

Sheet 8 Maths (Perry)

$$5^* \quad u = x^4 + 4x^2y^2 - 2x^2 + 2y^2 - 1$$

$$\frac{\partial u}{\partial x} = 4x^3 + 8xy^2 - 4x$$

$$\frac{\partial u}{\partial y} = 8x^2y + 4y$$

Require $\frac{\partial u}{\partial x} = 0$ and $\frac{\partial u}{\partial y} = 0$ for stationary pt

$$4x(x^2 + 2y^2 - 1) = 0 \quad (1)$$

$$4y(2x^2 + 1) = 0 \quad (2)$$

(2) $\Rightarrow y = 0$ and sub into (1) leaves $x(x^2 + 1) = 0 \Rightarrow 0, \pm 1$

$$(0, 0), (1, 0), (-1, 0)$$

$$E = u_{xy}^2 - u_{xx}u_{yy}$$

$$= (16xy)^2 - (12x^2 + 8y^2 - 4)(8x^2 + 4)$$

At $(0, 0)$ $E = 16$ saddle stationary

At $(\pm 1, 0)$ $E = -96$ max/min stationary

but at $(1, 0)$ $u_{xx} = 8 \Rightarrow$ min

" $(-1, 0)$ $u_{xx} = 8 \Rightarrow$ min

$\therefore$ 1 saddle, 2 mins