Sheet 4 Maths (Parity)

5*

$$L = \int dx \sqrt{1 + y'^2}$$

$$= \int dx \sqrt{1 + \left(\frac{3x^2}{a^2} - \frac{a^2}{12x^2}\right)^2} = \int dx \sqrt{1 + \left(\frac{9x^4}{a^4} + \frac{a^4}{144x^4} - \frac{1}{2}\right)}$$

$$= \int dx \sqrt{\frac{9x^4}{a^4} + \frac{a^4}{144x^4} + \frac{1}{2}} = \int dx \sqrt{\left(\frac{3x^2}{a^2} + \frac{a^2}{12x^2}\right)^2}$$

$$= \int_{a/2}^{a} dx \left(\frac{3x^2}{a^2} + \frac{a^2}{12x^2}\right) = \left[\frac{x^3}{a^2} - \frac{a^2}{12x}\right]_{a/2}^{a} = \frac{23a}{24}$$

6* let $x = \frac{\pi}{2} - y \Rightarrow dx = -dy$

$$I = -\int_{\pi/2}^{0} dy \frac{\sin^{1/3}\left(\frac{\pi}{2} - y\right)}{\sin^{1/3}\left(\frac{\pi}{2} - y\right) + \cos^{1/3}\left(\frac{\pi}{2} - y\right)}$$

$$= +\int_{0}^{\pi/2} dy \frac{\cos^{1/3} y}{\cos^{1/3} y + \sin^{1/3} y}$$

Note y is dummy variable $\therefore$ $I = \int_0^{\pi/2} dx \frac{\cos^{1/3} x}{\cos^{1/3} x + \sin^{1/3} x}$

Add 2 expressions for I

$$2I = \int_0^{\pi/2} dx \frac{\cos^{1/3} x + \sin^{1/3} x}{\cos^{1/3} x + \sin^{1/3} x} = \int_0^{\pi/2} dx = \frac{\pi}{2}$$