Mathematical Analysis 2017-8

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Tutorial problems 4 - Sequences

Question 1.

Prove the following using $\epsilon - N$ (ie. from first principles, don't assume any other results about convergent sequences);

1. The constant sequence (k_n), with $k_n = k$ for some $k \in \mathbb{R}$, converges to k.
2. Let $a_n = \frac{1}{n^2}$. Then $a_n \to 0$ as $n \to \infty$.
3. Let $b_n = n^2$. Then the sequence (b_n) does not converge.
4. Let $c_n \to c$ and $d_n \to d$ be convergent sequences. Let $x_n = 2c_n - 3d_n$. Then $x_n \to 2c - 3d$ as $n \to \infty$.
5. Let $w > 1$ and $u_n = w^{\frac{1}{n}}$. Then $u_n \to 1$ as $n \to \infty$.
6. Let $0 < z < 1$ and $v_n = z^{\frac{1}{n}}$. Then $v_n \to 1$ as $n \to \infty$.

Question 2.

Prove that the sequence, (x_n), defined by,

$$x_{n+1} = \frac{3x_n - x_{n-1}}{2} , \quad n \geq 2$$

converges for any real values of x_1, x_2.

[Hint: show it is a Cauchy sequence.]

Question 3.

Prove that the sequence, (x_n), defined by,

$$x_{n+1} = \arctan x_n , \quad n > 1$$

converges for any $x_1 \in \mathbb{R}$.

[Hint: Sketch a graph of arctan to figure out how this sequence behaves. You may use the fact that $\arctan x < x$ for $x > 0$.]