Sheet 7 Maths (Parry)

7*

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s}$$

$$= t\frac{\partial u}{\partial x} - \frac{2t}{(s-t)^2}\frac{\partial u}{\partial y} \qquad (1)$$

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t}$$

$$= s\frac{\partial u}{\partial x} + \frac{2s}{(s-t)^2}\frac{\partial u}{\partial y} \qquad (2)$$

Thus $\quad s\frac{\partial u}{\partial s} + t\frac{\partial u}{\partial t} = s(1) + t(2)$

$$= st\frac{\partial u}{\partial x} - \frac{2st}{(s-t)^2}\frac{\partial u}{\partial y} + st\frac{\partial u}{\partial x} + \frac{2st}{(s-t)^2}\frac{\partial u}{\partial y}$$

$$= 2st\frac{\partial u}{\partial x}$$

$$= 2x\frac{\partial u}{\partial x}$$

Similarly $\quad \frac{1}{s}\frac{\partial u}{\partial t} - \frac{1}{t}\frac{\partial u}{\partial s} = \frac{1}{s}(2) - \frac{1}{t}(1)$

$$= \left(\frac{\partial u}{\partial x} + \frac{2}{(s-t)^2}\frac{\partial u}{\partial y}\right) - \left(\frac{\partial u}{\partial x} + \frac{2}{(s-t)^2}\frac{\partial u}{\partial y}\right)$$

$$= \frac{4}{(s-t)^2}\frac{\partial u}{\partial y}$$

$$= \frac{4}{(s-t)^2}\frac{(s+t)}{(s+t)}\frac{\partial u}{\partial y} = \frac{4y}{(s^2-t^2)}\frac{\partial u}{\partial y}$$