

$$6^* \quad u = x \ln(x^2 + y^2) - 2y + \arctan \frac{y}{x}$$

Thus

$$\begin{aligned} \frac{\partial u}{\partial x} &= \ln(x^2 + y^2) + \frac{2x}{x^2 + y^2} + \frac{2y}{1 + \frac{y^2}{x^2}} \cdot \frac{y}{x^2} \\ &= \ln(x^2 + y^2) + 2 \end{aligned}$$

Similarly

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{2xy}{x^2 + y^2} - 2 + \arctan \frac{y}{x} - \frac{2y}{1 + \frac{y^2}{x^2}} \cdot \frac{1}{x} \\ &= -2 + \arctan \frac{y}{x} \end{aligned}$$

$$\begin{aligned} \text{Hence } xu_x + yu_y &= x \ln(x^2 + y^2) + 2x - 2y + \arctan y/x \\ &= u + 2x \quad \checkmark \end{aligned}$$

7\* Total derivative

$$\frac{\partial f}{\partial p} dp + \frac{\partial f}{\partial V} dV + \frac{\partial f}{\partial T} dT = 0$$

$$\text{If } dT = 0 \quad (\text{constant } T) \quad \frac{\partial f}{\partial p} dp + \frac{\partial f}{\partial V} dV = 0$$

$$\Rightarrow \left( \frac{\partial p}{\partial V} \right)_T = - \frac{(\partial f / \partial V)_{p,T}}{(\partial f / \partial p)_{V,T}}$$

If  $dV = 0$  (const  $V$ )

$$\frac{\partial f}{\partial p} dp + \frac{\partial f}{\partial T} dT = 0 \Rightarrow \left( \frac{\partial T}{\partial p} \right)_V = - \frac{(\partial f / \partial p)_{V,T}}{(\partial f / \partial T)_{V,p}}$$

If  $dp = 0$  (const  $p$ )

$$\frac{\partial f}{\partial V} dV + \frac{\partial f}{\partial T} dT = 0 \Rightarrow \left( \frac{\partial V}{\partial T} \right)_p = - \frac{(\partial f / \partial T)_{V,p}}{(\partial f / \partial V)_{p,T}}$$

Taking product all terms cancel

$$\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1$$

$$\frac{p}{x} \cdot \frac{p \Delta + \frac{x \Delta + (p+x) n}{p+x}}{\frac{p \Delta + \frac{x \Delta + (p+x) n}{p+x}}{p+x}} = \frac{p \Delta}{x \Delta}$$

$$\Delta + (p+x) n =$$

$$\frac{1}{x} \cdot \frac{p \Delta - \frac{p \Delta + \frac{x \Delta + (p+x) n}{p+x}}{x}}{\frac{p \Delta + \frac{x \Delta + (p+x) n}{p+x}}{p+x}} = \frac{p \Delta}{p \Delta}$$

$$\frac{p}{x} \Delta + \Delta - =$$

$$x \Delta + p \Delta - x \Delta + (p+x) n x = x p \Delta + x n \Delta$$

$$\Delta + n =$$

Total derivative

$$0 = T_b \frac{\Delta \delta}{T \delta} + V_b \frac{\Delta \delta}{V \delta} + q_b \frac{\Delta \delta}{q \delta}$$

$$0 = V_b \frac{\Delta \delta}{V \delta} + q_b \frac{\Delta \delta}{q \delta} \quad (T \text{ fixed}) \quad 0 = T_b \quad \text{I}$$

$$\frac{T_p(V \delta \Delta \delta)}{T_p(q \delta \Delta \delta)} = \left(\frac{q \delta}{V \delta}\right) \Delta \delta$$

$$(V \text{ fixed}) \quad 0 = V_b \quad \text{II}$$

$$\frac{V_p(q \delta \Delta \delta)}{V_p(T \delta \Delta \delta)} = \left(\frac{T \delta}{q \delta}\right) \Delta \delta \quad 0 = T_b \frac{\Delta \delta}{T \delta} + q_b \frac{\Delta \delta}{q \delta}$$

$$(q \text{ fixed}) \quad 0 = q_b \quad \text{III}$$

$$\frac{q_p(T \delta \Delta \delta)}{q_p(V \delta \Delta \delta)} = \left(\frac{V \delta}{T \delta}\right) \Delta \delta \quad 0 = T_b \frac{\Delta \delta}{T \delta} + V_b \frac{\Delta \delta}{V \delta}$$