Problem sheet 3 Maths (Parry)

Q 5*

a) write $I = \lim\limits_{N\to\infty} \int_1^N \ln x \, dx = \lim\limits_{N\to\infty} \int_1^N \underset{\substack{\uparrow \\ \frac{dv}{dx}}}{1} . \underset{\substack{\uparrow \\ u}}{\ln x} \, dx$ by parts

$$= \lim_{N\to\infty} \Big[x \ln x - 1 \Big]_1^N$$

$$= \lim_{N\to\infty} N \ln N - N + 1$$ <u>Diverges</u>

b) write $I = \lim\limits_{N\to\infty} \int_1^N e^{-ax} \sin bx \, dx$

Integral can be done by parts (requires 2 integrations) or using complex numbers

$$\int e^{-ax} \sin bx \, dx = \operatorname{Im} \int e^{(ib-a)x} dx = \operatorname{Im} \frac{e^{(ib-a)x}}{ib-a}$$

$$= \operatorname{Im} \cdot \frac{e^{-ax}}{a^2+b^2} (-a-ib)(\cos bx + i \sin bx)$$

$$\underline{I} = \left[\frac{e^{-ax}}{a^2+b^2} (-a \sin bx - b \cos bx) \right]_0^N \quad \text{with } N \to \infty$$

In $\lim N \to \infty$, e^{-aN} term vanishes leaving

$$\underline{I} = \frac{b}{a^2+b^2}$$ <u>converges</u>

Q 6*

a) rewrite $\dfrac{x^4}{x^2+1} = \dfrac{x^4-1+1}{x^2+1} = \dfrac{(x^2-1)(x^2+1)+1}{x^2+1}$

$$= x^2 - 1 + \frac{1}{x^2+1}$$

Thus Integral $= \dfrac{x^3}{3} - 1 + \tan^{-1} x + C$

b) let $u = \ln x$ $\therefore \displaystyle\int \frac{dx}{x \ln x} = \int \frac{du}{u} = \ln u + C$

$\dfrac{du}{dx} = \dfrac{1}{x}$ $\qquad = \ln \ln x + C$