Parry APS Q week 3 answer

(1) take out x term in square roots

$$\lim_{x\to\infty} x^{1/2}\left(\sqrt{4x+1}-\sqrt{x+1}-\sqrt{x-1}\right)$$

$$= \lim_{x\to\infty} x^{1/2}\left(2x^{1/2}\left(1+\frac{1}{4x}\right)^{1/2} - x^{1/2}\left(1+\frac{1}{x}\right)^{1/2} - x^{1/2}\left(1-\frac{1}{x}\right)^{1/2}\right)$$

$$= \lim_{x\to\infty} x\left(2\left(1+\frac{1}{4x}\right)^{1/2} - \left(1+\frac{1}{x}\right)^{1/2} - \left(1-\frac{1}{x}\right)^{1/2}\right)$$

Binomial expansion

$$= \lim_{x\to\infty} x\left(2\left(1+\frac{1}{8x}+O\left(\frac{1}{x^2}\right)\right) - \left(1+\frac{1}{2x}+O\left(\frac{1}{x^2}\right)\right) - \left(1-\frac{1}{2x}+O\left(\frac{1}{x^2}\right)\right)\right)$$

$$= \lim_{x\to\infty} x\left(2-1-1+\frac{1}{4x}-\frac{1}{2x}+\frac{1}{2x}+O\left(\frac{1}{x^2}\right)\right)$$

$$= \lim_{x\to\infty}\left(\frac{1}{4}+O\left(\frac{1}{x}\right)\right)$$

$$= \frac{1}{4}$$

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(2) "0/0" ∴ by L'H $= \lim_{x\to 1} \frac{nx^{n-1}+m}{mx^{m-1}+n} = \frac{m+n}{m+n} = 1$

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provided $m+n \neq 0$.

If $m+n=0$ still "0/0" ∴ use L'H again

$$= \lim_{x\to 1} \frac{n(n-1)x^{n-2}}{m(m-1)x^{m-2}} = \frac{n(n-1)}{m(m-1)} = \frac{n-1}{n+1}$$

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If $n=-m=-1$ denominator is identically zero and function not defined

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