PHYSICS 1: MATHEMATICAL ANALYSIS I.
PROBLEMS 3

1. Integrate by parts:

   (a) $x^3 \sin x$; (b) $\tan^{-1} x$.

2. Evaluate the integrals:

   (a) $\int_0^1 (1+x^2)^{-3/2} dx$ (b) $\int_0^\infty \left(1+e^{2x}\right)^{-1} dx$, (c) $\int_1^{3/2} (2-x)^{-1}(x-1)^{-1/2} dx$.

   Hint: In (a) substitute $x = \tan\theta$; in (b) substitute $u = e^{2x}$; in (c) use the substitution $(x-1) = u^2$.

3. Which of the following integrals are convergent?

   (a) $\int_0^1 \ln x \, dx$; (b) $\int_0^1 (x-1)^{-2} \, dx$;

4. Show that

   $$\int x^k \ln x \, dx = \frac{x^{k+1}}{(k+1)^2}\left[(k+1)\ln x - 1\right] + c$$

   where c is a constant and $k \neq -1$.

Questions for Tutorials (week 5)

5* Which of the following integrals are convergent?

   (a) $\int_1^\infty \ln x \, dx$; (b) $\int_0^\infty e^{-ax} \sin bx \, dx$, $(a > 0)$.

6* Integrate

   (a) $\dfrac{x^4}{x^2+1}$; (b) $\dfrac{1}{x \ln x}$.

Maths: Functions, multiple choice assessment

(to be submitted online by Midday 4th November)

Prof. Andy Parry

1) If $xe^y = \cos(xy)$, what is $\frac{dy}{dx}$ at x = 1, y=0?

a) 1

b) 0

c) -1

2) If $f(x) = \frac{2x^2-5x-25}{x^2+x-2}$, at what value of x is there a maximum?

a) -2

b) -5

c) -1

3) In question 2, at what value of x is there a minimum?

a) -2

b) -5

c) -1

4) What is the derivative of $y = x^x$ at x = 2?

a) 4

b) 4ln2

c) 4ln2 + 4

5) Consider the graph r = 1 – cosθ . At what positive value of x does the function have an infinite derivative $\frac{dy}{dx}$?

a) 1/4

b) 1/2

c) 1

All questions are worth 2 marks