Sheet 6 Maths (Parity)

6* $u = x\ln(x^2+y^2) - 2y\tan^{-1}\frac{y}{x}$

Thus
$$\frac{\partial u}{\partial x} = \ln(x^2+y^2) + \frac{2x^2}{x^2+y^2} + \frac{2y}{1+\frac{y^2}{x^2}}\cdot\frac{y}{x^2}$$

$$= \ln(x^2+y^2) + 2$$

Similarly
$$\frac{\partial u}{\partial y} = \frac{2xy}{x^2+y^2} - 2\tan^{-1}\frac{y}{x} - \frac{2y}{1+\frac{y^2}{x^2}}\cdot\frac{1}{x}$$

$$= -2\tan^{-1}\frac{y}{x}$$

Hence $xu_x + yu_y = x\ln(x^2+y^2) + 2x - 2y\tan^{-1}y/x$
$$= u + 2x \quad ✓$$

7* Total derivative

$$\frac{\partial f}{\partial p}dp + \frac{\partial f}{\partial V}dV + \frac{\partial f}{\partial T}dT = 0$$

If $dT = 0$ (constant T) $\frac{\partial f}{\partial p}dp + \frac{\partial f}{\partial V}dV = 0$

$$\Rightarrow \left(\frac{\partial p}{\partial V}\right)_T = -\frac{(\partial f/\partial V)_{p,T}}{(\partial f/\partial p)_{V,T}}$$

If $dV = 0$ (const V)

$$\frac{\partial f}{\partial p}dp + \frac{\partial f}{\partial T}dT = 0 \Rightarrow \left(\frac{\partial T}{\partial p}\right)_V = -\frac{(\partial f/\partial p)_{VT}}{(\partial f/\partial T)_{V,p}}$$

If $dp = 0$ (const p)

$$\frac{\partial f}{\partial V}dV + \frac{\partial f}{\partial T}dT = 0 \Rightarrow \left(\frac{\partial V}{\partial T}\right)_p = -\frac{(\partial f/\partial T)_{V,p}}{(\partial f/\partial V)_{p,T}}$$

Taking product all terms cancel)

$$\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1$$