and dielectric constant*

**Relative permittivity** is the ratio of charge stored with a dielectric to that without a dielectric, where  $\epsilon_r = Q/Q_0$ , also equal to  $C/C_0$ ,  $\epsilon/\epsilon_0$  or even  $I/I_0$ . It has no units because ultimately it is a **ratio**. The relative permittivity is also referred to as the **dielectric constant**, and the values of dielectric constants range from values like 2.3, to 81 for water.

If the capacitance of a capacitor before adding a dielectric of dielectric constant 4.2 were  $C$ , then the new capacitance would be  $4.2C$ .

### *Students should be able to describe the action of a simple polar molecule that rotates in the presence of an electric field*

In alternating electric fields, polar dipoles will rotate as the electric field constantly changes direction. Non-polar dipoles will oscillate in one direction and then the opposite direction.

### 3.7.4.3 Energy stored by a capacitor

#### Content

- Interpretation of the area under a graph of charge against pd.
- $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}Q/C^2$

*Interpretation of the area under a graph of charge against pd.  $E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}Q/C^2$*

Energy is stored as electric potential energy in a capacitor, and the amount of energy stored in a capacitor can be calculate from the area under a graph of charge against potential difference. Since work done is equal to the voltage multiplied by charge ( $W=QV$ ), then for a capacitor energy stored, E will be equal to  $\frac{1}{2} QV$  as from a graph of Q against V (straight line), this would be the area underneath.

The reason it is  $\frac{1}{2} QV$  (as opposed to  $QV$ ) is because as the capacitors discharge, the charge decreases so the potential difference across the capacitor decreases. So charges will be transferred over smaller and smaller voltages, and if you were to add up the total drop in charge, it turns out to be equal to  $\frac{1}{2} QV$ . On average, the overall charge is only dropped through half the initial voltage.

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}Q/C^2$$

The equations above all give the energy stored in a capacitor.





## AQA Jan 2010 Section B Unit 4 Q2bi)ii)

In order to produce a time delay, an intruder alarm contains a capacitor identical to the capacitor used in the experiment in part (a). This capacitor is charged from a 12 V (as opposed to 6V in the previous question) supply and then discharges through a 100 k $\Omega$  resistor, similar to the one used in the experiment.

**Question:**

State and explain the effect of this higher initial pd on the energy stored by this capacitor initially

**Answer:**

Initial energy stored is 4 greater

Because energy  $\propto V$  (and V is doubled) OR  $12^2/6^2 = 4$  (capacitance has not changed and  $E = CV^2/2$ )

**Question:**

State and explain the effect of this higher initial pd on the time taken for this capacitor to lose 90% of its original energy

**Answer:**

Time to lose 90% of energy is unchanged because time constant is unchanged (or depends only on R and C)

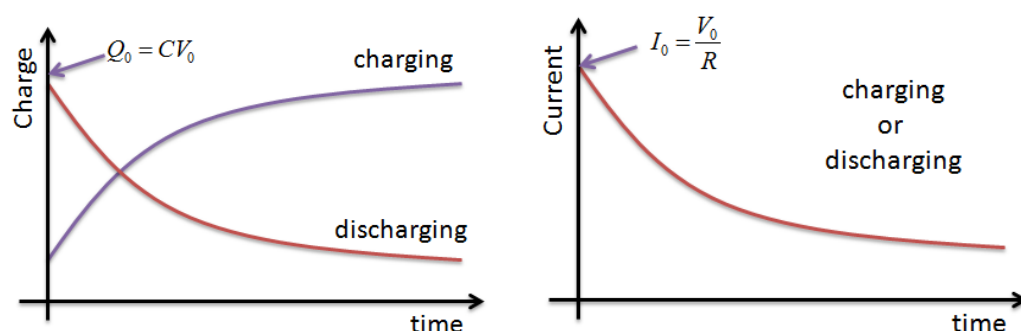
### 3.7.4.4 Capacitor charge and discharge

#### Content

- Graphical representation of charging and discharging of capacitors through resistors. Corresponding graphs for  $Q$ ,  $V$  and  $I$  against time for charging and discharging.
- Interpretation of gradients and areas under graphs where appropriate.
- Time constant  $RC$
- Calculation of time constants including their determination from graphical data.
- Time to halve,  $T_{1/2} = 0.69RC$
- Quantitative treatment of capacitor discharging,  $Q = Q_0 e^{-t/RC}$
- Use of the corresponding equations for  $V$  and  $I$
- Quantitative treatment of capacitor charge,  $Q = Q_0(1 - e^{-t/RC})$

#### *Graphical representation of charging and discharging of capacitors through resistors. Corresponding graphs for $Q$ , $V$ and $I$ against time for charging and discharging.*

The graph for charge against time for charging and discharging is shown below on the left, the two lines would be the same if the graph were voltage against time. To the right we have current against time, which is the same for charging or discharging.



#### Discharging a capacitor through a fixed resistor

In this process current will decrease quicker at first, and then more gradually to zero. The current decrease like this because the potential difference decreases across the capacitor as it loses charge, and  $I = V/R$  where  $R$  is constant. Moreover, this graph shows that the current will decrease exponentially with time, and thus according to the equation  $I = I_0 e^{-t/RC}$ . This means that after a time  $t = RC$  ( $RC$  is covered under the next heading),  $I = 0.37I_0$ , so the current decreases by approximately 63% of its original value. The same principle applies in the cases of both charge and potential difference.

#### Charging a capacitor through a fixed resistor

The graph showing the charging of a capacitor for current is identical to the graph showing discharging. The current in the circuit will begin high as there is a larger rate of flow of charge, then this rate of flow will decrease progressively throughout time. However according to the graph charge increases exponentially with time, and so too does the potential difference.

### *Interpretation of gradients and areas under graphs where appropriate*

The gradient of charge-time is current, because  $I = \Delta Q / \Delta t$ .

The area under a current-time graph is equal to the amount of charge stored on the plates of the capacitor as we know the equation  $Q = It$ .

We also know that the area under a voltage-time graph is equal to the energy stored in the capacitor, so  $E = 1/2 QV$ . We know this because  $W = E = QV$ . **This means that half of the energy is wasted as a result of resistance to the flow of charge in the circuit. This energy is then transferred to the surroundings as thermal energy for example.**

Other relationships can be formed between equations, and the gradients/area under of graphs can be determined using the equations governing capacitance. For example, the area under a graph of capacitance against voltage squared, will also yield the energy stored in the capacitor, equal to  $1/2 CV^2$ . However, the listed relationships above are the main ones.

### *Time constant RC*

The time constant is equal to the resistance multiplied by the capacitance, and the units are in seconds, which can be derived from the units of resistance and capacitance. If the time constant,  $RC$ , was 6 seconds in a circuit, then after 6 seconds the charge will have fallen to 63% its original value. Because  $Q = Qe^{-1}$ .

### *Calculation of time constants including their determination from graphical data.*

Take for instance a graph of charge-time for the discharging of a capacitor. Say the maximum value was  $1C$  and found the time that it took for the charge to fall from  $1C$  to  $0.37C$ , this time that you found would be equal to the time constant of the circuit. This is because according to  $Q = Q_0 e^{-t/RC}$ ,  $Q = 1e^{-1} \approx 0.37C$ . From knowing this value of the time constant, for example if it were 6s, you could calculate the resistance/capacitance of the circuit. This is because you know that  $6 = RC = \text{time constant}$ , so  $6/C = R$  and  $6/R = C$ .

### *Time to halve, $T_{1/2} = 0.69RC$*

If you were to substitute  $t \approx 0.69RC$  into  $Q = Q_0 e^{-t/RC}$ , you would get  $Q = Q_0 e^{-0.69}$  because  $0.69RC/RC = 0.69$ . And the result of this is  $Q \approx 0.5Q_0$ . This means that after a time equal to  $0.69RC$ , the charge falls to approximately half of its original value.

### *Quantitative treatment of capacitor discharging, $Q = Q_0 e^{-t/RC}$*

The discharging of a capacitor is modelled by the equation  $Q = Q_0 e^{-t/RC}$ . This equation also explains the exponential decrease on a charge – time and a voltage – time graph for the discharging of a capacitor.

Below shows the derivation of the above formula. The assumption we make is that the rate of decay of charge is proportional to the charge remaining. You may or may not be familiar with the integration technique used.

$$\frac{dQ}{dt} \propto Q$$

$$\frac{dQ}{dt} = -kQ \quad \text{where } k = \text{constant}$$

$$\int_{Q_0}^Q \frac{1}{Q} dQ = \int_0^t -k dt$$

$$\ln \frac{Q}{Q_0} = -kt - 0$$

$$e^{-kt} = \frac{Q}{Q_0}$$

$$\text{so } Q = Q_0 e^{-kt}$$

$$\text{where } k = + \frac{1}{RC}$$

*Use of the corresponding equations for  $V$  and  $I$*

*Quantitative treatment of capacitor charge,  $Q = Q_0(1 - e^{-t/RC})$*

The capacitor charges exponentially too, and is modelled by  $Q = Q_0(1 - e^{-t/RC})$ . This same equation describes the way that the potential difference changes as a capacitor charges, by using the substitutions  $Q = V$  and  $Q_0 = V_0$ , so  $V = V_0(1 - e^{-t/RC})$ .

## AQA June 2004

- 3** A capacitor of capacitance  $330\text{ }\mu\text{F}$  is charged to a potential difference of  $9.0\text{ V}$ . It is then discharged through a resistor of resistance  $470\text{ k}\Omega$ .
- (a)** Calculate:
- (i) the energy stored by the capacitor when it is fully charged, (2 marks)
  - (ii) the time constant of the discharging circuit, (1 mark)
  - (iii) the pd across the capacitor  $60\text{ s}$  after the discharge has begun. (3 marks)
- (b)** The capacitor is charged using a  $9.0\text{ V}$  battery and negligible internal resistance in series with a  $1.0\text{ k}\Omega$  resistor. Calculate:
- (i) the time constant for this circuit, and sketch graphs to show how the capacitor pd and the current changed with time during charging, (5 marks)
  - (ii) the energy supplied by the battery and the energy supplied to the capacitor during the charging process, and explain the difference. (3 marks)
- AQA, 2004

3 (a) (i)	<p>Charge stored <math>Q = CV</math>  <math>= 330 \times 10^{-6} \times 9.0 = 2.97 \times 10^{-3} \text{ C}</math></p> <p>Energy stored <math>E = \frac{1}{2} QV</math>  <math>= \frac{1}{2} \times 2.97 \times 10^{-3} \times 9.0 = 1.34 \times 10^{-2} \text{ J}</math></p>	1 1	<p><i>Alternatively:</i> energy stored  <math>E = \frac{1}{2} CV^2 = \frac{1}{2} \times 330 \times 10^{-6} \times 9.0^2</math>  <math>= 1.34 \times 10^{-2} \text{ J}</math></p>
3 (a) (ii)	<p>Time constant <math>= RC</math>  <math>= 470 \times 10^3 \times 330 \times 10^{-6} = 155 \text{ s}</math></p>	1	A large value for the time constant indicates that the discharge will be slow.
3 (a) (iii)	<p>When <math>t = 60 \text{ s}</math>, <math>Q = Q_0 e^{-t/RC}</math> gives  <math>Q = 2.97 \times 10^{-3} e^{-60/155}</math>  <math>= 2.02 \times 10^{-3} \text{ C}</math></p> <p>pd across capacitor <math>V = \frac{Q}{C} = \frac{2.02 \times 10^{-3}}{330 \times 10^{-6}}</math>  <math>= 6.11 \text{ V}</math></p>	1 1 1	<p>Part (c) is a good test of your ability to use a calculator to find an exponential quantity.</p> <p><i>Alternative approach:</i>          using the equation established in Question 2 (c)(ii) above,  <math>V = V_0 e^{-t/RC}</math> gives  <math>V = 9.0 e^{-60/155} = 6.11 \text{ V}</math></p>
3 (b) (i)	<p>time constant <math>= RC = 1.0 \times 10^3 \times 330 \times 10^{-6}</math>  <math>= 0.33 \text{ s}</math></p> <p>Graphs to show:</p> <ul style="list-style-type: none"> <li>• correct axes and labels on both</li> <li>• exponential growth for pd</li> <li>• exponential decay for current</li> <li>• value dropped to 0.37 of maximum after RC for current, and increased to 0.63 of maximum after RC for pd.</li> </ul>	1 1 1 1	
3 (b) (ii)	<p>Energy supplied to capacitor <math>= \frac{1}{2} QV</math>  <math>= 0.5 \times 330 \times 10^{-6} \times 9.0 = 1.485 \times 10^{-3} \text{ J}</math></p> <p>Energy supplied by the battery <math>= QV</math>  <math>= 330 \times 10^{-6} \times 9.0 = 2.97 \times 10^{-3} \text{ J}</math></p> <p>Half the energy from the battery is lost to the resistance in the circuit/dissipated in the resistor.</p>	1 1 1	



## 3.7.5 Magnetic Fields

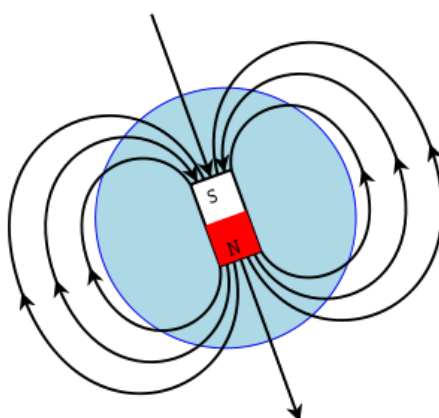
### 3.7.5.1 Magnetic flux density

#### Content

- Force on a current-carrying wire in a magnetic field:  $F = BIl$  when field is perpendicular to current.
- Fleming's left hand rule.
- Magnetic flux density  $B$  and definition of the tesla

***Force on a current-carrying wire in a magnetic field:  $F = BIl$  when field is perpendicular to current.***

A magnetic field is a force field that surround a magnet or a wire that has a current flowing through it. Like gravitational and electric fields, magnetic fields also have associated field lines on their north and south poles. The field lines are shown below.



The magnetic fields are strongest at the north and south poles. The line in which a free north pole would move in a field is referred to as the line of force of the magnetic field.

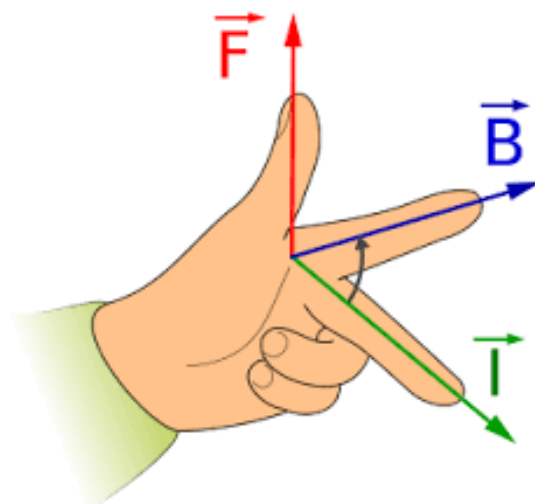
This magnetic field causes a force to be exerted on a current-carrying wire, but only when the field is perpendicular to the current. This force  $F = BIl$  where  $B$  is the magnetic flux density (T),  $I$  is the current (A) and  $l$  (m) is the length of the wire in contact with the magnetic field. Furthermore, a current-carrying wire placed in a magnetic field will experience a force that acts perpendicular to the wire and to the lines of force of said magnetic field. However, a force will only be experienced if the wire is placed at a non-zero angle. This force will be highest when the wire is at right angles to the magnetic field, and zero when the wire is parallel.

Also, another way of looking at the field lines is that the line of force of a magnetic field is a line along which a free north pole would move in the field

#### ***Fleming's left hand rule***

This rule can be used to determine the direction of current, of the magnetic field or of the force in a current-carrying wire. If two of the variables are known the third can be worked out

using this rule. The rule clearly uses your left hand, where your thumb denotes **m**otion, which also gives the direction of the force. The **f**irst finger represents the **f**ield direction and the **s**econd finger represents the direction of the **c**urrent. You can remember it as the m in 'thumb' representing motion, the f in 'first finger' denoting field direction, and 'second finger' for current in the wire.



Fleming's left hand rule is used for **electric motors** to describe something called the motor effect. It is used when dealing with situations involving the forces experienced by current carrying wires passing through a magnetic field.

### *Magnetic flux density $B$ and definition of the tesla*

Magnetic flux density, denoted by the symbol  $B$  can also be looked at as the magnetic field strength. It has the units Tesla (T). The direction of the force it exerts, and the direction of the field is found using Fleming's left hand rule. The SI units of the Tesla are  $\text{Nm}^{-1}\text{A}^{-1}$ .



### 3.7.5.2 Moving charges in a magnetic field

#### Content

- Force on charged particles moving in a magnetic field,  $F = BQv$  when the field is perpendicular to velocity.
- Direction of force on positive and negative charged particles.
- Circular path of particles; application in devices such as the cyclotron

#### *Force on charged particles moving in a magnetic field, $F = BQv$ when the field is perpendicular to velocity.*

In the last topic, the force experienced on a current-carrying wire in a magnetic field was studied, in this topic the force on charged particles moving in a magnetic field is the focus. The force can be calculated using the equation  $F = BQv$ , where  $B$  is the magnetic flux density (magnetic field strength), denoted by  $-$  (T),  $Q$  is the charge (C) and  $v$  is the velocity ( $\text{ms}^{-1}$ ). However, this only stands true when the field is perpendicular to the velocity. An example is an electron beam fired through a vacuum tube, you can observe that in the presence of a magnetic field, the electrons will experience a force and be deflected in a certain direction.

#### *Direction of force on positive and negative charged particles.*

Fleming's left hand rule can be used in this situation as well in order to work out the direction of the force on a moving charge. If the charge in question is an electron, then it is important to remember that the convention for the direction of flow of current is in the opposite direction to the direction of flow of the electrons. Therefore, the direction of force on positive and negative charged particles can be found using Fleming's left hand rule.

Moreover, if you have a charge  $Q$  and  $-Q$  travelling in identical magnetic fields at the same speed, then  $F_Q = BQV$  and  $F_{-Q} = -BQV$ , so the force acts in the opposite direction.

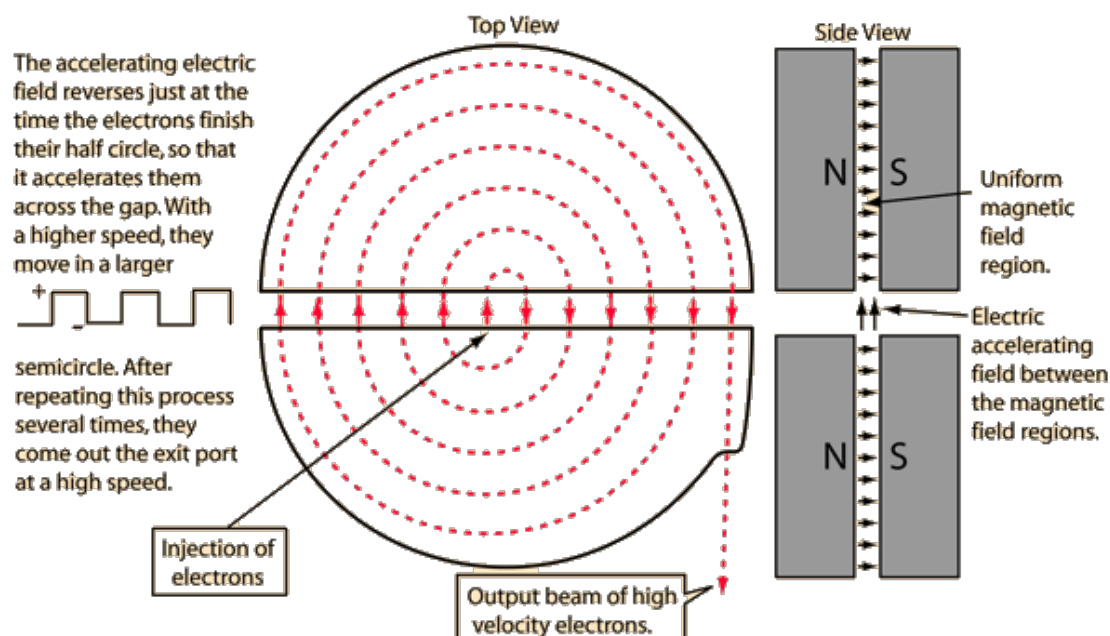
#### *Circular path of particles; application in devices such as the cyclotron*

Magnetic fields can be used to control the path of beams of charged particles, for example in televisions. Since the force on a moving charged particle acts perpendicular to the direction of motion of the particle, a charged particle moving in a magnetic field will follow a circular path. Remembering back to circular motion, for a particle to maintain circular motion, its tangential velocity must act perpendicular to its direction of motion.

The consequences of this are that no work is done on the particle by the field as the force is always acting at right angles to the velocity. To exemplify this,  $W = Fd$  where  $d$  is the distance travelled in the direction of the force, however for a charged particle travelling in a magnetic field there is no movement in the direction of the force, so  $W = 0$ .

The radius of the path of the charged particles can be calculated by equating  $F = BQv$  and  $F = mv^2/r$ . So you are left with  $BQv = mv^2/r$ , which when rearranged for  $r$  gives  $r = mv/BQ$ .

An example of the application of magnetic fields is in a device called the cyclotron. This device produces high-energy beams that can be used in hospitals for treatments like radiation therapy. Its structure is shown below, but essentially the cyclotron acts to accelerate particles.



Charged particles enter the cyclotron at a site called a dee. Across the dees a high-frequency AC is applied, and a magnetic field is applied at right angles to the plane the dee is in. The particles within the cyclotron, usually electrons gain speed each time they pass from one dee to another, and so its radius increases. Once the radius is the same as the radius of the dee, the particle will leave the cyclotron.

Magnetic fields in mass spectrometry are also used in the analysis of the types of atoms present in a sample. The ions pass through velocity selectors that only allow an ion of a given velocity to pass. The velocity selector is comprised of a magnet and a pair of parallel plates. To find the velocity that is allowed to pass you must equate the force on the particle due to the magnetic field and the electric field. Therefore,  $VQ/d = BQv$ , so the accepted velocity is  $v = V/Bd$ . If the velocity is too high or too low, then the ion will either pass above or below the gap. Once the ion passes through the gap into the magnetic field, it will follow a path with a radius  $r = mv/BQ$ . This radius can be measured and so knowing the charge of the particle, the magnetic field strength and velocity of the particle, its mass can be calculated. This mass can then be used to identify the type of atoms present in a sample.

## Jan 2010 Q22 Unit 4 Section A

### Question:

An electron moves due North in a horizontal plane with uniform speed. It enters a uniform magnetic field directed due South in the same plane. Which one of the following statements concerning the motion of the electron in the magnetic field is correct?

- A. It accelerated due West.
- B. It slows down to zero speed and then accelerates due South.
- C. It continues to move North with its original speed.
- D. It is accelerated due North.

### Answer:

If you draw a diagram of what is happening here, you will notice that the electron is moving parallel to the magnetic field.

As we already know, if a charge moves parallel to a magnetic field it will feel no force, so the answer is C.

***MAGNETIC FIELDS JUNE 2012 Q3 onwards***

### 3.7.5.3 Magnetic flux and flux linkage

#### Content

- Magnetic flux defined by  $\Phi = BA$ , where  $B$  is normal to  $A$
- Flux linkage as  $N\Phi$  where  $N$  is the number of turns cutting the flux
- Flux and flux linkage passing through a rectangular coil rotated in a magnetic field
- Flux linkage  $N\Phi = BAN\cos\theta$

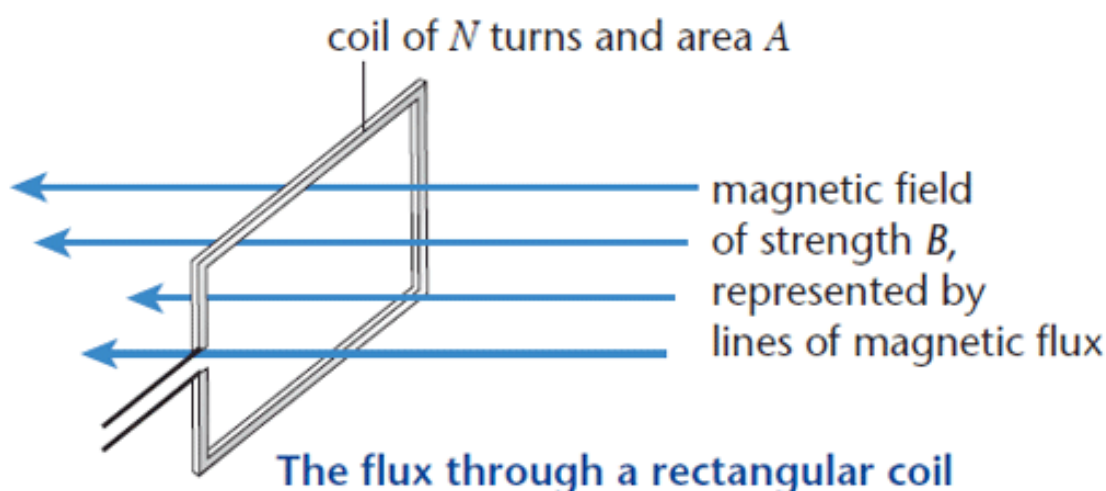
*Magnetic flux is defined by  $\Phi = BA$ , where  $B$  is normal to  $A$*

Magnetic flux is defined as the total magnetic field which passes through a given area. It is calculated by using the equation  $\Phi = BA$ , where  $\Phi$  is magnetic flux with the units, Weber, denoted by the symbol - Wb.  $B$  is the magnetic field strength (T), and  $A$  is the area penetrated by the magnetic field ( $m^2$ ), which must be perpendicular to the magnetic field.

*Flux linkage as  $N\Phi$  where  $N$  is the number of turns cutting the flux*

Flux linkage is denoted by  $N\Phi$ , where  $N$  is the number of turns cutting the flux (cutting the magnetic field).

*Flux and flux linkage passing through a rectangular coil rotated in a magnetic field*



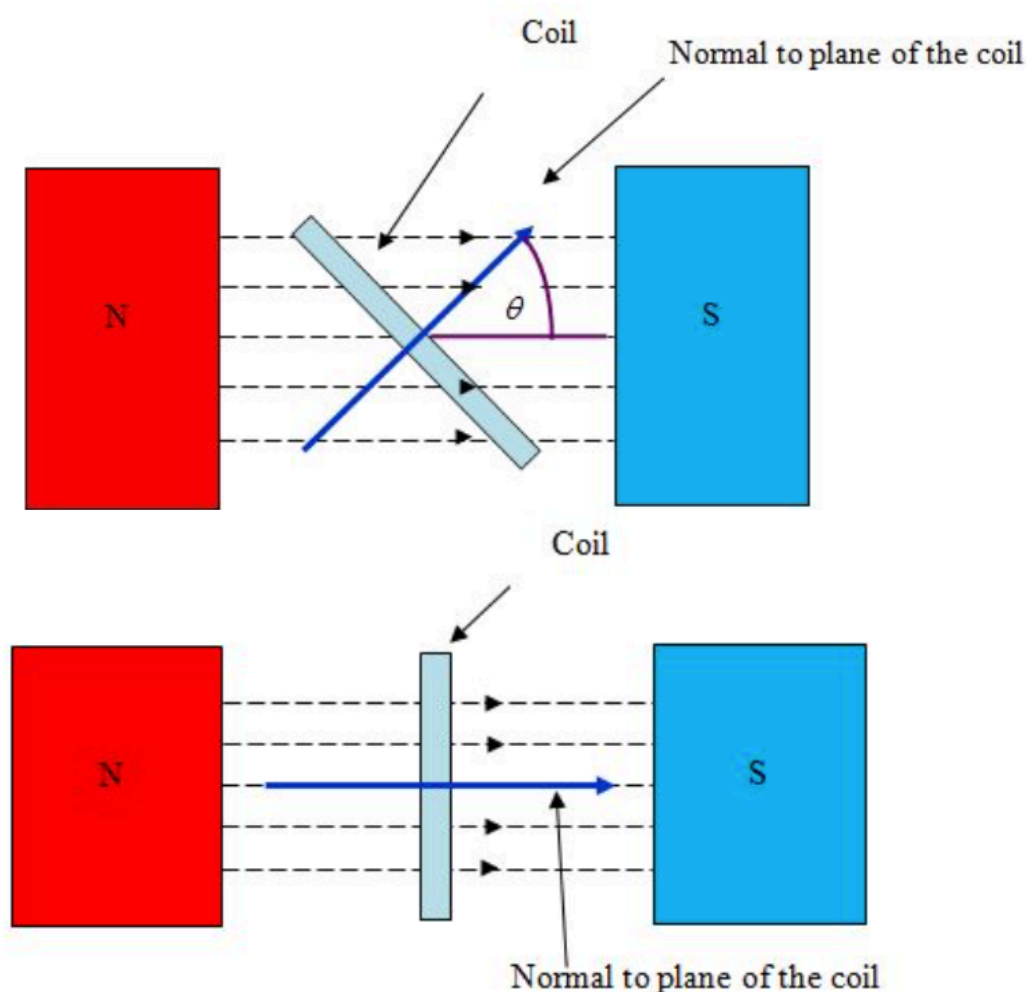
Source: <https://revisionworld.com/a2-level-level-revision/physics/fields-0/electromagnetic-induction>

The above diagram shows the flux and flux linkage through a rectangular coil in a magnetic field. Rotating the coil clockwise will result in less flux passing through the coil. This reduction of area in contact with the magnetic field will also reduce the flux linkage.

*Flux linkage  $N\Phi = BAN\cos\theta$*

Flux linkage as mentioned before, is the magnetic flux linked to a coil. Also aforementioned, the flux linkage and flux passing through a coil will change as a result of rotating it in the magnetic field. This means that the overall flux linkage will decrease.

The magnetic field must be acting perpendicular to the area in the equation  $\Phi = BA$ , so in calculating flux linkage, this condition must also be met. To meet this condition, you must find the angle between the normal to the coil, and magnetic field. The diagram below demonstrates this concept.



Source: [http://www.antonine-education.co.uk/Pages/Physics\\_4/Magnetism/MAG\\_04/Mag\\_field\\_4.html](http://www.antonine-education.co.uk/Pages/Physics_4/Magnetism/MAG_04/Mag_field_4.html)

You will get the greatest value of flux linkage when  $\cos\theta = 1$ , ie  $\theta = 0$ , thus a minimum at  $\theta = 90$ . At this minimum value, the field is parallel to the coil area, so no field lines pass through the coil area.

You can increase this value of flux linkage by increasing strength of the magnetic field. Or you can increase the 'linked' area. Another thing worth mentioning is that if you rotate a coil through  $180^\circ$ , the flux linkage is equal to  $-BAN$ .

### 3.7.5.4 Electromagnetic induction

#### Content

- Simple experimental phenomena.
- Faraday's and Lenz's laws.
- Magnitude of induced emf = rate of change of flux linkage  $\varepsilon = N\Delta\Phi/\Delta t$
- Applications such as a straight conductor moving in a magnetic field.
- Emf induced in a coil rotating uniformly in a magnetic field  $\varepsilon = BAN\omega\sin(\omega t)$

#### Simple experimental phenomena

Generating electricity can be done experimentally, using a magnet and a conducting wire. When the wire moves through the magnetic field, (or vice versa), and cuts the field lines (flux lines), an emf will be induced. If there is a complete circuit, then this emf will force a current to flow as electrons will be forced around the circuit. This effect is called **electromagnetic induction**. Moreover, to build on what I previously stated, if the wire moves parallel to the field lines, no emf will be induced.

Electromagnetic induction was discovered by Michael Faraday in 1832. It had been established that magnetic fields are produced in current-carrying wires, and Faraday wanted to know if you could use magnet to generate a current.

Other experiments to show this phenomenon can be done where the coil spins and magnet remains fixed, or the magnet spins and coil remains fixed. For example, in a dynamo.

- A dynamo is an electrical generator that works to convert mechanical energy into electrical energy. This is achieved by rotating a magnet within a coil of wire. When connected to a lamp, the rotation induces an emf, which causes a current to flow thus powering the lamp. The faster the rotation of the magnet, the greater the induced current and brighter the lights. This is utilised in bicycles that use this energy to power lamps.

The power is the work done per unit time. The induced emf multiplied by the current is equal to the rate of transfer of energy (power) from the source.

#### Faraday's and Lenz's laws

**Faraday's law** = "the induced emf in a circuit is equal to the rate of change of magnetic flux linkage through the circuit".

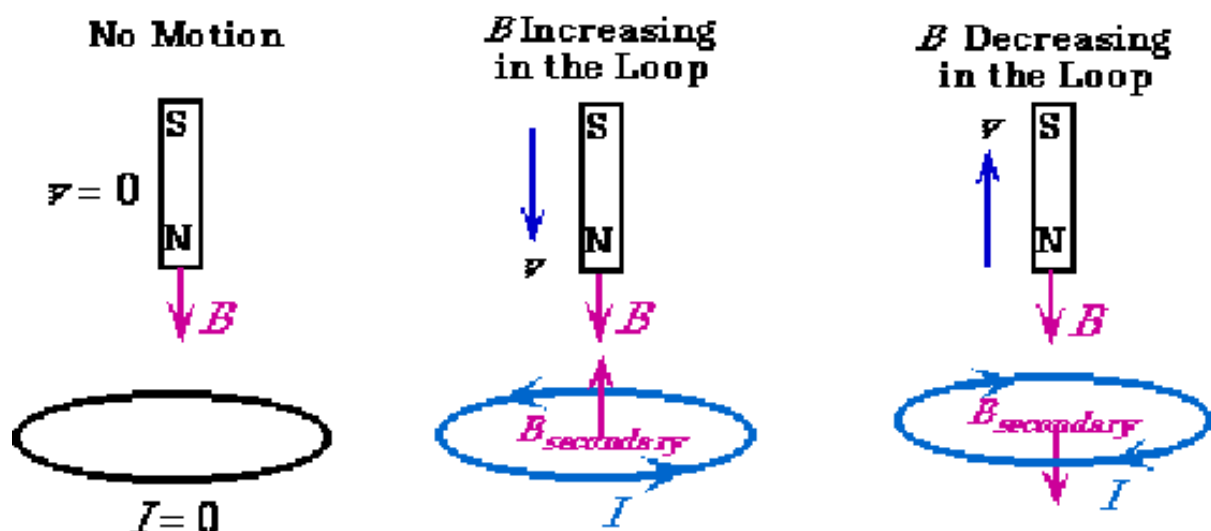
**Lenz's law** = "the direction of the induced current is always such that it opposes the change that causes the current".

We already know that an emf is produced when a conducting wire is moved in a magnetic field. To be more precise, and according to **Faraday's law**, this potential difference is produced as a result of a change in the motion. The induced voltage is dependent on the rate of change of magnetic flux linkage through a circuit.

This is represented by the equation  $\varepsilon = N\Delta\Phi/\Delta t$ . Where  $\varepsilon$  is the emf (V),  $N$  = number of turns,  $\Phi$  is magnetic flux (Wb), and  $t$  is time (s). There should be a negative sign in front of the equation which is as a result of Lenz's law, which will be covered next, which states the induced emf acts in such direction as to oppose the change that causes it.

If you have a set up of a coil connected to a meter measuring current, when a bar magnet is pushed into this coil, there will be a deflection on the meter. If you were to pull the magnet out of the coil in the opposite direction to that which it entered, the meter will deflect in the opposite direction.

We already know the definition of Lenz's law: the diagram below shows its application.



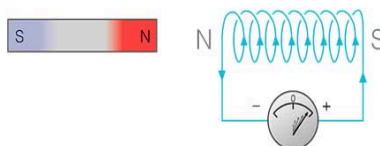
Source: <https://astarmathsandphysics.com/ib-physics-notes/118-electricity/1271-lenz-s-law.html>

When a permanently magnetised magnet passes through a coil that is connected to a meter there will be a deflection shown on the meter. This is because the coil of wire is seeing a changing magnetic field, thus a current is induced in the coil. This induced current creates a magnetic field in the coil that serves to oppose the incoming north pole (as shown above). It is important that this happens, because if it did not, energy, both electrical and kinetic would not be conserved. Energy would not be conserved as if the magnetic field supported the direction of movement of the magnet, then the north pole would be pulled in faster, which would increase the induced current and make the pole move in faster still. This would mean both kinetic energy and electrical energy would be created, thus not conserved. **This rule is also in accordance with Newton's third law of motion, that in short states for every action there will be an equal and opposite reaction.**

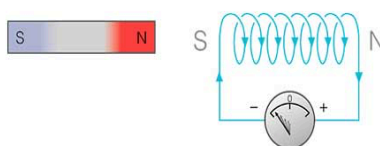
The **dynamo** rule applies to electric generators, and is other-wise known as Fleming's right hand rule. To remember that his right hand rule is for generators, you could use riGht-hand rule for **G**enerators.

### Explanation of Lenz's Law

For understanding Lenz's law, consider two cases : **CASE-I** When a magnet is moving towards the coil.



When the north pole of the magnet is approaching towards the coil, the magnetic flux linking to the coil increases. According to Faraday's law of electromagnetic induction, when there is change in flux, an emf and hence current is induced in the coil and this current will create its own magnetic field. Now according to Lenz's law, this magnetic field created will oppose its own or we can say opposes the increase in flux through the coil and this is possible only if approaching coil side attains north polarity, as we know similar poles repel each other. Once we know the magnetic polarity of the coil side, we can easily determine the direction of the induced current by applying right hand rule. In this case, the current flows in anticlockwise direction. **CASE-II** When a magnet is moving away from the coil



When the north pole of the magnet is moving away from the coil, the magnetic flux linking to the coil decreases. According to Faraday's law of electromagnetic induction, an emf and hence current is induced in the coil and this current will create its own magnetic field. Now according to Lenz's law, this magnetic field created will oppose its own or we can say opposes the decrease in flux through the coil and this is possible only if approaching coil side attains south polarity, as we know dissimilar poles attract each other. Once we know the magnetic polarity of the coil side, we can easily determine the direction of the induced current by applying right hand rule. In this case, the current flows in clockwise direction.

Also, to distinguish between using the left and right hand rules, you can remember that:

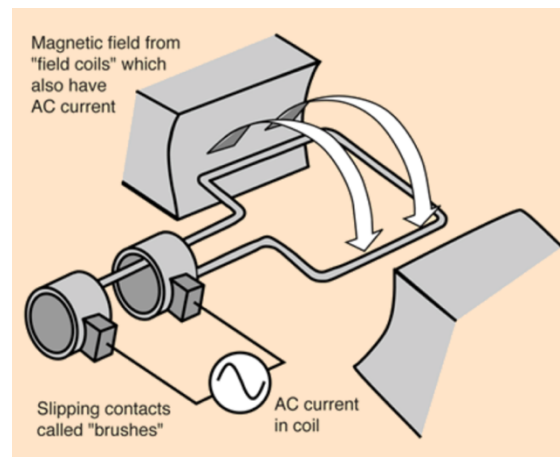
- For a **motor** the input energy is electrical energy and the useful output energy is mechanical energy.
- For a **generator** the input energy is mechanical energy and the useful output energy is electrical energy.

**Below is a summary of the action of both motors and generators.**



## Generators vs Motors

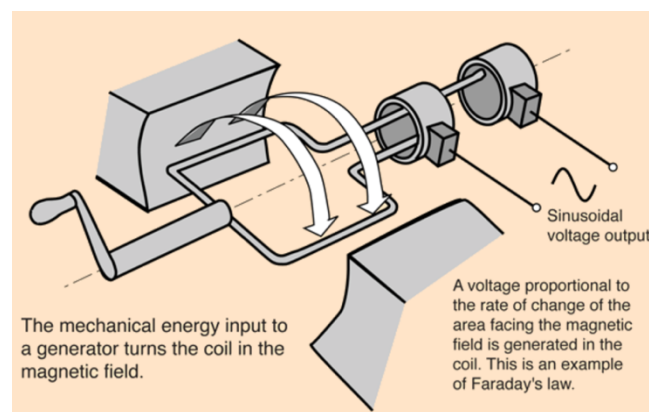
The first diagram below shows an electric **motor**. In the motor a current is passed through the coil, generating a force on the coil and thus torque. This causes the rotational movement depicted in the image. This follows the point above that states electric motors use electrical energy as input energy, which is converted to useful mechanical output energy.



<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motorac.html>

In an electric motor you will also get an emf **that** is induced that will oppose the potential difference  $V$  applied to the motor, this in accordance with Lenz's law. This then gives the equation  $V - \varepsilon = IR$ , as the back emf must be subtracted from the source pd. When the coil spins slowly, this induced emf is small thus current is high, and vice versa. Also, the equation  $V - \varepsilon = IR$  can be multiplied by  $I$ , to give  $VI - I\varepsilon = I^2R$ . This tells you that the source power ( $VI$ ) is equal to the power wasted due to resistance in the circuit added to the power **dissipated** by the back emf.

On the contrary, in **generators** the turning of the coils in the magnetic field produce emfs that force currents on both sides of the coil. The generated electrical energy can be described using Faraday's Law of electromagnetic induction. Again, this follows the point above that stated useful mechanical energy is used to produce useful output electrical energy.



<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motorac.html>

***Magnitude of induced emf = rate of change of flux linkage  $\varepsilon = N\Delta\Phi/\Delta t$***

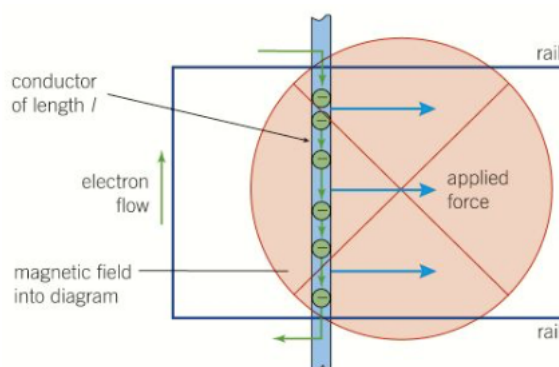
This point has already been covered, and gives the equation for calculating the magnitude of the induced emf  $\varepsilon = N\Delta\Phi/\Delta t$ . As also previously mentioned, this equation should actually have a negative sign at the beginning, as a result of Lenz's Law.

## INCREASE EMF INDUCED IN A COIL

- By increasing the number of turns in the coil i.e  $N$ - From the formulae derived above it is easily seen that if number of turns of coil is increased, the induced emf also gets increased.
- By increasing magnetic field strength i.e  $B$  surrounding the coil- Mathematically if magnetic field increases, flux increases and if flux increases emf induced will also get increased. Theoretically, if the coil is passed through a stronger magnetic field, there will be more lines of force for coil to cut and hence there will be more emf induced.
- By increasing the speed of the relative motion between the coil and the magnet - If the relative speed between the coil and magnet is increased from its previous value, the coil will cut the lines of flux at a faster rate, so more induced emf would be produced.

### *Applications such as a straight conductor moving in a magnetic field.*

An emf will be induced in a straight conductor moving in a magnetic field only if the conductor cuts the magnetic field lines.



Source: Kerboodle A Level Physics Textbook

The diagram above shows a straight conductor moving in a magnetic field. The magnitude of the emf induced  $\varepsilon = E/Q$ , which can be used to derive Faradays' equation. This equation tells us that the work done divided by the charge is equal to the induced emf.

We know that work done is equal to,  $E = F \times \Delta s$ , and we also know that the force on the conductor is  $F = BIl$ . Thus work done is equal to  $BIl\Delta s$ . We also know that the charge transferred is  $Q = I\Delta t$ .

So we can say that  $\varepsilon = BIl\Delta s/I\Delta t = Bl\Delta s/\Delta t$ . Also knowing that  $l \times \Delta s$  is equal to the area swept out (length of the wire multiplied by distance moved by the wire), then we can write

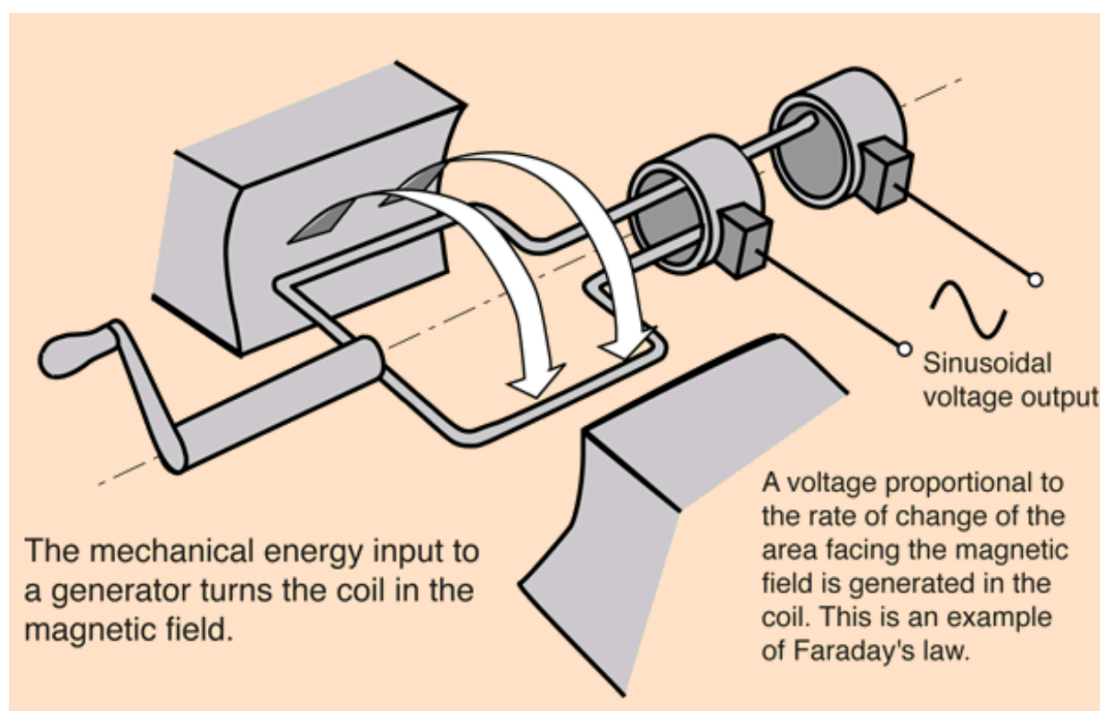
the induced emf  $\varepsilon = BA/\Delta t$ , where  $BA$  equals magnetic flux ( $\Phi$ ). However, we also know that velocity  $= \Delta s/\Delta t$ , so it can also be said that  $\varepsilon = Blv$ , where  $B$  is the magnetic field strength,  $l$  is the length of the conductor, and  $v$  is the velocity of the conductor, all with their usual units.

There are other examples that can be used such as a fixed coil subject to a changing magnetic field. Or you could look at the example of a rectangular coil moving in a uniform magnetic field.

### *Emf induced in a coil rotating uniformly in a magnetic field $\varepsilon = BAN\omega \sin(\omega t)$*

In a simple AC generator, a rectangular coil is forced to spin in a magnetic field (as shown below). The coil sees a change in magnetic flux through it as it spins, generating an emf that drives the current. If the coils spin faster, at a higher frequency, ie there is a faster rate of change of flux, or there is a greater number of turns in the coil then the larger the peak emf will be. Also you could use a stronger magnet, or a bigger coil.

In this coil the angle is changing at the angular frequency  $\omega$ , so the equation  $\omega t$  would give you the angle between the normal to the area and the magnetic field lines at a given time. This means that the flux linkage  $N\Phi = BAN\cos(\omega t)$ . It can be then shown mathematically that  $\varepsilon = BAN\omega \sin(\omega t)$  by differentiating  $\cos(\omega t)$  with respect to time.

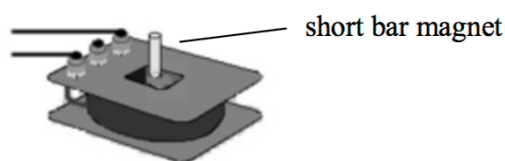


<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/motorac.html>

## OCR A Level Specimen Paper 3 2014 Q3

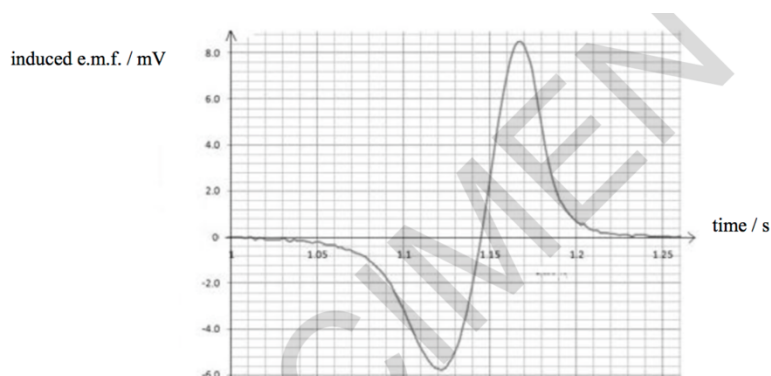
### Question:

**Fig. 3.1** shows a short bar magnet being dropped vertically through a small horizontal coil.



**Fig. 3.1**

**Fig. 3.2** shows the graph of how the e.m.f. induced in the coil varies with time, as the magnet passes through the coil.



**Fig. 3.2**

Identify and explain the main features of the peaks of induced emf shown on **Fig. 3.2**, in terms of Faraday's law of electromagnetic induction

**Answer:**

### Features of induced peaks to be explained

- Sense of each peak opposite
- Amplitude of 2nd peak larger because greater speed or greater  $(-)N \Delta\Phi / \Delta t$
- Area under peaks is equal because  $\Delta t = (-)N \Delta\Phi$

### Vocabulary guidelines

- For top level marks vocabulary should be in terms of changing flux linkage  $N \Phi$  with coil

### Level 3 (5–6 marks)

All 3 features fully explained: sense and amplitude explained in terms of changes of flux linking coil. Explanations involve reference to Faraday's Law or  $\varepsilon = (-) N \Delta\Phi / \Delta t$ .

**Sense:** increase in  $N \Delta\Phi$  is + ve and decrease – ve. **Amplitude:** peak occurs when rate of change of flux linkage is greatest, may be mathematically expressed.

**Area:** equated to total change of flux linkage with coil =  $\Sigma \varepsilon \Delta t = (-) N \Delta\Phi$  or sum of strips and same flux links coil on way in as unlinks from coil on way out.



#### ***3.7.5.5 Alternating currents***

## Content

- Sinusoidal voltages and currents only; root mean square, peak and peak-to-peak values for sinusoidal waveforms only.

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} ; V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

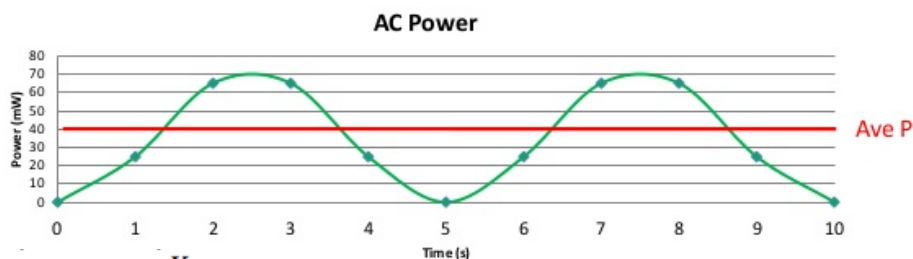
- Application to the calculation of mains electricity peak and peak-to-peak voltage values.
- Use of an oscilloscope as a dc and ac voltmeter, to measure time intervals and frequencies, and to display ac waveforms.
- No details of the structure of the instrument are required but familiarity with the operation of the controls is expected.

### *Sinusoidal voltages and currents only; root mean square, peak and peak-to-peak values for sinusoidal waveforms only*

An alternating current is one in which the current is constantly changing direction. The rate at which these cycles occur per unit time (1s) is called the frequency. The frequency,  $f = 1/T$ , where  $T$  is time period (s). The units of frequency are hertz (Hz). The alternating currents studied in this specification are sinusoidal, ie can be modelled by sine waves. Since they can be modelled as sine waves, you can measure peak-to-peak values for the wave.

You can observe alternating currents by connecting a signal generator to an LED. If you make the frequency very low, you are able to see the lamp light up and fade out regularly. When the lamp lights up to its brightest, the peak value of the current is reached. This happens twice every cycle, when the current is at its peak value in either direction. The frequency is increased until the lamp flickers so fast that to the eye, there is no observable 'flashing', and it seems the intensity is constant.

Obviously an alternating current will produce an alternating power output, where this output will vary between 0 and  $I^2R$ . For this alternating power output as shown below, there will be an average power output. If you took this average power output, and tried to find the corresponding constant current value that would give this power output value, you would find that the value is the root mean square of the alternating current, denoted by  $I_{\text{rms}}$ . So it can be said that 'The root mean square value of an alternating current is the value of direct current that would give the same heating effect as the alternating current in the same resistor.' The root mean square value is as it sounds, and is the root of the mean of the square of the original values.

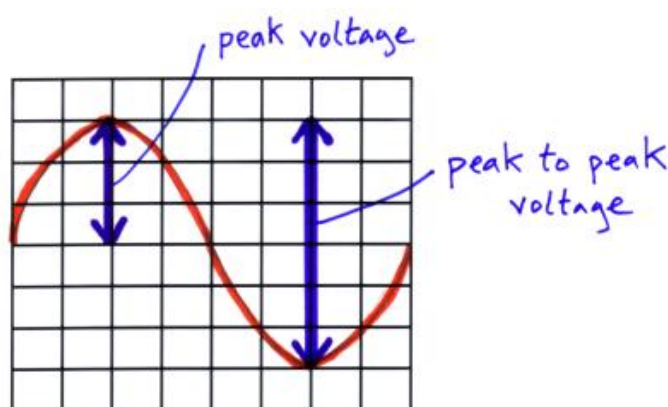


$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} ; V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$$

The rms value of both the current and potential difference can be calculated by multiplying the peak value ( $I_0/V_0$ ) by  $1/\sqrt{2}$ .

*Application to the calculation of mains electricity peak and peak-to-peak voltage values.*

The UK mains electricity runs at about 230V, with a frequency of 50Hz. The peak to peak voltage of the mains supply is around 650V.



Source: <http://physicsnet.co.uk/a-level-physics-as-a2/current-electricity/alternating-current-ac/>

If we know the y-gain sensitivity of an oscilloscope, we are able to work out the peak to peak voltage. If the y-gain were set at 20V, then the peak to peak voltage would be 120V. Thus the peak voltage would be 60V, and from this the  $V_{\text{rms}}$  value could be calculated. We can then also find the peak current value.

*Use of an oscilloscope as a dc and ac voltmeter, to measure time intervals and frequencies, and to display ac waveforms.*

The point above illustrates how an oscilloscope can be used as an AC voltmeter. A DC signal would be a horizontal line, so the value of voltage can be calculated again, if the y gain is known. Time intervals between waveforms can be used to calculate the frequency of an AC supply.

*No details of the structure of the instrument are required but familiarity with the operation of the controls is expected*



## AQA Jan 2011 Unit 1 Q5a

### Question:

Domestic users in the United Kingdom are supplied with mains electricity at a *root mean square voltage* of 230 V.

State what is meant by root mean square voltage.

### Answer:

- the square root of the mean of the squares of all the values of the voltage in one cycle
- or the equivalent dc/steady/constant voltage that produces the same heating effect/power

### 3.7.5.6 The operation of a transformer

#### Content

- The transformer equation:  $N_S/N_P = V_S/V_P$
- Transformer efficiency =  $I_S V_S / I_P V_P$
- Production of eddy currents.
- Causes of inefficiencies in a transformer.
- Transmission of electrical power at high voltage including calculations of power loss in transmission lines.

#### *The transformer equation: $N_S/N_P = V_S/V_P$*

Alternating current is used widely for many reasons, for example it is easily produced by generators, the maximum voltage can be changed easily using a transformer, it can be controlled by a variety of different components, also it has a regular frequency which is useful for timing.

Transformers gain their name from their action, as they transform voltage and current from one level to another. Thus in transformers, the magnitude of the alternating current can be increased or decreased. The structure of a transformer comprises of a primary and a secondary coil linked by an iron core. The iron core serves to concentrate the magnetic field and can be magnetised/demagnetised easily to ‘transfer’ the magnetic field from one core to the other. Although, ‘transfer’ would not strictly be the most accurate term to use, but is useful for understanding the concept.

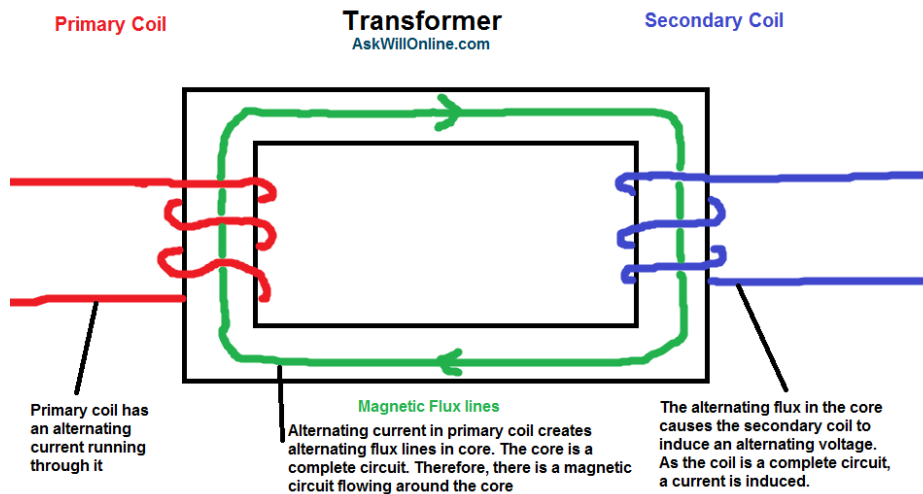
The action of a transformer is as follows: an alternating current is applied to the primary core and thus generates a constantly changing magnetic field in this core. This magnetic field then flows through the iron core.

Transformers can be step-up or step-down transformers, where step-up transformers step up the voltage and step down the current. The ratio of turns in the primary to the secondary coil determines whether the transformer is step-up or step-down.

- A **step-up** transformer has less turns on the primary coil, thus more on the secondary coil. This means the secondary voltage is increased from the primary voltage
- A **step-down** transformer has more turns on the primary coil, thus less turns on the secondary coil.

The equation  $N_S/N_P = V_S/V_P$  can be used to describe the action of transformers.  $N_S$  is the number of turns in the secondary coil and  $N_P$  is the number of turns in the primary coil.  $V_S$  is the voltage in the secondary coil,  $V_P$  is that in the primary coil. The equation is telling you that the ratio of output voltage to input voltage is equal to the ratio of turns of wire around the secondary coil, to the ratio of turns around the wire in the primary coil.

The diagram in the picture below gives a good outline of the action of a transformer. The transformer shown would not be a step-up or step-down transformer, as the number of turns on both sides is the same. This type of transformer can be used to restrict the amount of direct electrical connections, as the two coils are linked only via the magnetic field.



**Transformer efficiency** =  $I_S V_S / I_P V_P$

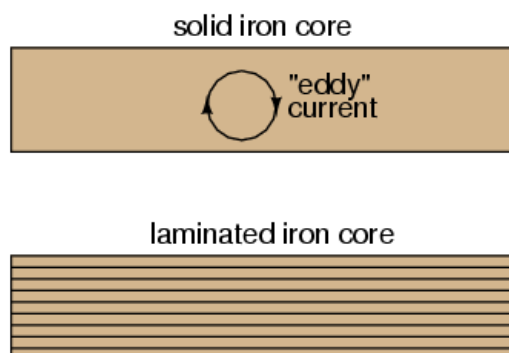
To calculate the efficiency of a transformer you can use the equation above, where  $I_S$  is the secondary current,  $V_S$  is the secondary voltage.  $I_P$  is primary current and  $V_P$  the primary voltage. This equation is telling you that the efficiency of a transformer is the ratio of the power delivered by the secondary coil to the power supplied by the primary coil.

### **Production of eddy currents**

Eddy currents are induced currents that are generated in the core itself. Iron is used for its ability to allow the magnetic field to flow in a circuit. However, it also conducts electricity, so there will be induced currents in the iron core itself, just like you get in the secondary coil. These currents can be thought of as ‘swirling’ currents circulating perpendicular to the primary or secondary turns on the core. Their circular motion has given them a name that resembles ‘eddies’ in a stream of water that circulate as opposed to moving in a straight line. In the core, these eddies create induced magnetic fields that oppose the change of that of the original magnetic field, in accordance with Lenz’s law.

### **Causes of inefficiencies in a transformer.**

1. **Eddy currents.** Iron is not as good a conductor of electricity as copper or aluminium, of which the coils are usually made. As a consequence, the eddy currents within the core receive a lot of resistance to their circulation. In overcoming this resistance, they dissipate power in the form of heat.
  - a. Mitigation against these eddy currents involves using a laminated core as opposed to a solid iron core. Between each layer there is also a layer of insulation. The culmination of the insulation and separation of layers provides extra resistance to the eddy currents that are attempting to flow.



2. **Resistance** in the coils. More power will be dissipated in higher resistance coils, due to the heating effect of the current
  - a. The coils are made of a material with a low resistance
3. **Hysteresis** in the core. The inclination to stay magnetised is called hysteresis, thus energy is used in overcoming this opposition to change each time the magnetic field produced in the primary coil changes polarity, ie 100 times a second from mains electricity (remember there are two peaks in a full AC cycle).
  - a. To lessen these effects, a core with low hysteresis should be chosen.  
Essentially, a '**soft**' core that can be magnetised easily and demagnetised easily, like iron, should be used.

The inefficiencies within the transformer will usually increase in significance as the **frequency** is increased. As frequency increases, the effective resistance increases so more power is dissipated as heat from the windings. Within the magnetic core, losses will also increase as frequency increases, in terms of the eddy currents and hysteresis effects. Thus most large transformers have been designed so that they operate very efficiently across a very tiny array of frequencies.

### *Transmission of electrical power at high voltage including calculations of power loss in transmission lines.*

In the UK, the national grid system is the supplier of electricity to most homes. The electricity is transmitted at high voltages, and then these voltages are stepped down so that they are safe to be transmitted into homes. Electrical power is more efficiently transferred at high voltages. This can be shown because  $P = VI$ , so for a given power output,  $P$ , the current required to reach the required power is decreased if voltage is increased. We already know that the current has a heating effect, so lower currents reduce this heating effect, thus reduce power dissipated (wasted) due to heating.

The current required to deliver this power output,  $P$  is given by the current,  $I = P/V$ . We also know that power is given by  $I^2R$ , so if we want the power dissipated by the current, we can say that the power dissipated by the cables  $= I^2R = P^2R/V^2$ . If you were transmitting 10MW of power at 100kV through cables with a resistance of  $200\Omega$ , the current required would be 100A ( $P=VI$ ). This power dissipated by this 100A current would therefore be 2MW. Thus higher voltages would be required as this power loss is quite significant.

## AQA June 2011 Unit 4 Q5bii

### Question:

Explain why the secondary windings of a step-down transformer should be made from thicker copper wire than the primary windings.

### Answer:

- to reduce heating) loss [or energy/power/copper loss]
- (because)  $I_s > I_p$
- and R is reduced (by use of thicker wire)

## AQA Jan 2010 Unit 4 Section B Q4

### Question:

Outline the essential features of a step-down transformer when in operation.

### Answer:

- Primary coil with more turns than secondary coil
- (wound around) a core/ or input is ac

### Question:

Describe **two** causes of the energy losses in a transformer and discuss how these energy losses may be reduced by suitable design and choice of materials.

The quality of your written communication will be assessed in this question.

### Answer:

1. When a transformer is in operation, there are ac currents in the primary and secondary coils. The coils have some resistance and the currents cause heating of the coils, causing some energy to be lost. This loss may be reduced by using low resistance wire for the coils. This is most important for the high current winding (the secondary coil of a step-down transformer). **Thick copper wire** is used for this winding, because thick wire of low **resistivity** has a low **resistance**.
2. The ac current in the primary coil magnetises, demagnetises and re- magnetises the core continuously in opposite directions. Energy is required both to magnetise and to demagnetise the core and this energy is wasted because it simply heats the core. The energy wasted may be reduced by choosing a material for the core which is easily magnetised and demagnetised, ie a magnetically soft material such as iron, or a special alloy, rather than steel. This reduces energy losses to hysteresis.
3. The magnetic flux passing through the core is changing continuously. The metallic core is being cut by this flux and the continuous change of flux induces emfs in the core. In a continuous core these induced emfs cause currents known as eddy currents,

which heat the core and cause energy to be wasted. The eddy current effect may be reduced by laminating the core instead of having a continuous solid core; the laminations are separated by very thin layers of insulator. Currents cannot flow in a conductor which is discontinuous (or which has a very high resistance).

4. If a transformer is to be efficient, as much as possible of the magnetic flux created by the primary current must pass through the secondary coil. This will not happen if these coils are widely separated from each other on the core. Magnetic losses may be reduced by adopting a design which has the two coils close together, eg by better core design, such as winding them on top of each other around the same part of a common core which also surrounds them.

## 3.8 Nuclear physics

### 3.8.1 Radioactivity

#### 3.8.1.1 Rutherford scattering

##### Content

- Qualitative study of Rutherford scattering.
- Appreciation of how knowledge and understanding of the structure of the nucleus has changed over time

##### *Qualitative study of Rutherford scattering.*

It is thought that the Greeks were the first to theorise the existence of ‘atoms’, and the word ‘atom’ itself comes from the Greek for indivisible. The first model put in place was called the plum pudding model, proposed by J.J Thompson. He said that atoms are discrete objects made up of pieces of positive and negative charge, with very small electron particles within an overall positive charge. This model represented known knowledge at the time, but was revolutionised after Rutherford’s scattering experiment. At this point, atoms were also thought of as solid objects.

Rutherford’s experiment consisted of firing a high energy beam of alpha particles (of the same  $E_K$ ) at a very thin gold foil. At the time of his experiment, the three types of radiation were known, thus alpha radiation was used. From the current knowledge on the atom he expected a beam of alpha particles should be scattered, but not by much, due to this ‘spread of positive charge’. However, he observed that some of the particles rebounded back in the same direction.

In his experiment, most alpha particles passed through the foil with almost no deflection, only around 1 in 2000 were deflected. Although around 1 in 10000 were deflected over angle of more than  $90^\circ$ .

This experiment led him to propose the idea of a concentrated nucleus of positive charge within the atom (because positive charge was deflected when passing too close). It also led to the conclusion that the nucleus is extremely small relative to the size of the atom.

From these results Rutherford also estimated the size of the nucleus. It can be said the probability of an alpha particle being deflected by an atom is 1 in 10000n, n being the number of layers of atoms. This probability comes from the ratio of the cross-section of the nucleus to that of the atom. So you get  $1/4\pi d^2 / 1/4\pi D^2 = 1 / 10000n$ , where d = diameter of the nucleus, and D the diameter of the atom. You can rearrange this equation to give you  $d^2 = D^2 / 10000n$ . Then, if you assume a standard value for the number of layers of atoms, you can take  $n = 10000$ . Substitute this value of n in and solve for d to get  $d = D/10000$  which gives you the size of a nucleus relative to an atom.

*Appreciation of how knowledge and understanding of the structure of the nucleus has changed over time*



### 3.8.1.2 $\alpha$ , $\beta$ and $\gamma$ radiation

#### Content

- Their properties and experimental identification using simple absorption experiments; applications eg to relative hazards of exposure to humans.
- Applications also include thickness measurements of aluminium foil paper and steel.
- Inverse-square law for radiation:  $I=k/x^2$
- Experimental verification of inverse-square law
- Applications eg to safe handling of radioactive sources.
- Background radiation; examples of its origins and experimental elimination from calculations.
- Appreciation of balance between risk and benefits in the uses of radiation in medicine.

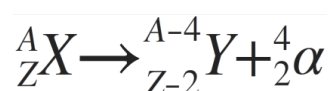
#### *Their properties and experimental identification using simple absorption experiments; applications eg to relative hazards of exposure to humans.*

There are three types of radiation, alpha, beta and gamma radiation.

The general equation for radioactive decay involves a nuclide  ${}_Z^AX$  where A is the mass number and Z is the proton number.

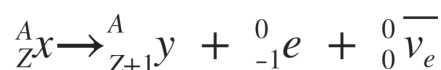
Alpha ( $\alpha$ ) radiation is the emission of a helium nucleus – two protons and two neutrons. This emission occurs from the nucleus of an unstable atom. Alpha radiation has a relatively large mass and charge relative to beta and gamma radiation. Due to **these properties**, alpha radiation is the least penetrating of the three types of radiation, however it is the most ionising and readily knocks electrons off nearby atoms. Alpha radiation is absorbed by paper/skin amongst many other materials.

A general equation representing alpha decay is shown below:



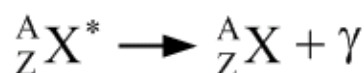
Beta ( $\beta$ ) radiation is the emission of an electron from an unstable nucleus. Clearly the nucleus does not contain any electrons, so a neutron is changed into a proton, or vice versa, for  $\beta^-$  or  $\beta^+$  decay respectively. The  $\beta$  radiation can either be  $\beta^-$  or  $\beta^+$ , where the beta minus decay involves emission of an electron, and beta plus, a positron (a positron is the electrons antiparticle). In beta minus decay the emission is accompanied by an antineutrino, and in beta plus decay a neutrino (the neutrino ensures conservation of energy). Moreover, beta radiation is more penetrating than alpha radiation, but less ionising than it. The beta particles produce fewer ions per unit length along their path than an alpha particle, provided the length is small as alpha particles have a short range in air. Beta particles are absorbed by around 5 centimetres of metal.

A general equation showing beta minus decay is given below. Beta plus decay is identical, except a positron is released (beta plus particle), and an electron neutrino (as opposed to the antielectron neutrino)



Gamma ( $\gamma$ ) radiation occurs in conjunction with alpha and beta decay. After alpha or beta decay, the nucleus is left in an excited state, expelling this excess energy in the form of a gamma ray (photon). These photons are highly penetrating and are not stopped, however can be absorbed. Gamma radiation has the lowest ionisation abilities, as they simply pass through most substances without any interaction. The inverse-square law can be applied to gamma radiation, as intensity decreases proportionally to the square of the distance ie  $I = k 1/r^2$ , where k is a constant to be determined. Gamma radiation is absorbed by a few centimetres of lead.

A general equation for gamma decay is shown below



These decays are summarised, with their respective equations, in the picture below.

Decay	Equation
<b><math>\alpha</math> decay</b> = in heavy nuclei	${}^A_ZX \rightarrow {}^{A-4}_{Z-2}Y + {}^4_2\alpha$
<b><math>\beta^-</math> decay</b> = in neutron rich nucleus	${}^A_ZX \rightarrow {}^A_{Z+1}Y + {}^0_{-1}\beta + \bar{\nu}_e$
<b><math>\beta^+</math> decay</b> = in proton rich nucleus	${}^A_ZX \rightarrow {}^A_{Z-1}Y + {}^0_{+1}\beta + \nu_e$
<b><math>e^-</math> capture</b> = in proton rich nucleus, inner $e^-$ captured (releasing x-rays)	${}^A_ZX + {}^0_{-1}e \rightarrow {}^A_{Z-1}Y + \nu_e$
<b>Gamma Decay</b>	${}^A_ZX \rightarrow {}^A_ZX + \gamma$

Source: <http://www.physbot.co.uk/nuclear-physics.html>

The properties of the three types of radiation are given below:

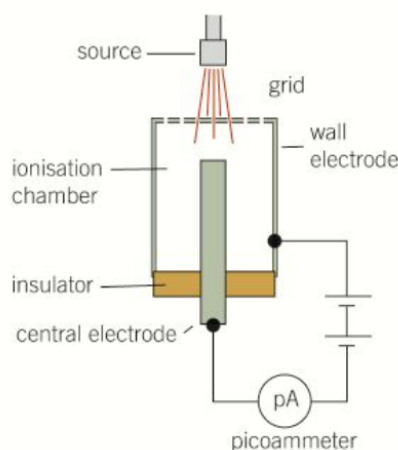
	$\alpha$ radiation	$\beta$ radiation	$\gamma$ radiation
nature	2 protons + 2 neutrons	$\beta^-$ = electron ( $\beta^+$ = positron)	photon of energy of the order of MeV
range in air	fixed range, depends on energy, can be up to 100 mm	range up to about 1 m	follows the inverse square law
deflection in a magnetic field	deflected	opposite direction to $\alpha$ particles, and more easily deflected	not deflected
absorption	stopped by paper or thin metal foil	stopped by approx 5 mm of aluminium	stopped or significantly reduced by several centimetres of lead
ionisation	produces about $10^4$ ions per mm in air at standard pressure	produces about 100 ions per mm in air at standard pressure	very weak ionising effect
energy of each particle/photon	constant for a given source	varies up to a maximum for a given source	constant for a given source

Source: <https://www.kerboodle.com/api/courses/17892/interactives/137640.html>

Rutherford also undertook studies into radioactivity. At this time, only alpha and beta types had been discovered. He realised that radiation ionised air, so conducts electricity. He could therefore make a detector to detect this ionising ability. Then he conducted further tests to show the deflection of alpha and beta radiation, and from the direction of deflection, the positive charge of alpha radiation was determined, and the negative nature of beta radiation. (Although now we know that there is also a beta plus decay). Also, this experiment shows gamma rays are uncharged as they are not affected by a magnetic field.

A **Geiger counter** can be used to investigate the absorption of radiation by different materials. You must place a Geiger tube at a fixed distance from a radioactive source, dependent on the type of radiation being studied. You must first, using the Geiger tube, measure the background count rate with no source present as this will need to be subtracted from your count rates for the source. Then, placing different absorbers in front of varying types of radiation ie paper and alpha radiation, you can investigate the effect of different materials on absorption.

You can use the set up shown below to investigate the **ionising** abilities of each type of radiation.



<https://www.kerboodle.com/api/courses/17892/interactives/137640.html>

In this experiment, as ions are created, the ions are attracted to the electrode holding an opposite charge. They are then discharged and electrons are able to form a current in the circuit, the value of which can be determined by the picoammeter. The current produced is proportional to the number of ions per second created in the chamber.

**Cloud chambers** can be used to determine the properties of radiation as well. Alpha and beta particles both ionise air, so leave a trail of water vapour droplets. The observed results show that all alpha particles from the same source have the same range, as they all travel the same distance in the cloud chamber. You can infer from this that each source gives the alpha particles a fixed amount of kinetic energy. Beta particles do not ionise air as well as alpha particles, thus their tracks are less easy to follow. Also, beta particles are released with a range of kinetic energies up to a maximum, so will all travel different distances.

*Applications also include thickness measurements of aluminium foil paper and steel.*

Radiation can be used in determining the thickness of materials like aluminium foil paper and steel. Thicker materials will absorb more radiation, so less radiation would reach a detector placed behind the material. This information can be used to determine the thickness of the material. The preferred type of radiation in these applications is beta minus. Alpha particles are not suitable as they are stopped by paper. Also, gamma rays would pass through the materials every time so it would not be useful to use.

*Inverse-square law for radiation:  $I=k/x^2$*

Any physical quantity that follows the inverse square law will decrease with inverse proportion to the square of the distance from the source. Radiation intensity follows the inverse square law, this means that  $I$  is proportional to  $1/x^2$ , therefore  $I = k/x^2$  where  $k$  is a constant.  $I$  represents intensity ( $\text{Wm}^{-2}$ ),  $x$  represents distance (m). The units of  $k$  can then be found to be the watt (W), because  $\text{Wm}^{-2} \times \text{m}^2 = \text{W}$ .

*Experimental verification of inverse-square law*

To verify the inverse square law, you can use a Geiger-Muller tube, a metre ruler and a radioactive source. First you must measure the background count at a significant distance from the source. Then you could start with the source at 10cm from your zero point, and increase this linearly up to 100cm for example. You would need to subtract the background count from the measured count rates at each distance and then plot a graph of this corrected-count rate against distance. From your graph you can check that as the distance doubles, the count rate decreases by a factor of four. You could also then draw a graph of count rate vs distance<sup>2</sup>, which should yield a straight line. Also it should be noted that count rate is used as a substitute to intensity.

*Applications eg to safe handling of radioactive sources*

There are procedures that must be followed when attempting to handle radioactive sources safely. For example, if the source you are using is solid, it must be handled using equipment

like tongs. This is to ensure that the source is as far away from the person as possible, thus in an effort to reduce exposure ie beyond the range of alpha/beta radiation, or to reduce the intensity of gamma radiation as low as possible. Moreover, you must use sealed containers made of lead for example, to store the radioactive sources, whether they are in solid, liquid or gas form. This is to ensure that a radioactive gas could not be breathed in, or a liquid could not come into contact with skin, or be drunk. It is also vital that the radioactive sources are not used for longer than necessary, as to reduce exposure time and reduce the dose of radiation received. Also, there are regulations that ensure any time a radioactive source is used, it must be recorded in a log book.

### *Background radiation; examples of its origins and experimental elimination from calculations.*

Background radiation occurs naturally all around us from various sources. For example, you have the uniform microwave radiation (cosmic rays) that still remain from the Big Bang. Also certain rocks are radioactive and give off radon gas. Plants will also absorb radioactive materials from the soil and these are able to pass up the food chain. You will get background radiation from nuclear fallout ie Chernobyl, and from the use of nuclear weapons. Pilots are exposed to higher doses of background radiation as levels are higher as you increase in altitude from the surface of the earth.

In calculations regarding the activity (amount of nuclei of the isotope that decay per second), the background radiation count must be removed from the data obtained. For example if you used a Geiger counter to find the activity of a gamma source, you would first obtain a measurement of the background radiation level at a distance from your source (so the source does not interfere with your measured value). You can repeat this multiple times and calculate a mean, for experimental accuracy and then from your measured count for the source, subtract the background count from each value to give a truer value of the actual activity of your source.

### *Appreciation of balance between risk and benefits in the uses of radiation in medicine.*

Ionising radiation is able to destroy the membranes of cells, which causes cell death. It can also damage important biological molecules such as DNA, which can cause cancerous tumours as it can stimulate cells to divide uncontrollably. It can cause genetic effects if it causes a mutation in a sex cells as these could be passed on to offspring. Also it can have an affect on living cells other than reproductive cells (somatic cells), thus somatic effects which can be detrimental to a person's health. High doses will simply kill cells, although mutation of cells can occur at both high and low doses.

This means that people using equipment that produces ionising radiation, or people at higher exposure to ionising radiation, wear something called a 'film badge' to display exposure levels. The various types of radiation can be detected, and if levels are too high action must be taken.

In medicine these risks are very real as nuclear radiation is used for example, in sterilising medical equipment or helping to diagnose and treat cancer. Gamma rays are able to pass

through medical equipment like syringes, so can be used to inactivate viruses and kill bacteria.

Radioactive tracers can be used to find areas of disease and investigate the inside of a person's body without requiring surgery. Although doses must be limited due to the risks of the radiation. Radioactive tracers must have a half-life long enough for the measurements that are required, but short enough so that it will decay quickly after use and can be disposed of.

Gamma rays can also be directed onto tumours (groups of potentially cancerous cells), which then destroys these cells. Although it is crucial exposure time is limited, otherwise too many normally functioning body cells will be killed in the process.

## AQA Jan 2010 Unit 4 Section B Q3bi)ii)iii)

### Question:

A  $\gamma$  ray detector with a cross-sectional area of  $1.5 \times 10^{-3} \text{ m}^2$  when facing the source is placed 0.18 m from the source.

A corrected count rate of  $0.62 \text{ counts s}^{-1}$  is recorded.

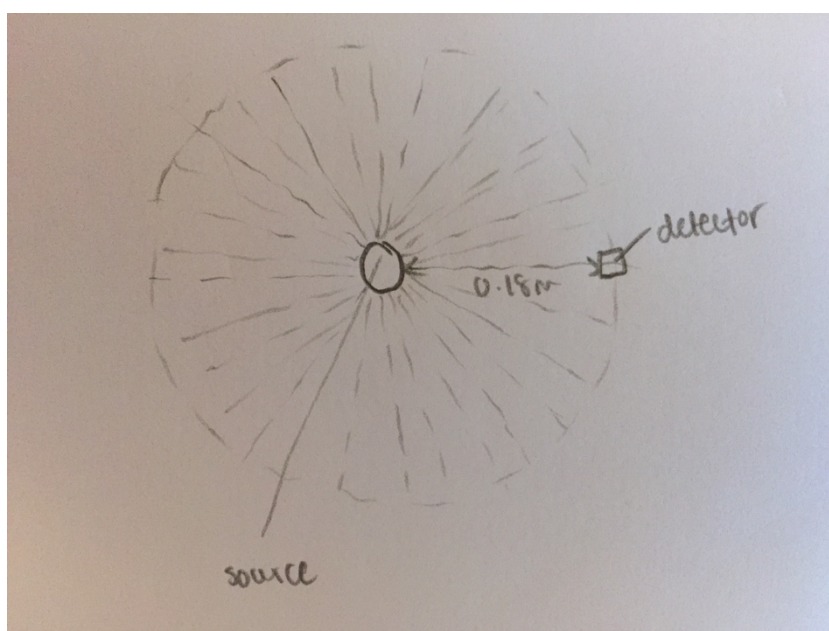
Assume the source emits  $\gamma$  rays uniformly in all directions.

Show that the ratio of number of  $\gamma$  photons incident on detector to the number of  $\gamma$  photons produced by the source is about  $4 \times 10^{-3}$ .

### Answer:

If you imagine a radioactive source emitting  $\gamma$  rays uniformly in all directions, you could represent it like is shown in the picture below.

The area that you are interested in is the surface area confined by the radius 0.18m. Any radiation outside of this radius we can ignore for the purposes of finding the ratio. Essentially you are finding the proportion of the surface area occupied by the detector. If you had a detector that could cover  $360^\circ$  then the ratio would be 1:1.



The mark scheme says:

(ratio of area of detector to surface area of sphere) ratio =  $0.0015$  (surface area of detector) /  $4\pi(0.18)^2$  (surface area of the sphere)

=  $0.0037$  = approximately  $4 \times 10^{-3}$ .



**Question:**

The  $\gamma$  ray detector detects 1 in 400 of the  $\gamma$  photons incident on the facing surface of the detector.

Calculate the activity of the source. State an appropriate unit

**Answer:**

Actual count rate that should be detected by a 100% accurate detector would therefore be  $0.62 \times 400 = 248$  counts per second. However, this is only the count rate per second for the area of  $1.5 \times 10^{-3}\text{m}^2$ , which is 0.368% of the total area (we know this from the previous question). So we can do  $248/0.0037 =$  approximately  $67000 \text{ Bq/s}^{-1}$ .

**Question:**

Calculate the corrected count rate when the detector is moved 0.10 m further from the source.

**Answer:**

Since  $I = k/x^2$ , and  $I$  for the source at 0.18m = 0.62. This means that  $k = 0.020088$ . Therefore, for a distance of 0.28m from the source (0.1 + 0.18),  $I = 0.020088/ (0.28)^2 = 0.256$  so = 0.26 count per second (2sf).



## Edexcel June 2010 Unit 6 Q4a

### Question:

*You are to plan an experiment to investigate the ability of gamma rays to penetrate lead. You are then to analyse a set of data from such an experiment.*

You have a source of radiation and a detector and counter. Describe briefly a simple experiment to confirm that the source emits gamma radiation.

### Answer:

- Record background count (rate)
- Place thick aluminium/thin lead between source & detector **OR** Distance greater than 25 cm between source and detector
- Count rate detected above background value shows gamma radiation.

### Question:

You are provided with sheets of lead and apparatus to support them safely between the source and the detector.

The thickness of lead affects the count rate. Describe the measurements you would make to investigate this.

Your description should include:

- A variable you will control to make it a fair investigation
- How you will make your results as accurate as possible
- One safety precaution

### Answer:

Keep distance between the source and detector constant

Any **four** from:

- Record count (rate) for different thicknesses
- Record count for a specified time
- Subtract background count
- Take several readings at each thickness
- Measure thickness with micrometer screw gauge/Vernier calipers

One of:

- Keep people away from source
- Use tongs to handle source
- Use tongs to handle lead sheets

- Ensure source is held securely
- Limit exposure time to source

### 3.8.1.3 Radioactive decay

#### Content:

- Random nature of radioactive decay; constant decay probability of a given nucleus;  $\Delta N/\Delta t = -\lambda N$ .
- The equation:  $N = N_0 e^{-\lambda t}$
- Use of activity,  $A = \lambda N$
- Modelling with constant decay probability
- Questions may be set which require students to use  $A = A_0 e^{-\lambda t}$
- Questions may also involve use of molar mass or the Avogadro constant
- Half-life equation:  $T_{1/2} = \ln 2/\lambda$
- Determination of half-life from graphical decay data including decay curves and log graphs
- Applications eg relevance to storage of radioactive waste, radioactive dating etc.

#### *Random nature of radioactive decay; constant decay probability of a given nucleus; $\Delta N/\Delta t = -\lambda N$ .*

Radioactive decay is a completely random process, and is carried out by alpha/beta/gamma emission in order to make an unstable nucleus more stable. Radioactive decay can be modelled by the equation  $\Delta N/\Delta t = -\lambda N$ . In the equation  $N$  = number of nuclei present and  $t$  is time (s).  $\lambda$  is called the decay constant, carrying the unit  $s^{-1}$ .

It can be said that the rate of disintegration of the number of nuclei present is proportional to the number of nuclei present, ie  $-\Delta N/\Delta t \propto N$  (the negative sign represents the decrease in number). The constant of proportionality is called the decay constant, denoted by lambda, thus giving the equation  $\Delta N/\Delta t = -\lambda N$ .

#### *The equation $N = N_0 e^{-\lambda t}$*

This equation can be used to find  $N_0$  (the initial number of nuclei present). It can be derived using the equation in the previous point, although its derivation is not required, but is shown below as it can help with the understanding of the equation. The derivation requires integration techniques you may or may not be familiar with.

IT SHOULD SAY  $\lambda dt$  not  $\lambda t$

The image shows a handwritten derivation of the radioactive decay equation. It starts with the differential equation  $\frac{dN}{dt} = -\lambda N$ . This is rearranged to  $\int_{N_0}^N \frac{1}{N} dN = \int_0^t -\lambda dt$ . Integrating both sides gives  $[\ln N]_{N_0}^N = [-\lambda t]_0^t$ . This simplifies to  $\ln N - \ln N_0 = -\lambda t$ , which can be written as  $\ln \frac{N}{N_0} = -\lambda t$ . Taking the exponential of both sides yields  $e^{-\lambda t} = \frac{N}{N_0}$ . Finally, multiplying both sides by  $N_0$  gives the equation  $N_0 e^{-\lambda t} = N$ .

This equation can therefore be used to find the number of nuclei present at a given time, if you know the initial value  $N_0$ , the decay constant, and time elapsed.

### *Use of activity, $A = \lambda N$*

The activity of a sample is defined as ‘the number of nuclei of the isotope that decay per unit time (ie per unit second)’, or it can be looked at as the rate of disintegration of a nucleus, thus  $A = -\Delta N/\Delta t$ . And since we already know that  $-\Delta N/\Delta t = \lambda N$ , then it must be true to say  $A = \lambda N$ . The units of activity are the Becquerel (Bq)

### *Modelling with constant decay probability*

The probability of decay is a constant (specific for each isotope).

### *Questions may be set which require students to use $A = A_0 e^{-\lambda t}$*

The derivation of the equation above is shown below. Again, it is not required, but useful for understanding the concept.  $A_0$  represents initial activity (ie activity at  $t = 0$ ).

Handwritten derivation of the activity equation:

$$N = N_0 e^{-\lambda t}$$

sure  $A = \lambda N$

$$A_0 = \lambda N_0$$

$$\frac{A_0}{\lambda} = N_0$$

so  $N = \frac{A_0}{\lambda} e^{-\lambda t}$

$$\lambda N = A_0 e^{-\lambda t}$$

sure  $A = \lambda N$

$$A = A_0 e^{-\lambda t}$$

### *Questions may also involve use of molar mass or the Avogadro constant*

An element with mass number  $A$  will have a molar mass equal to its mass number in grams, and one mole of the element contains  $6.023 \times 10^{23}$  atoms, which is called the Avogadro constant ( $N_A$ ). So for carbon 12 ( $^{12}\text{C}$ ),  $6.023 \times 10^{23}$  atoms of carbon-12 = 12g. Its molar mass (ie the mass of one mole of carbon) is 12g.

### *Half-life equation: $T_{1/2} = \ln 2/\lambda$*

The half-life ( $T_{1/2}$ ) of a radioactive isotope is defined as ‘the time taken for the mass of the isotope to decrease to half the initial mass, or, time taken for the activity to half’

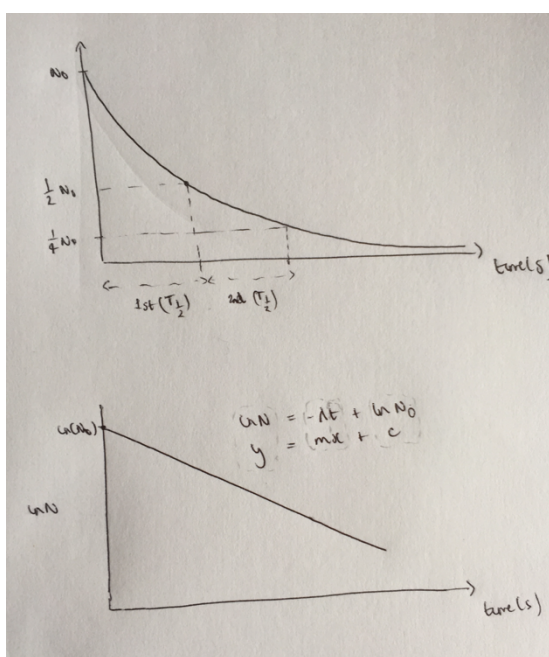
From this definition, we can say that the half-life occurs when  $N = 0.5N_0$ . We can substitute this value of  $N$  into the equation  $N = N_0 e^{-\lambda t}$  to derive a value of  $t$  that will be the time taken for half-life to occur (so when  $t = T_{1/2}$ ). In the derivation I have used a lowercase  $t$ , however it should be uppercase.

$$\begin{aligned}
 0.5N_0 &= N_0 e^{-\lambda t_{1/2}} \\
 \ln \frac{1}{2} &= -\lambda t_{1/2} \\
 \ln 1 - \ln 2 &= -\lambda t_{1/2} \\
 t_{1/2} &= \frac{\ln 2}{\lambda}
 \end{aligned}$$

### *Determination of half-life from graphical decay data including decay curves and log graphs*

From a graph of  $N$  vs  $t$ , where  $N = N_0/2$ , the corresponding value of  $t$  is the first half life. The time taken for this value to halve will then be the second half life time, which should be identical. However, if you are asked to calculate the half life from this type of graph, always find as many values for  $T_{1/2}$  as the graph will allow you, and calculate a mean value.

For the  $\ln(N)$  graph, the gradient is equal to  $-\lambda$ , thus to calculate half life,  $T_{1/2} = \ln 2 / \lambda$ , so substitute  $\lambda$  into this equation to find half life.



### *Applications eg relevance to storage of radioactive waste, radioactive dating etc.*

Half-life is an important concept when examining the safety of radioactive waste, or use in radioactive dating.

**Carbon dating** can be used to find the age of an object containing organic material. It only applies to matter which was once living, and in an equilibrium with the atmosphere. Cosmic rays lead to the formation of the isotope carbon -14 by bombarding nitrogen nuclei. This carbon isotope then combines with oxygen, forming carbon dioxide which can be incorporated into living things ie through photosynthesis in plants/trees etc. The rate of production of carbon -14 appears to be constant. So if you measure the activity of a sample then compare its activity to that of the same mass of living material now, you can calculate an estimate for the time that should have passed for its activity to decrease to your measured value. This is also made reliable by the fact that once a tree for example, has died, it will take no more carbon dioxide in.

Example: A sample of dead organic material is found to have an activity of 0.5Bq. An equal mass of living organic material of the same type, is found to have an activity 1.8Bq. Estimate the age of the sample given that the half life of carbon - 14 is 5570 years. Give your answer to a suitable number of significant figures.

So an equation that will give us a value of time is  $N = N_0 e^{-\lambda t}$  and we also know  $A = \lambda N$ . So we can calculate the value of N for the living material, and the value of N for the dead material. The value of N we find for the living material (providing the rate of carbon - 14 production, and its relative concentration in the atmosphere, has remained constant), will give us the value of N that the sample would have started at when it was living. Another way of writing this is  $N_0$  (when  $t = 0$ ). The decay constant also needs to be calculated, which can be done using  $\lambda = \ln 2 / T_{1/2}$ .

So now we have all of the information we require to solve for t, which is shown below.

Handwritten calculations for carbon dating:

$$\lambda = \frac{\ln 2}{T_{1/2}}$$
$$\lambda = \frac{\ln 2}{5570}$$
$$= 1.244 \times 10^{-4}$$

Living material

$$A = \lambda N$$
$$\frac{A}{\lambda} = N$$
$$\frac{1.8}{1.244 \times 10^{-4}} = 14464.96 \dots = N_0$$

Dead material

$$\frac{A}{\lambda} = N$$
$$\frac{0.5}{1.244 \times 10^{-4}} = 4017.9 \dots = N$$

So  $N = N_0 e^{-\lambda t}$

$$\ln \frac{N}{N_0} = -\lambda t$$
$$t = \frac{\ln N/N_0}{-\lambda}$$
$$= \frac{\ln 4017.9/14464.96}{-1.244 \times 10^{-4}}$$
$$= 10293.3 \text{ years}$$
$$= 10,000 \text{ years (2sf)}$$

**Potassium-Argon dating** can also be used to estimate the age of samples ie rocks. It is useful because argon is chemically unreactive, so would not be expected to be found naturally inside of a rock, so any found is very likely to be from radioactive decay of potassium into argon.

Potassium decays by either beta decay, or by electron capture. It is 8x more likely to decay by beta emission to form the calcium isotope  $^{40}\text{Ca}$ , than to decay via electron capture. This means that in a sample, if you now have 4 potassium-40 atoms to every 1 argon-40 atom, then  $N = 4$  now, but  $N_0 = 13$  because there would have originally been 13 potassium-40 atoms, 8 of which decayed into the calcium-40 isotope, 1 of which to argon-40, leaving 4. This can be then substituted into  $N = N_0 \dots$ , alongside  $\lambda = \ln 2 / T_{1/2}$  to give the age of the sample as over 2000 million years.

## OCR (B) A Level Specimen 1 2014 Q38b

### Question:

Here are two correct statements:

- Radioactive decay is a random process
- The decay curve of a radioisotope can be predicted mathematically.

Use your understanding of the decay constant to explain how both statements can be true for sources containing large numbers of atoms. Explain how you expect the scatter of the results shown in **Fig. 38.1** to change as the count rate falls

### Answer:

#### **Indicative scientific points may include:**

##### **Randomness**

- cannot know when an individual nucleus will decay
- explanation of the meaning of the decay constant (e.g. probability of decay of individual nucleus in unit time)
- $\lambda$  as the probability related to  $dN/dt$
- discussion of an analogue (e.g. coins or dice)

##### **The exponential curve as a model**

- reference in correct context to  $N = N_0 e^{-\lambda t}$   
**or**
- linking to  $dN/dt = -\lambda N$

##### **The effect of the number of nuclei present**

- for fixed  $\lambda$  the number of nuclei decaying in a given time can be predicted given sufficiently large sample
- as count rate falls, the number of nuclei that may decay also falls
- as the number of nuclei falls the variation from the predicted outcome will increase





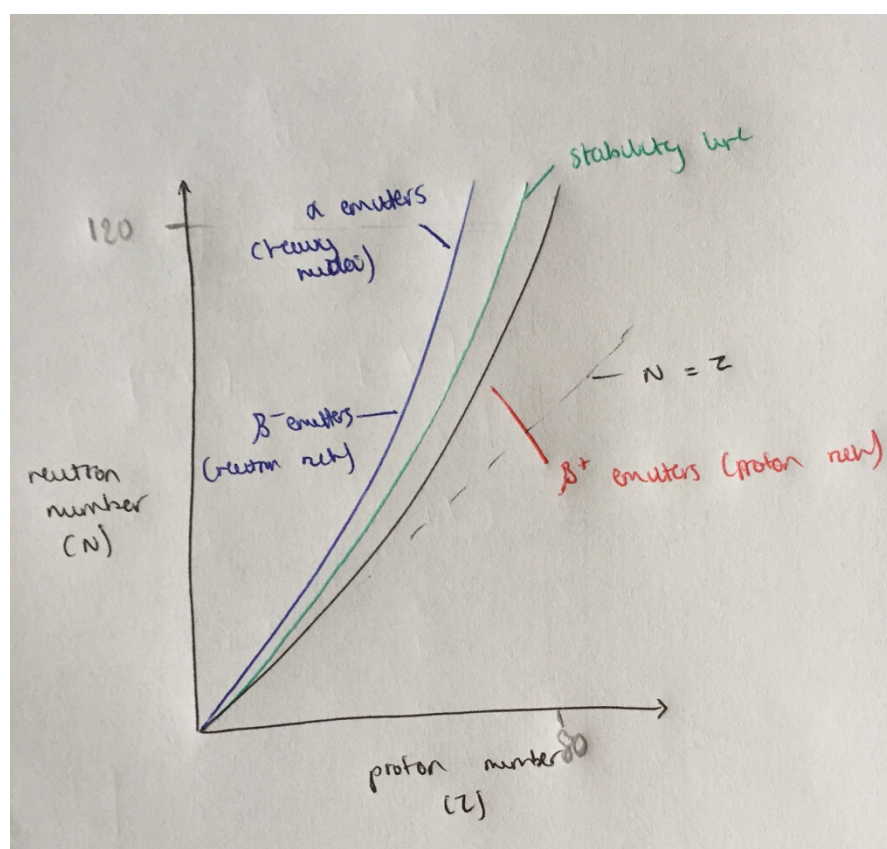
### 3.8.1.4 Nuclear instability

#### Content:

- Graph of  $N$  against  $Z$  for stable nuclei.
- Possible decay modes of unstable nuclei including  $\alpha$ ,  $\beta^-$ ,  $\beta^+$  and electron capture.
- Changes in  $N$  and  $Z$  caused by radioactive decay and representation in simple decay equations. Questions may use nuclear energy level diagrams.
- Existence of nuclear excited states;  $\gamma$  ray emission; application eg use of technetium - 99m as a  $\gamma$  source in medical diagnosis

#### Graph of $N$ against $Z$ for stable nuclei.

The graph above shows the curve for stable nuclei. The stable nuclei lie between the origin and around  $N = 120$ ,  $Z = 80$ , where  $N$  = neutron number,  $Z$  = proton number.



- For  $Z < 20$ , the stable nuclei follow the line  $N = Z$ .
- For  $Z > 20$ , the stable nuclei have more neutrons than protons. These extra neutrons serve to help nucleons bind together without adding more electrostatic repulsion between protons.
- For  $Z \geq 60$ , you will get alpha emitters. Usually for these alpha emitters  $Z > 80$ , and  $N > 120$ . These nuclei are too large to be stable because the strong nuclear force becomes insufficient to overcome the electrostatic repulsion between the vast amount of protons.

### *Possible decay modes of unstable nuclei including $\alpha$ , $\beta$ , $\beta^+$ and electron capture.*

Alpha omission reduces both the N and Z value by 2, so alpha emitters move diagonally downwards, to the left.

$\beta^-$  emitters are found to the left of the N-Z stability curve because they are neutron rich. They convert a neutron to a proton in the nucleus to become more stable and move closer to the stability curve (diagonally downwards to the right).

The opposite can be said for  $\beta^+$  emitters as these are found to the right of the stability curve, since they are proton rich. These emitters will then move diagonally upwards to the left.

Another type of decay for an unstable nucleus is electron capture. This occurs when an unstable nucleus pulls an orbital electron from the first shell into its nucleus. Here, the electron combines with a proton, forming a neutron and neutrino. The neutrino is ejected from the nucleus, the neutron remains, an X-ray is also released in this process. An example of electron capture is given below. The nucleus formed will now have one more neutron and one less proton, thus becoming more stable. On the N-Z graph, it would cause a movement diagonally upwards, to the left, like the translation you would get from beta plus emission.



### *Changes in N and Z caused by radioactive decay and representation in simple decay equations. Questions may use nuclear energy level diagrams.*

The changes in N and Z caused by radioactive decay for alpha, beta plus and minus are given below. Also, simple decay equations are given for them.

Alpha decay = Z - 2, N - 2

$\beta^-$  decay = N - 1, Z + 1

$\beta^+$  decay = N + 1, Z - 1 (also the same for electron capture)

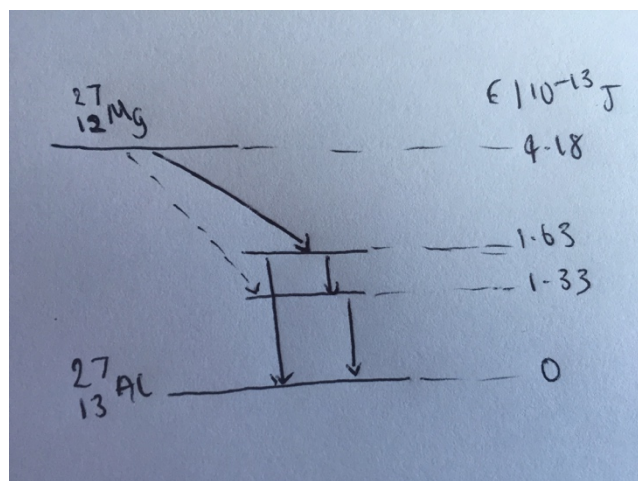
Gamma = N + 0, Z + 0

Decay	Equation
<b><math>\alpha</math> decay</b> = in heavy nuclei	${}_Z^AX \rightarrow {}_{Z-2}^{A-4}Y + {}_2^4\alpha$
<b><math>\beta^-</math> decay</b> = in neutron rich nucleus	${}_Z^AX \rightarrow {}_{Z+1}^AY + {}_{-1}^0\beta + \bar{\nu}_e$
<b><math>\beta^+</math> decay</b> = in proton rich nucleus	${}_Z^AX \rightarrow {}_{Z-1}^AY + {}_{+1}^0\beta + \nu_e$
<b><math>e^-</math> capture</b> = in proton rich nucleus, inner $e^-$ captured (releasing x-rays)	${}_Z^AX + {}_{-1}^0e \rightarrow {}_{Z-1}^AY + \nu_e$
<b>Gamma Decay</b>	${}_Z^AX \rightarrow {}_Z^AX + \gamma$

### *Existence of nuclear excited states; $\gamma$ ray emission; application eg use of technetium-99m as a $\gamma$ source in medical diagnosis*

Just like we can model electrons as having different excited states, we can model the nucleus in the same way. Unstable nuclei may emit a  $\gamma$  ray after alpha/beta decay or possibly electron capture. In this way, if the 'daughter' nucleus is given energy to move into an excited state, it is able to lose energy by releasing a photon of energy, allowing the nucleus to move to its ground state (lowest energy state). The term 'daughter nucleus' refers to the nucleus remaining after emitting an alpha or beta particle, or undergoing electron capture.

The energy levels of nuclei will usually be represented as shown below. The magnesium-27 has decayed by  $\beta^-$  decay to produce aluminium-27, and has decayed to leave its nucleus in an excited state. Just like with the electron energy levels, the nucleus may have a few energy levels. Take for example the diagram below, the nucleus has two energy levels, and can decay straight from  $1.63 \times 10^{-13} \text{ J}$  to 0J at ground state in one process. Or it can move down one energy level and release  $0.3 \times 10^{-13} \text{ J}$ , and then fall down to ground state from the slightly lower energy level, thus releasing two photons of energy.



Another key term associated with these nuclear excited states are **metastable** states. These states are excited states of an atom that have a much longer lifetime than that of the other states, but a shorter lifetime than the stable ground state. So atoms could stay in this metastable state for a considerable amount of time, long enough for them to be separated from the parent isotope. This concept is utilised in medical diagnosis, using technetium-99m as a gamma source.

Nuclei of the technetium-99 form in a metastable state, hence the notation **technetium-99m**. It is also written as  $^{99}\text{Tc}^{\text{m}}$  to indicate this metastable state. This  $^{99}\text{Tc}^{\text{m}}$  is formed by beta minus decay of  $^{99}\text{Mo}$ . This technetium-99m has a half life of around 6 hours, and when this technetium decays to ground state it releases a  $\gamma$  ray. Once this decays to its ground state, it forms a stable product with a half life of over 500000 years. This means that virtually only  $\gamma$  rays are obtained, which is useful in medical diagnosis because:

- You can use it to monitor blood flow through the brain

- There is such thing called a  $\gamma$  camera, which can be used to gain insight into internal organs and bones by detecting  $\gamma$  rays from places the  $^{99}\text{Tc}^{\text{m}}$  is located inside of the body.

### 3.8.1.5 Nuclear radius

#### Content

- Estimate of radius from closest approach of alpha particles and determination of radius from electron diffraction.
- Knowledge of typical values for nuclear radius.
- Students will need to be familiar with the Coulomb equation for the closest approach estimate.
- Dependence of radius on nucleon number:
- $R = R_0 A^{1/3}$  derived from experimental data.
- Interpretation of equation as evidence for constant density of nuclear material.
- Calculation of nuclear density.
- Students should be familiar with the graph of intensity against angle for electron diffraction by a nucleus.

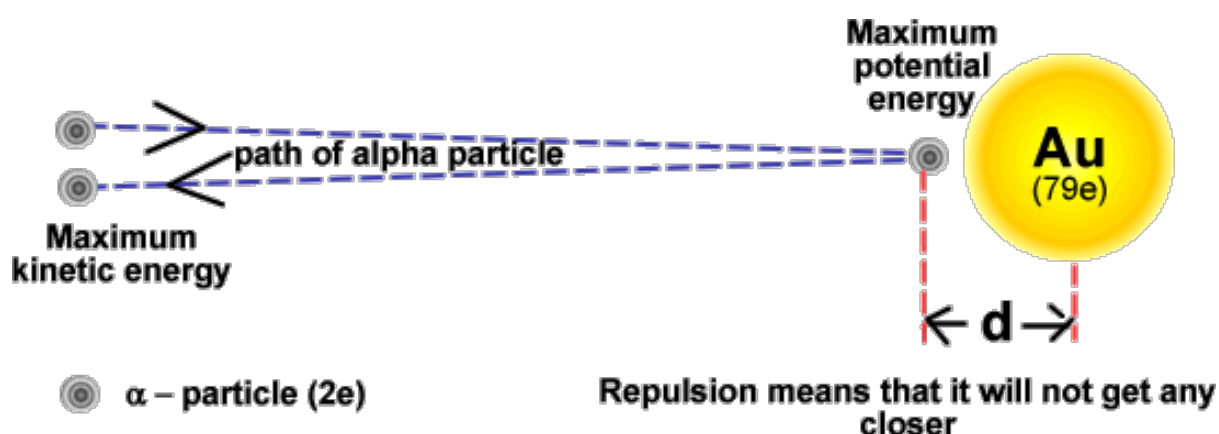
#### *Estimate of radius from closest approach of alpha particles and determination of radius from electron diffraction.*

The **closest approach** of an alpha particle to a nucleus can be used to give an estimate for the radius of a nucleus.

Essentially, as an alpha particle approaches a nucleus (for example a gold atomic nucleus), it will have a known kinetic energy  $= mv^2/2$ . It experiences repulsion by the positively charged gold nucleus which increases as it gets closer, so the alpha particle will reach a certain distance and then reverse its path.

When the alpha particle reaches its closest approach (to the nucleus), all of its kinetic energy has been converted to electric potential energy. At this point,  $E_k = kqQ/d$ , where  $k = 1/4\pi\epsilon_0$ . The equation can be rearranged to get  $d = kqQ/E_k$ . Where  $q$  is the charge on the alpha particle, and  $Q$  is the charge on the (gold) nucleus.

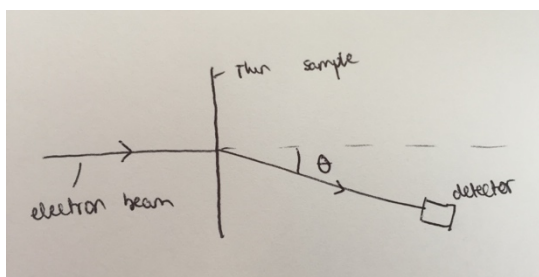
The distance  $d$  calculated will give an estimate for the size of the nucleus, however the alpha particle will not come into direct contact with it, so the value calculated will be slightly too large.



Source: [http://www.cyberphysics.co.uk/topics/atomic/Rutherford/Rutherford\\_calcs.htm](http://www.cyberphysics.co.uk/topics/atomic/Rutherford/Rutherford_calcs.htm)

**Electron diffraction** can be used to determine the diameter of a nucleus. You can use the diffraction grating equation  $d\sin\theta = n\lambda$  to determine the diameter of the slit, ie the diameter of a nucleus. If you take the first order, then the equation becomes  $d\sin\theta = \lambda$ , so  $d = \lambda/\sin\theta$ , clearly you need to know the angle of diffraction. Also you need to know the wavelength of the electron, which can be calculated using  $\lambda = h/mv$ . Since the electrons are being accelerated to incredibly high potentials, their speed can be approximated as 'c', thus  $\lambda = h/mc$ . Also, since  $E = mc^2$ , we can write  $\lambda = hc/E$ . In this way, we can calculate the wavelength of the electron, substitute in our values and calculate an estimate for the nuclear diameter, which can be used to determine the radius of the nucleus.

The diagram below shows a simplified version of electron diffraction to estimate the diameter of a nucleus.



### *Knowledge of typical values for nuclear radius.*

The nuclear radius usually takes a value of around  $5 \times 10^{-16}$ , as a typical value for the nuclear diameter is around 1 angstrom ( $10^{-10}\text{m}$ ).

### *Students will need to be familiar with the Coulomb equation for the closest approach estimate.*

This has been covered in the first point. It is linking the equation for electric potential energy  $= kQq/r$  to the kinetic energy of the alpha particle.

### *Dependence of radius on nucleon number:*

The radius of nuclei depends on the nucleon number of the atom. This should be a fairly logical argument to visualise, as more nucleons should result in more space occupied by the nucleus, thus a larger radius.

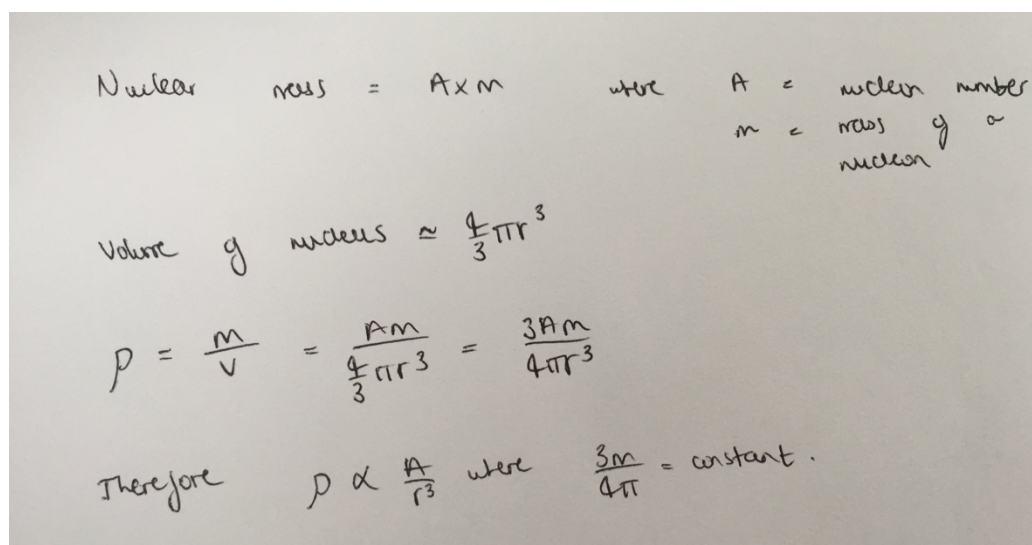
### *$R = R_0 A^{1/3}$ derived from experimental data.*

This equation was derived from experimental data as opposed to theory. A is the nucleon number (mass number), R is the radius. If you draw a graph of R against  $A^{1/3}$ , you will get a straight line with the gradient equal to  $R_0$ , where  $R_0$  is a constant  $= 1.05\text{fm}$ . Also, a graph of  $\ln R$  against  $\ln A$  yields a straight line where the gradient is equal to  $1/3$ , and the y intercept is  $\ln R_0$ .

### *Interpretation of equation as evidence for constant density of nuclear material.*



If you make the assumption that the nucleus is a perfect sphere, then you can say that its volume is  $\frac{4\pi r^3}{3}$ . From this you can determine that its density is proportional to mass number divided by the radius<sup>3</sup>, thus the density will be constant.



Handwritten derivation showing the relationship between nuclear mass, volume, and density.

$$\text{Nuclear mass} = A \times m \quad \text{where } A = \text{nucleon number}$$

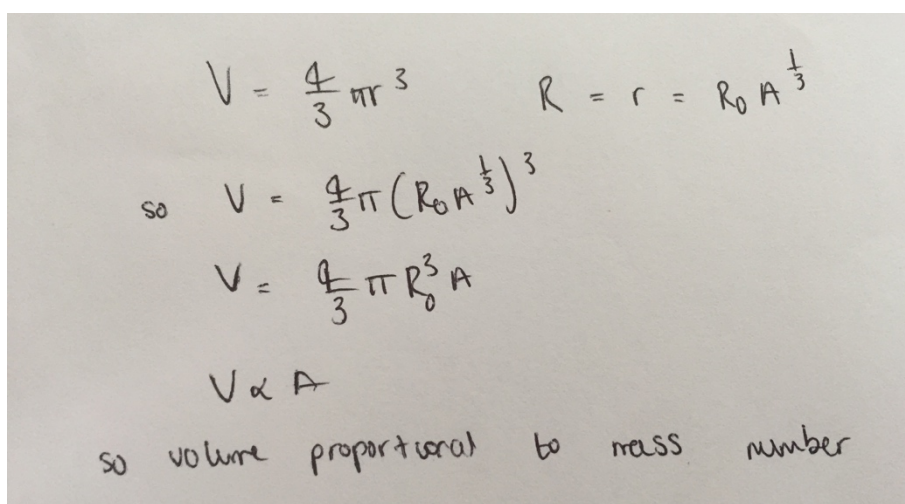
$$m = \text{mass of a nucleon}$$

$$\text{Volume of nucleus} \approx \frac{4\pi r^3}{3}$$

$$\rho = \frac{m}{V} = \frac{Am}{\frac{4\pi r^3}{3}} = \frac{3Am}{4\pi r^3}$$

$$\text{Therefore } \rho \propto \frac{A}{r^3} \quad \text{where } \frac{3m}{4\pi} = \text{constant.}$$

The volume can be shown to be proportional to the mass number, which also alludes to the fact the density of the nucleus is a constant, independent of radius. This means the nucleons are evenly distributed throughout the nucleus.



Handwritten derivation showing the volume of a nucleus is proportional to its mass number.

$$V = \frac{4\pi}{3} r^3 \quad R = r = R_0 A^{\frac{1}{3}}$$

$$\text{so } V = \frac{4\pi}{3} (R_0 A^{\frac{1}{3}})^3$$

$$V = \frac{4\pi}{3} R_0^3 A$$

$$V \propto A$$

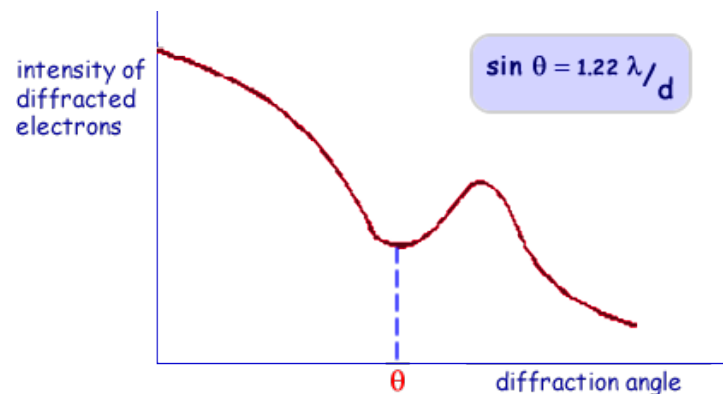
so volume proportional to mass number

### Calculation of nuclear density

We can say that  $V = \frac{4\pi(R_0)^3 A}{3}$ , so density  $= m/v = \frac{3Am}{4\pi(R_0)^3}$ . We know the value of  $R_0$  to be 1.05fm, we can take  $A = 1$  ie for Hydrogen (although any atom will yield the same result), and we can say  $m = u$  (atomic mass unit ie mass of one nucleon). If we plug in those values, you will find the nuclear density to be approximately  $3.4 \times 10^{17} \text{ kgm}^{-3}$ .



*Students should be familiar with the graph of intensity against angle for electron diffraction by a nucleus.*



Source: [http://www.cyberphysics.co.uk/topics/atomic/electron\\_diffraction.html](http://www.cyberphysics.co.uk/topics/atomic/electron_diffraction.html)

It says  $\sin \theta = 1.22 \lambda / d$ , the 1.22, for this specification, can be ignored (unless specified otherwise).

The graph shows intensity of diffracted electrons vs diffraction angle. You should make yourself familiar with this graph as you may be asked to draw it.

Another thing worth noting is the value of  $\theta$  shown on the graph is  $\theta_{\min}$ .

## AQA Unit 5 June 2012 Q5c

### Question:

Nuclear radii have been investigated using  $\alpha$  particles in Rutherford scattering experiments and by using electrons in diffraction experiments.

Make comparisons between these two methods of estimating the radius of a nucleus. Detail of any apparatus used is not required.

For each method your answer should contain:

- The principles on which each experiment is based including a reference to an appropriate equation
- An explanation of what may limit the accuracy of each method
- A discussion of the advantages and disadvantages of each method

The quality of your written communication will be assessed in your answer

### Answer:

#### Principles

- $\alpha$  scattering involves coulomb or electrostatic repulsion
- Electron diffraction treats the electron as a wave having a de Broglie wavelength
- Reference to first minimum for electron diffraction

#### Accuracy

- $\alpha$ 's only measure the least distance of approach, not the radius
- $\alpha$ 's have a finite size which must be taken into account
- Electrons need to have high speed/kinetic energy
- To have a small wavelength or wavelength comparable to nuclear diameter, the wavelength determines the resolution
- The wavelength needs to be of the same order as the nuclear diameter for significant diffraction
- Requirement to have a small collision region in order to measure the scattering angle accurately
- Importance in obtaining monoenergetic beams
- Cannot detect alpha particles with exactly  $180^\circ$  scattering
- Need for a thin sample to prevent multiple scattering

#### Advantages and disadvantages

- $\alpha$ -particle measurements are disturbed by the nuclear recoil
- Mark for  $\alpha$ -particle measurements are disturbed by the SNF when coming close to the nucleus or electrons are not subject to the strong nuclear force.
- A second mark can be given for reference to SNF if they add electrons are leptons or alpha particles are hadrons.
- $\alpha$ 's are scattered only by the protons and not all the nucleons that make up the nucleus

- Visibility – the first minimum of the electron diffraction is often difficult to determine as it superposes on other scattering events

### 3.8.1.6 Mass and energy

#### Content

- Appreciation that  $E = mc^2$  applies to all energy changes,
- Simple calculations involving mass difference and binding energy.
- Atomic mass unit, u.
- Conversion of units;  $1 \text{ u} = 931.5 \text{ MeV}$ .
- Fission and fusion processes.
- Simple calculations from nuclear masses of energy released in fission and fusion reactions.
- Graph of average binding energy per nucleon against nucleon number.
- Students may be expected to identify, on the plot, the regions where nuclei will release energy when undergoing fission/fusion.
- Appreciation that knowledge of the physics of nuclear energy allows society to use science to inform decision making.

#### *Appreciation that $E = mc^2$ applies to all energy changes*

The equation  $E = mc^2$  is probably one of the most famous equations of all time, where  $E$  = energy,  $m$  = mass, and  $c$  = speed of light. It applies to any situation where energy is released or gained from an object. For example, a light bulb radiating 5W of power for a year, would release  $P\Delta t = \text{Work done} = 1.6 \times 10^8 \text{ J}$ . Converting this to equivalent mass,  $E/c^2 = 1.7 \times 10^{-9} \text{ kg}$ , so negligible compared to the mass of the bulb. This example however, serves to show that it can be applied to any energy change. Moreover, an antiparticle and particle annihilating produces 2 photons, where each photon has energy  $mc^2$ .

The energy changes are usually only important in nuclear reactions, as only on this scale are they significant. Any change where energy is released, the total mass after will be less than total mass before (some of the mass is converted to energy).

#### *Simple calculations involving mass difference and binding energy.*

The binding energy of a nucleus is defined as ‘the amount of work that must be done to separate a nucleus into its constituent neutrons and protons.’ When you get a nucleus from individual nucleons, energy is released because the strong nuclear force does work to pull them together. This energy released is what is called the binding energy of the nucleus, and because of this energy released when the nucleus forms from separate nucleons, the mass of a nucleus is marginally less than the mass of the separate nucleons ( $E=mc^2$ ). On the contrary, separating the nucleus into its individual constituents requires a lot of input energy, which is converted back to mass to restore the nucleons to their usual mass outside of the nucleus.

The mass defect ( $\Delta m$ ) of a nucleus is defined as the difference between the mass of the separated nucleons and the mass of the nucleus. The mass defect is equal to the mass of individual protons added to the mass of individual neutrons (outside of the nucleus), subtract the mass of the nucleus (this mass will be lower).

This mass defect is due to the energy released after a nucleus forms from separate nucleons, so you can convert this 'mass defect' to energy using  $E = \Delta mc^2$ , and this will give you the binding energy of a nucleus.

### *Atomic mass unit, u.*

The atomic mass unit is  $1/12^{\text{th}}$  the mass of an atom of carbon-12. Carbon-12 has 6 protons and 6 neutrons, so  $1/12^{\text{th}}$  the mass is the average of the proton and the neutron rest mass. It has the value of approximately  $1.661 \times 10^{-27}\text{kg}$ .

### *Conversion of units; $1 u = 931.5 \text{ MeV}$ .*

The equation  $E = mc^2$  can be used to find equivalent energy value of  $1.661 \times 10^{-27}\text{kg}$ . So it can be said  $E = (1.661 \times 10^{-27}) \times (3 \times 10^8)^2$ . Then to convert this value of E to MeV, we simply divide  $E/1.6 \times 10^{-19} \times 10^6 = 931.5 \text{ MeV}$ .

### *Fission and fusion processes*

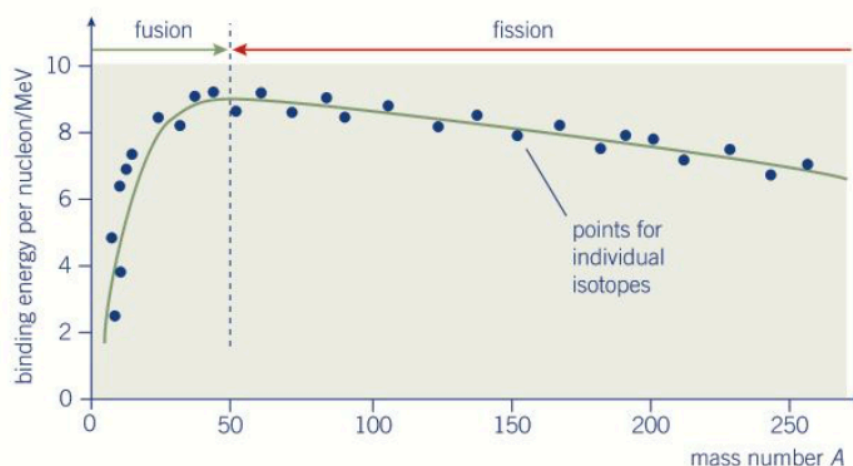
**Nuclear Fission:** this occurs when an unstable, heavy nucleus splits into two fragments of similar size. Fission does not usually occur naturally, and it is induced fission that we can make use of. Induced fission is a process that occurs in U-235 when it is bombarded with neutrons. The only other fissionable isotope is Pu-239.

**Nuclear Fusion:** this is when two nuclei combine to form a larger nucleus. To combine two nuclei together, you must provide huge amounts of energy. This energy is the binding energy, and when two lighter nuclei fuse together, the binding energy per nucleon increases, as nucleus becomes more stable (up to the nuclear mass of iron).

Energy released in nuclear fusion/fission is equal to the change in binding energy.

### *Simple calculations from nuclear masses of energy released in fission and fusion reactions.*

### *Graph of average binding energy per nucleon against nucleon number.*



Source: Kerboodle A Level Physics Textbook

The graph above shows the average binding energy per nucleon vs nucleon number (mass number). Binding energy per nucleon is defined as ‘the average work done per nucleon to remove all of the nucleons from a nucleus.’ Nuclei with a higher binding energy per nucleon are more stable, this is because it takes more energy to separate out the constituent nucleons.

The maximum value on the graph is **8.7 MeV** per nucleon, and this occurs between  **$A = 50$  and  $A = 60$** , nuclei within this range are most stable.

In **fission** – the binding energy per nucleon increases for the two produced fragments, so they are more stable.

In **fusion** – the parent nucleus formed from two daughter nuclei has more binding energy per nucleon. This means it is more stable. This holds true as long as the nucleon number of the nucleus produced is not larger than around 50.

*Students may be expected to identify, on the plot, the regions where nuclei will release energy when undergoing fission/fusion.*

The graph shows the areas of fusion and fission. Fusion takes place until around  $A = 50$ , at around 8.7 MeV.

*Appreciation that knowledge of the physics of nuclear energy allows society to use science to inform decision making.*

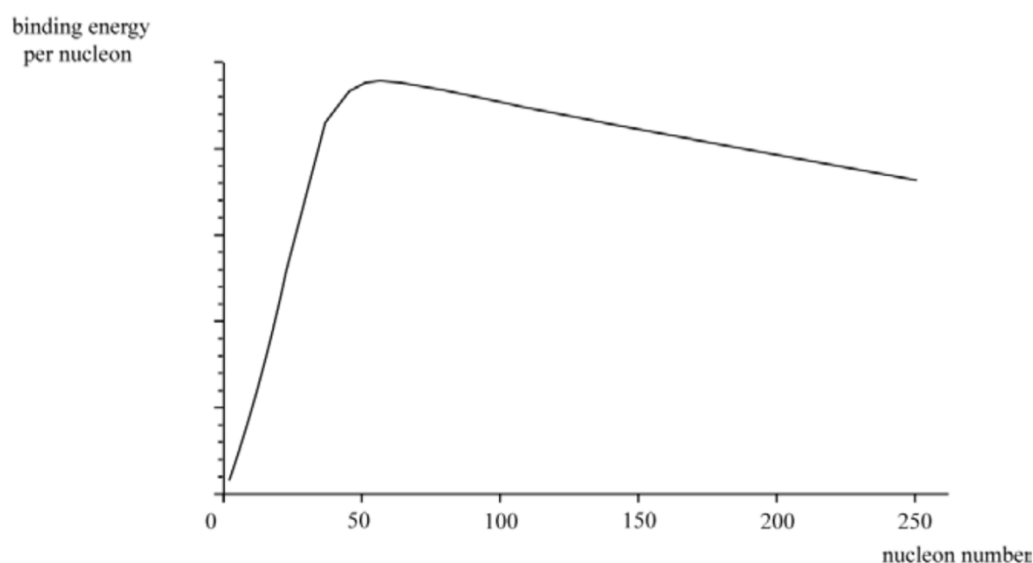
## AQA June 2011 Unit 5

### Question:

**3 (a)** Sketch a graph of binding energy per nucleon against nucleon number for the naturally occurring nuclides on the axes given in **Figure 2**.

Add values and a unit to the binding energy per nucleon axis.

### Answer:



- Peak 8.7 in MeV
- At nucleon number 50 – 60

### Question:

Use the graph to explain how energy is released when some nuclides undergo fission and when other nuclides undergo fusion.

### Answer:

- Energy is released/made available when binding energy **per nucleon** is increased
- In fission a (large) nucleus splits and in fusion (small) nuclei join
- The most stable nuclei are at a peak
- Fusion occurs to the left of peak and fission to the right

## AQA International 2018 Specimen Paper 6.1

### Question:

Describe the changes made inside a nuclear reactor to reduce its power output and explain the process involved.

### Answer:

- insert control rods (further) into the nuclear core / reactor
- which will absorb (more) neutrons (reducing further fission reactions)

### Question:

State the main source of the highly radioactive waste from a nuclear reactor.

### Answer:

Fission fragments / daughter products or spent / used fuel / uranium rods (allow) plutonium (produced from U-238)

### Question:

In a nuclear reactor, neutrons are released with high energies. The first few collisions of a neutron transfers sufficient energy to excite the nuclei of atoms in the reactor.

Describe and explain the nature of the radiation that may be emitted from an excited nucleus

### Answer:

- $\gamma$  (electromagnetic radiation is emitted)
- as the energy gaps are large (in a nucleus) as the nucleus de-excites down discrete energy levels to allow the nucleus to get to the ground level / state

### Question:

The subsequent collisions of a neutron with the moderator are elastic.

Describe what happens to the neutron as a result of these subsequent collisions with the moderator.

### Answer:

- momentum / kinetic energy is transferred (to the moderator atoms)  
or
- a neutron slows down / loses kinetic energy (with each collision)



- (eventually) reaching speeds associated with thermal random motion or reaches speeds which can cause fission (owtte)



### *3.8.1.7 Induced fission*

#### **Content**

- Fission induced by thermal neutrons; possibility of a chain reaction; critical mass.
- The functions of the moderator, control rods, and coolant in a thermal nuclear reactor.
- Details of particular reactors are not required.
- Students should have studied a simple mechanical model of moderation by elastic collisions.
- Factors affecting the choice of materials for the moderator, control rods and coolant. Examples of materials used for these functions.

#### *Fission induced by thermal neutrons; possibility of a chain reaction; critical mass.*

Induced fission was discovered when two scientists attempted to bombard uranium (the heaviest naturally occurring element) with thermal neutrons (neutrons not bound within an atomic nucleus) in an attempt to make it heavier, but noticed that in fact the nucleus split into two fragments of similar size, and two or three more neutrons were also released.

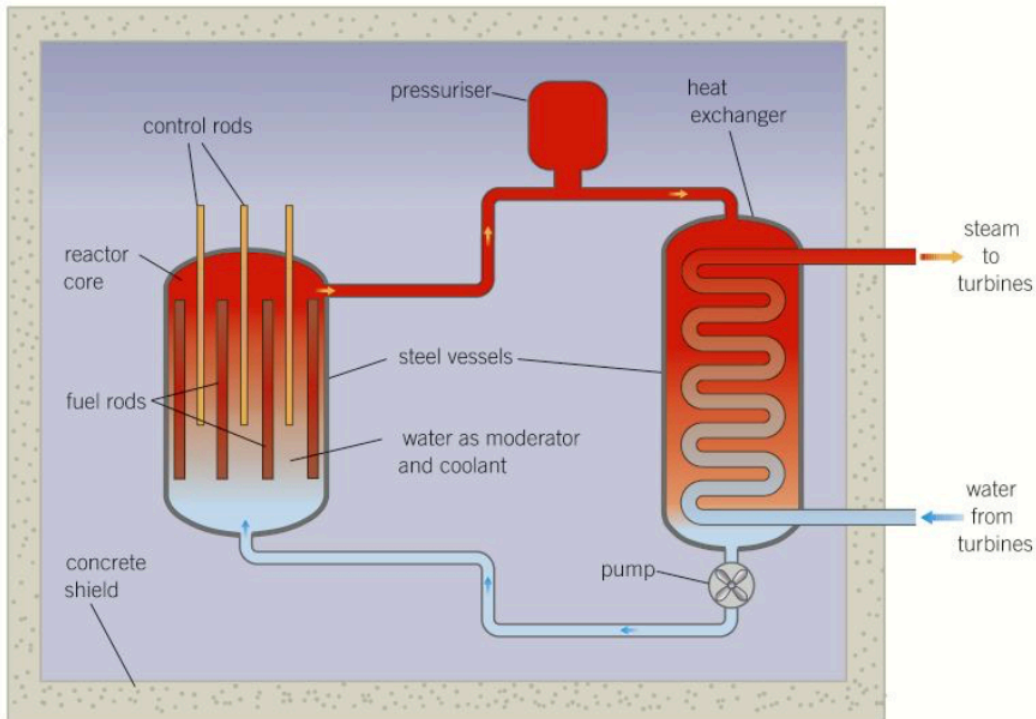
The process of fission releases energy which is utilised in power stations. It begins a chain reaction, as when induced fission occurs in U-235, it splits into two fragments and releases fission neutrons which can go on to cause a further fission event. This ultimately causes a chain reaction where these fission neutrons cause more fission events, which release more fission neutrons that cause more fission events, so can become uncontrollable under certain conditions. Each event releases a vast amount of energy that can be used in nuclear power stations.

For a chain reaction to occur, a ‘critical mass’ must be reached, ie the mass of fissionable material must be greater than this value of mass. This is because not all fission neutrons go on to cause fission, as some are absorbed by other nuclei without fission, and some escape the material without causing fission. Essentially, the point where the chain reaction can become self-sustaining is called the critical mass, where neutrons are not lost at a faster rate than they are formed.

#### *The functions of the moderator, control rods, and coolant in a thermal nuclear reactor.*

An example of a typical thermal nuclear reactor is shown below. Only knowledge of the moderator, control rods and coolant within the reactor is required for this specification. Although it is useful to learn further details to boost understanding.

The reactor core contains fuel rods, control rods and a coolant (water for example). This core is connected to a pressurizer and then heat exchanger where it can then be used to produce steam that can drive turbines and ultimately produce electricity. A pump is used to push the coolant into the core.



Source: Kerboodle A Level Physics Textbook

The fuel rods are so called because they contain the fuel, namely the fissionable U-235, but almost 98% U-238 (which is not fissionable).

The role of the control rods is to absorb neutrons. The depth of the control rods is varied remotely in order to regulate the number of fission neutrons in the reactor core so that the chain reaction does not become out of control. It is regulated to aim towards an average one fission neutron per fission will go on to produce another fission event. This also means the energy produced per unit time takes a fairly constant value. To absorb more neutrons, you must simply lower your control rods.

Also, if the neutrons travel too fast they are unable to cause fission, they need to be travelling at the correct speed which is regulated by the moderator. The moderator in this type of nuclear power station is water, which has the role of slowing neutrons down. The neutrons collide with the atoms of the moderator (water) thus slowing them down.

It is described as a 'thermal' nuclear reactor because fission neutrons are slowed to kinetic energies similar to that of the moderator molecules (to achieve a thermal equilibrium).

The water also acts as a coolant alongside its role as a moderator.

As mentioned already, the mass of fissionable material (U-235) must be at or above the critical mass.

*Details of particular reactors are not required.*

*Students should have studied a simple mechanical model of moderation by elastic collisions.*

In certain nuclear power stations, graphite is used as the moderator. It turns out that the most effective moderators are ones where the mass of the moderator atom is very similar to the mass of the neutron. This is because when they are similar, the neutrons kinetic energy after the collision is similar, but slightly smaller than before the collision as the moderator atoms gain kinetic energy. (Overall kinetic energy remains the same in an elastic collision before and after the event). A good analogy is a snooker table, where one ball strikes another of identical mass, with the second moving off with kinetic energy, and the cue ball stopping dead in its tracks, or moving with much less kinetic energy.

*Factors affecting the choice of materials for the moderator, control rods and coolant. Examples of materials used for these functions.*

The moderator used most commonly is water, but graphite can also be used. Moderators with atoms that have a similar mass to that of the neutron are chosen, for the reasons outlined in the above point. Control rods are chosen for their ability to absorb lots of neutrons without fissioning themselves ie boron or silver. The coolants are chosen to have a large specific heat capacity, so that they can absorb lots of energy without showing a significant change in temperature, hence the reason water is used. CO<sub>2</sub> gas is also used.

### *3.8.1.8 Safety aspects*

#### **Content**

- Fuel used, remote handling of fuel, shielding, emergency shut-down.
- Production, remote handling, and storage of radioactive waste materials.
- Appreciation of balance between risk and benefits in the development of nuclear power.

#### *Fuel used, remote handling of fuel, shielding, emergency shut-down.*

Nuclear reactors have various safety features, as they are potentially catastrophic. Take for example the Chernobyl disaster as evidence of this.

The reactor core must be made of very thick steel that can withstand extreme temperatures and high pressures. The thick steel vessel absorbs neutrons from the core as well as beta radiation and some gamma radiation. The actual core of the whole set up must be built with very thick concrete walls, as to absorb neutrons and the gamma radiation that escapes. Every reactor must also have an emergency shut-down, this inserts the control rods into the core to completely inhibit the fission process. Moreover, the handling of fuel must be done completely remotely. After use they emit beta and gamma radiation which is very dangerous, and before use they emit alpha radiation.

#### *Production, remote handling, and storage of radioactive waste materials.*

Radioactive waste comes in three forms, low, intermediate or high risk. The used radioactive waste can no longer be dispersed into the sea after dilution with water.

**High-level** waste like the used fuel rods are, as mentioned previously, removed remotely and then stored underwater to cool them. This occurs for over a year as they still decay radioactively, so still release heat. They are then stored in containers after the unused uranium/plutonium is extracted. They are usually stored in deep trenches/underground bunkers of sorts.

**Intermediate-level** waste can usually be sealed in containers enclosed by concrete. These are stored in buildings made with more reinforced concrete.

**Low-level** waste can be stored in small metal containers and buried in trenches.

# AQA June 2014 Unit 5 Q5ab

## Question:

A nuclear reactor core is contained in a steel vessel surrounded by concrete. State and explain the purpose of the concrete other than its structural function.

## Answer:

- It forms a (biological) shield to reduce the (intensity of) radiation from/ for protection from
- Neutron (and gamma) radiation

## Question:

A quantity of highly active waste removed from a nuclear reactor consists of similar amounts of two radioisotopes, X and Y.

X has a half-life of about 20 days and emits  $\gamma$  rays and  $\beta$  particles. Y has a half-life of about 20 years and emits  $\alpha$  particles. Assume that both X and Y become relatively stable after their initial decays.

Discuss the problems of storing the waste until it is safe and describe and explain the way in which the waste would normally be treated.

Your account should include details of:

- A comparison of the storage problems associated with X and Y in both the short term and the long term
- How the waste is treated initially at the reactor site and how it could be stored safely for a long time.

The quality of your written communication will be assessed in your answer.

## Answer:

X group

X ( $\beta$  or  $\gamma$ ) needs significant screening(allow lead here)

is highly active

therefore produces heat

as activity  $\propto 1/\text{half-life}$  (only counted once as a mark regardless of which group it is in)

so lasts for a short time quoted as 80 days or more

Y group

Y ( $\alpha$ ) is easy to screen with metal container (if metal is quoted it must be realistic ie not lead)

as activity  $\propto 1/\text{half-life}$  (only counted once)

is active for a very long time quoted as 80 years or more  
problems over container fatigue

Treatment group

By remote control remove waste

initially place in a cooling pond/water tank the water acts as a shield

water dissipates heat/lowers temperature cooling pond is on site/close to source

as activity  $\propto 1/\text{half-life}$ (only counted once) keep for 1 – 3 years –

it will then be cooler

highly active waste will be greatly reduced make suggestions for longer term storage – vitrify  
the active material (to prevent leaking) store underground storage/salt mines in barrels / steel  
containers

geological considerations etc



## Sources:

[http://www.met.reading.ac.uk/pplato2/h-flap/phys6\\_4.html](http://www.met.reading.ac.uk/pplato2/h-flap/phys6_4.html)

[http://abyss.uoregon.edu/~js/glossary/radio\\_telescope.html](http://abyss.uoregon.edu/~js/glossary/radio_telescope.html)

<http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/raylei.html>

<http://www.physicsclassroom.com/class/estatics/Lesson-4/Electric-Field-Intensity>

[https://en.wikipedia.org/wiki/Synchronous\\_orbit](https://en.wikipedia.org/wiki/Synchronous_orbit)

<http://hyperphysics.phy-astr.gsu.edu/hbase/electric/dielec.html>

<http://www.physbot.co.uk/capacitance.html>

<http://www.physbot.co.uk/gravity-fields-and-potentials.html>

[https://en.wikipedia.org/wiki/Magnetic\\_field#/media/File:Earths\\_Magnetic\\_Field\\_Confusion.svg](https://en.wikipedia.org/wiki/Magnetic_field#/media/File:Earths_Magnetic_Field_Confusion.svg)

<http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/cyclot.html>

