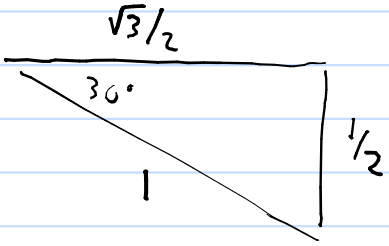
1. (a) $(3-i)(2+i) = 6+1+i(-2+3) = \underline{7+i}$ [1]

(b) $i(2-i)^* = i(2+i) = \underline{-1+2i}$ [1]

(c) $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^3 = \left(e^{-i\pi/6}\right)^3$

$= e^{-i\pi/2} = \underline{-i}$



(d) $\operatorname{arc\,tanh}(-i) = z$ [1]

$-i = \tanh z = \frac{e^{2z}-1}{e^{2z}+1} \Rightarrow e^{2z}(1+i) = 1-i$

$z - i m\pi = \frac{1}{2}\ln\left(\frac{1-i}{1+i}\right) = \frac{1}{2}\ln e^{-i\pi/2} = \underline{-i\frac{\pi}{4}}$. [1]

$z = -i\frac{\pi}{4} + i m\pi$

2. $e^{iz} = 3i \Rightarrow i(z - 2k\pi) = \ln 3 + i\frac{\pi}{2}$

$\Rightarrow \underline{z = \frac{\pi}{2} - i\ln 3 + 2k\pi}$ [1]

3. $\tan z = i \Rightarrow \frac{e^{iz} - e^{-iz}}{i(e^{iz}+e^{-iz})} = i \Rightarrow e^{iz} = 0$

$\underbrace{e^{-y}}_{\neq 0}\underbrace{(\cos x + i\sin x)}_{\neq 0} = 0$

Hence no value of z. [1]

4. $\int e^{-(a+ib)x}\,dx = \frac{e^{-(a+ib)x}}{-(a+ib)}$

$= \frac{-(a-ib)e^{-ax}(\cos bx - i\sin bx)}{a^2+b^2}$

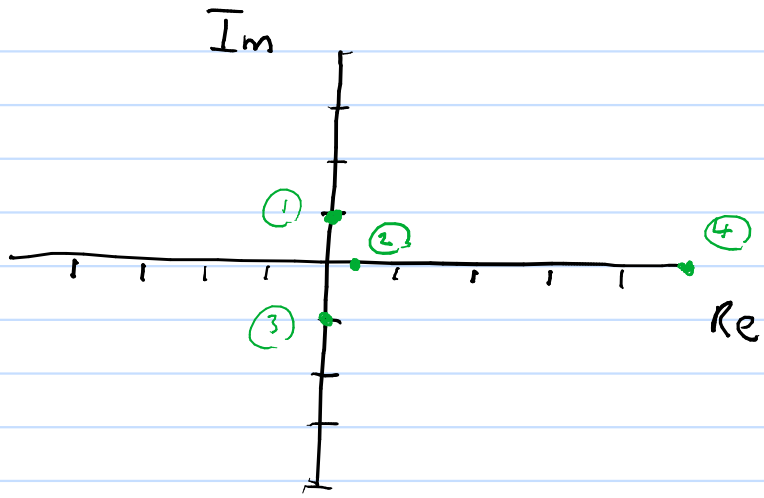
(a) Real part $\rightarrow \int e^{-ax}\cos bx\,dx = \frac{e^{-ax}(-a\cos bx + b\sin bx)}{a^2+b^2}$ [1]

(b) Imag part $\rightarrow \int e^{-ax} \sin bx \, dx = -\dfrac{e^{-ax}(a\sin bx + b\cos bx)}{a^2+b^2}$ [1]

5. (a) $i = e^{i\pi/2}$ ① , $i^i = e^{-\pi/2}$ ② ; $i^{i^i} = e^{-i\pi/2}$ ③ ,

$i^{i^{i^i}} = e^{\pi/2}$ ④ , $i^{i^{i^{i^i}}} = e^{i\pi/2} = i$ ⑤ $= 1$ [1]

(b)



① (0, 1)

② (0·21, 0)

③ (0, −1)

④ (4·8, 0) [1]

Total [10]