Problem sheet 2 Maths (Perry) **marksphysicshelp** MPH

Q 6*

- write $u = \dfrac{x}{1+x}$, $y = \sin^{-1} u$

Chain rule $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = \dfrac{1}{\sqrt{1-u^2}} \dfrac{1}{(1+x)^2}$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{(1+x)\sqrt{1+2x}}}$$

- $y = \sec^{-1} x \Rightarrow \sec y = x$

Thus $\sec y \tan y \dfrac{dy}{dx} = 1 \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\sec y \sqrt{\sec^2 y - 1}}$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}}$$

Q 7* Factorise

$$f(x) = \frac{(2x+5)(x-5)}{(x+2)(x-1)}$$

- Thus f is 0 at $x = 5$, $x = -5/2$

- near $x = -2$ $\quad f \approx \dfrac{7}{3(x+2)} \Rightarrow \to +\infty$ as $x \to -2^+$
  $\to -\infty$ as $x \to -2^-$

  near $x = 1$ $\quad f \approx \dfrac{-28}{3(x-1)} \Rightarrow \to -\infty$ as $x \to 1^+$
  $\to +\infty$ as $x \to 1^-$

- For large x behaviour write

$$f(x) = 2\left(1+\frac{5}{2x}\right)\left(1-\frac{5}{x}\right)\left(1+\frac{2}{x}\right)^{-1}\left(1-\frac{1}{x}\right)^{-1}$$

i.e. pull out "x" in each term

Expand $= 2\left(1+\dfrac{5}{2x}\right)\left(1-\dfrac{5}{x}\right)\left(1-\dfrac{2}{x}+\cdots\right)\left(1+\dfrac{1}{x}+\cdots\right)$

$= 2 - \dfrac{7}{x} + \cdots$ Thus $f \to 2^+$ as $x \to -\infty$

2^- as $x \to +\infty$

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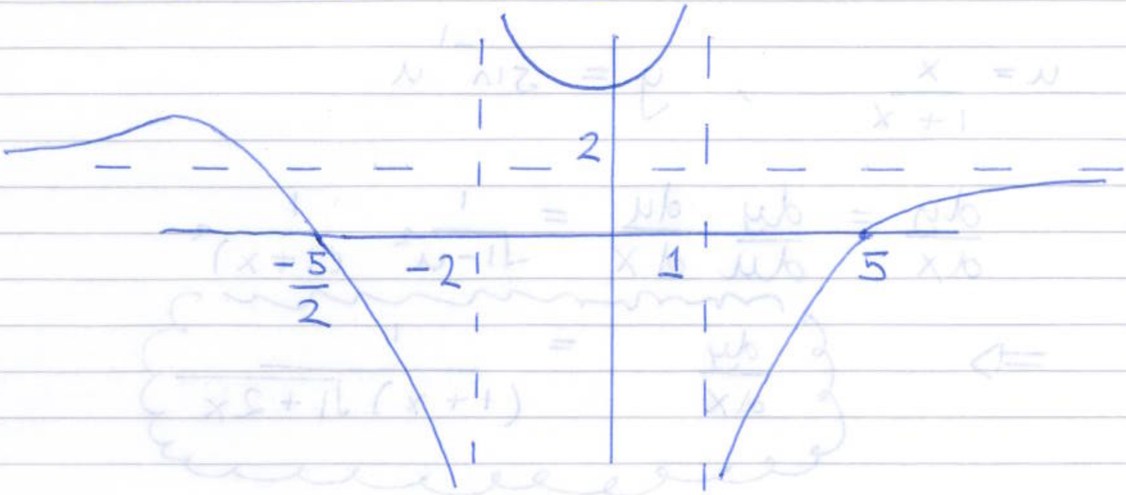
Putting together all above $f(x)$ must look like



$f(0) = \frac{25}{2}$

Must have one maximum & one minimum as shown

Location found from $f'(x) = 0$

writing $f(x) = \dfrac{2x^2 - 5x - 25}{x^2 + x - 2} = \dfrac{u}{v}$

$\Rightarrow f' = \dfrac{vu' - uv'}{v^2} = \dfrac{(x^2+x-2)(4x-5) - (2x^2-5x-25)(2x+1)}{(x^2+x-2)^2}$

numerator simplifies to $7x^2 + 42x + 35$

$\Rightarrow f'(x) = 0 \Leftrightarrow 7(x+1)(x+5) = 0$

Max^m at $x = -5$

min^n at $x = -1$