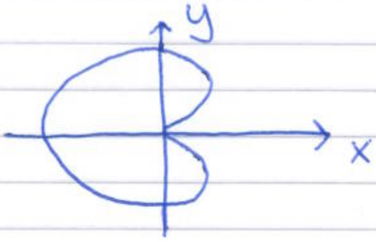
Sheet 5 Maths (Parry 1)

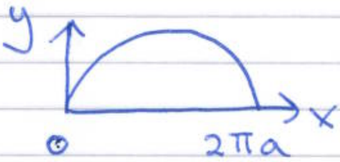
5* $$L = \int_0^{2\pi} d\theta \sqrt{r^2 + \left(\frac{\partial r}{\partial \theta}\right)^2} = 2\int_0^{\pi} d\theta \sqrt{a^2(1+\cos^2\theta - 2\cos\theta) + a^2\sin^2\theta}$$

$$= 2a\int_0^{\pi} d\theta \sqrt{2(1-\cos\theta)} = 2a\int_0^{\pi/2} d\theta \sqrt{4\sin^2\frac{\theta}{2}}$$

$$= 4a\int_0^{\pi/2} d\theta \sin\frac{\theta}{2} = 8a\left[-\cos\frac{\theta}{2}\right]_0^{\pi} = 8a$$



6*



Length $$L = \int_0^{2\pi} dt \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$= 2\int_0^{\pi} dt \sqrt{a^2(1-\cos t)^2 + a^2\sin^2 t} = 2a\int_0^{\pi} dt \sqrt{2(1-\cos t)}$$

same integral as in Q 5* $= 8a$

Surface of revolution $S = 2\pi \int dx \, y \sqrt{1 + y'^2}$

Parametrically $S = 2\pi \int dt \frac{dx}{dt} y \sqrt{1 + \frac{\dot{y}^2}{\dot{x}^2}}$

$$= 2\pi \int_0^{2\pi} dt \, y \sqrt{\dot{x}^2 + \dot{y}^2}$$

$$= 4\pi a^2 \int_0^{\pi} dt \, (1-\cos t)\sqrt{2(1-\cos t)} = 16\pi a^2 \int_0^{\pi} dt \sin^3\frac{t}{2}$$

$$= 32\pi a^2 \int_0^{\pi/2} dz \sin^3 z = 32\pi a^2 \int_0^{\pi/2} dz \sin z \, (1-\cos^2 z)$$

$$= 32\pi a^2 \left[-\cos z + \frac{\cos^3 z}{3}\right]_0^{\pi/2} = \frac{64\pi a^2}{3}$$