Mathematical Analysis 2017-8
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Tutorial problems 1 - Sets and proof

Question 1.

Consider the following sets $A = \emptyset$, $B = \{\emptyset\}$, $C = \{\emptyset, \{\emptyset\}\}$, $D = \{\emptyset, \{\emptyset\}, \{2,3\}\}$, $E = \{\{1\}, 2, 3\}$, $F = \{2,3\}$, $G = \{\{2,3\}\}$.

Are the following statements true or false;

a) $\{1\} \subseteq E$

b) $F \subset E$

c) $2 \in D$

d) $F \in D$

e) $D \setminus F = C$

f) $A \cup B = C$

Question 2.

Use contradiction to prove the following;

Proposition: Given that $n \in \mathbb{N}^+$ is not a prime number, then one of its prime factors is at most $\sqrt{n}$.

[Note: this is an important fact when designing prime factoring algorithms, such as in cryptography.]

Question 3.

Prove using induction that the sum of the cubes of 3 consecutive natural numbers is divisible by 9.

Question 4.

For a finite set A, prove $|2^A| = 2^{|A|}$ by induction.

For fun only:

If you have time, have a look at a very peculiar logic problem and a possible solution by induction!

https://terrytao.wordpress.com/2011/04/07/the-blue-eyed-islanders-puzzle-repost/