Vectors Questions (to supplement 'M1 Questions by Topic : Vectors')

- ☐ M1 June 2017 Q7 (IAL)
- ☐ M1 June 2008 Q5
- ☐ M1 June 2014 Q6
- ☐ M1 Jan 2017 Q3 (IAL)
- ☐ M1 Jan 2006 Q4
- ☐ M1 Oct 2017 Q4 (IAL)

questions that require method ① or method ②

vector diagram

using vectors and their magnitudes

resolving

using components of vectors (usually horizontal & vertical components)

- ☐ M1 June 2017 Q1
- ☐ M1 June 2015 Q1 (IAL)
- ☐ M1 June 2009 Q2
- ☐ M1 Jan 2012 Q3
- ☐ M1 June 2016 Q5 (IAL)
- ☐ M1 Jan 2014 Q7 (IAL)
- ☐ M1 Jan 2009 Q6

questions requiring algebra predominantly (no vector addition / subtraction diagram).

☐ Extra notes and questions at the end: M1 Jan 2007 Q1 for comparison / explanation.
challenge p96 A2

For these questions (except Jan 2006 Q4) we are given a vector diagram (we know we are dealing with vectors because the letters are bold in the question). Given one of these problems we have 2 ways of solving.

① Draw a vector diagram

→ This requires knowledge of how to add vectors (tip-to-tail method) and also ow to form a vector that represents one vector subtracted from another ($\underline{A} - \underline{B}$). e must also know that the length of a vector is its magnitude. nd we must know the sine and cosine rule.

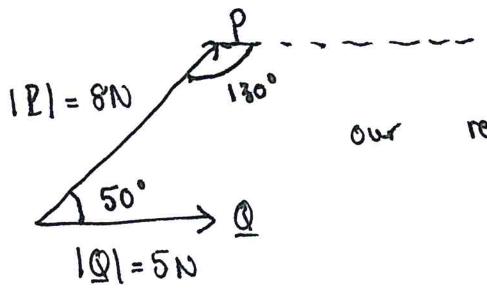
② Resolve in 2 directions (horizontal & vertical usually).

→ This method is much less intuitive than method ①. I will provide some xamples of how to solve using method ②. But it really is worth etting your head around method ①, it turns out to be much simpler nd much quicker.

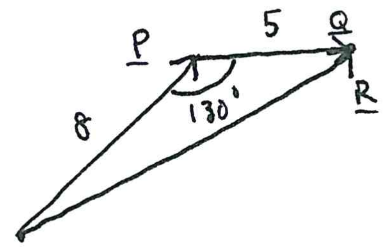
n the example Jan 2006 Q4, you must recognise that the question is leading ou to draw a diagram. Also you are given vectors and their magnitudes, so e have enough information to use method ① (involving the sine & cosine rule usually)

M1 June 2017 Q7 (IAL)

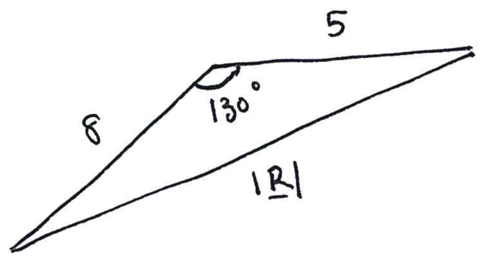
see 'vector addition' online if you are unsure



our resultant vector $\underline{R} = \underline{P} + \underline{Q}$, and we add vectors like this

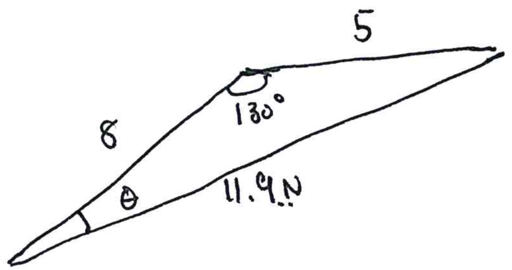


o we end up with a triangle that we can apply the sine and cosine rule on to find
ur missing sides (magnitudes) and angles.



Hence $|\underline{R}| = 8^2 + 5^2 - 2(8)(5)\cos 130 = 11.9\,N$ (3sf)

reads 'magnitude of $\underline{R}$'



Hence $\dfrac{\sin\theta}{5} = \dfrac{\sin 130}{11.85..}$

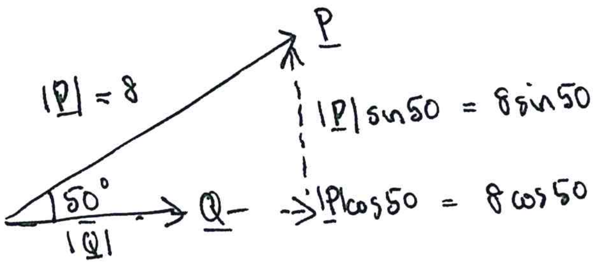
$\sin\theta = \dfrac{5}{11.85..}\sin 130$

$\theta = \sin^{-1}\left(\dfrac{5\sin 130}{11.85..}\right)$

$= 19°$. (2sf)

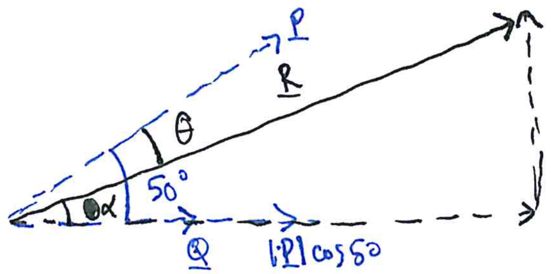
Alternative method : method (2)

We could find $|\underline{R}|$ and θ (angle between $\underline{R}$ and $\underline{P}$) in a different way (a slightly more complicated way IMO.)



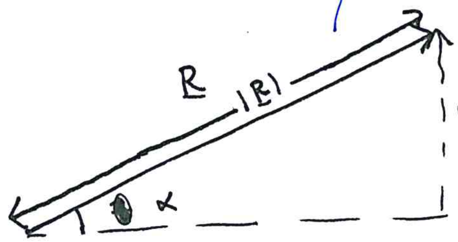
We can see intuitively that our resultant vector is going to have a horizontal component $8\cos 50 + |\underline{Q}|$ and vertical component $8\sin 50$.

It will look like this :



$\Rightarrow$

remember the length of a vector is the magnitude of that vector.



$|\underline{R}|\sin\alpha = |\underline{P}|\sin 50 = 8\sin 50$

$|\underline{R}|\cos\alpha = |\underline{Q}| + |\underline{P}|\cos 50$
$= 5 + 8\cos 50$

Hence (by pythagoras) $(|\underline{R}|\cos\alpha)^2 + (|\underline{R}|\sin\alpha)^2 = |\underline{R}|^2$

$$|\underline{R}|^2 = (5 + 8\cos 50)^2 + (8\sin 50)^2 = (10.14^2 + 6.13^2)$$

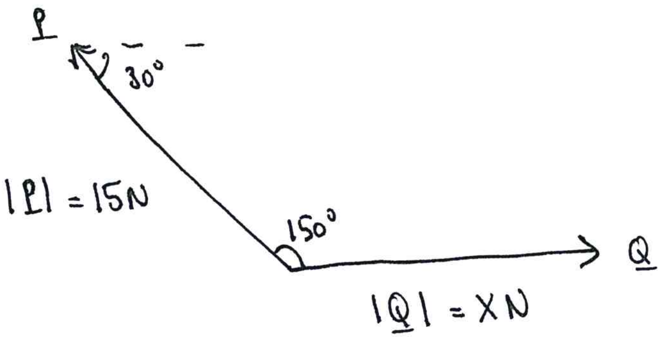
$$|\underline{R}| = 11.9\text{N} \quad (3\text{sf})$$

$\tan\alpha = \dfrac{8\sin 50}{5 + 8\cos 50} \Rightarrow \alpha = 31.1\ldots^\circ$ Hence $\theta = 50 - \alpha = 19^\circ$ (2sf)

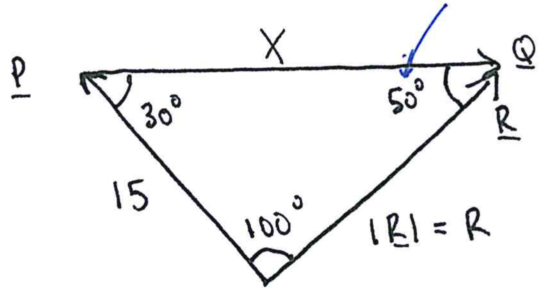
from diagram.

M1 June 2008 Q5

They tell us this in the question (angle between Q and R)



$\Rightarrow$



$$\frac{R}{\sin 30} = \frac{15}{\sin 50}$$

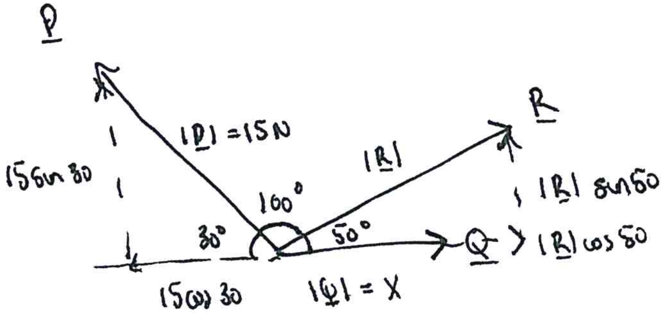
$$R = 9.79\text{N} \quad (3sf)$$

$$\frac{X}{\sin 100} = \frac{15}{\sin 50} \quad \left(= \frac{R}{\sin 30} \right)$$

$$X = 19.3\text{N} \quad (3sf)$$

Alternative method : (essentially we resolve horizontally and vertically)

method ②



What we have done here is looked at a particular direction (horizontal or vertical) and resolved the forces. ie we have said ΣF = resultant force (in a particular direction)

So $(\rightarrow)$ $X - 15\cos 30 = |\underline{R}|\cos 50$ ①

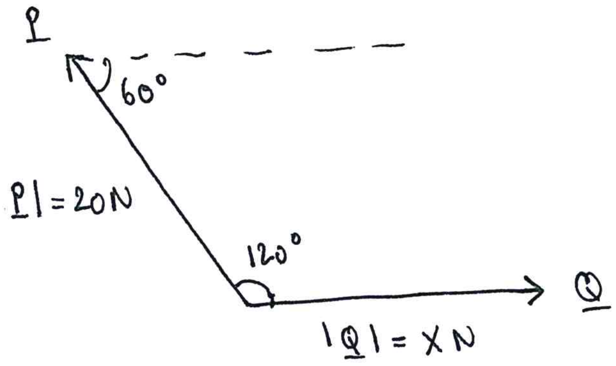
$(\uparrow)$ $15\sin 30 = |\underline{R}|\sin 50$ ② from ② $|\underline{R}| = 9.79\text{N}$ (3sf)

Hence, for X we sub $|\underline{R}|$ into eq ① for $X = 19.3\text{N}$ (3sf)

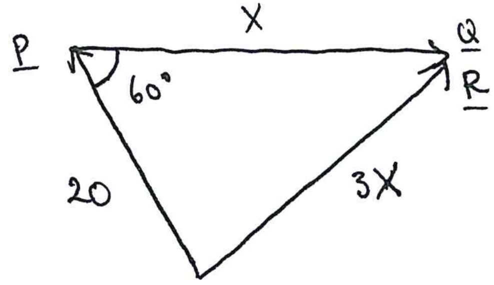
This method I would only recommend as a back-up. Use your knowledge of how to add (or subtract) vectors and then use the sine rule or cosine rule to work out missing sides. (Note that the length of a vector represents its magnitude.)

'Vectors, what even are they? | Essence of linear algebra, chapter 1' provides a great explanation of vectors and vector addition.

M1 June 2014 Q6



$\Rightarrow$ $\underline{P} + \underline{Q}$ is drawn like this



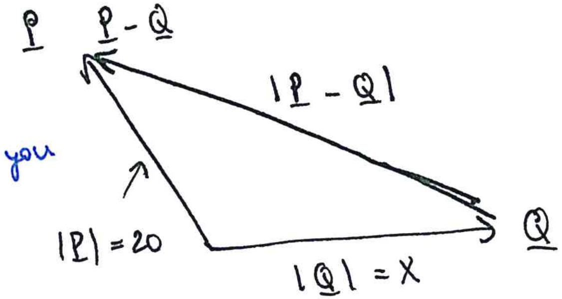
$$(3X)^2 = (X)^2 + (20)^2 - 2(20)(X)\cos 60$$

$$9X^2 = X^2 + 400 - 20X$$

$$\Rightarrow 8X^2 + 20X - 400 = 0$$

$$2X^2 + 5X - 100 = 0 \quad \Rightarrow \quad X = \frac{-5 \pm \sqrt{25 - 4(2)(-100)}}{2(2)} = 5.93 \ (3sf)$$

Draw $\underline{P} - \underline{Q}$ $\Rightarrow$



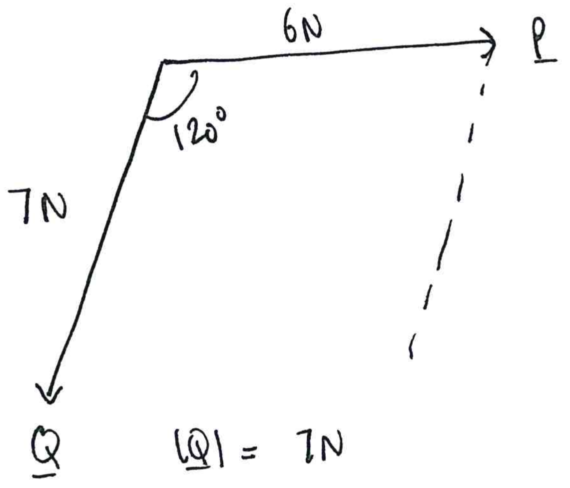
(see 'vector subtraction' online if you are unsure)

so
$$|\underline{P} - \underline{Q}|^2 = (20)^2 + (X)^2 - 2(X)(20)\cos 120$$

$$|\underline{P} - \underline{Q}| = \sqrt{20^2 + X^2 - 40X\cos 120}$$

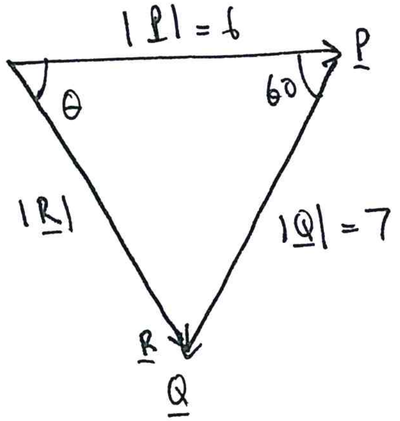
$$= 23.5\,N \;(3sf)$$

Jan 2017 Q3 IAL



$|\underline{P}| = 6N$

$|\underline{Q}| = 7N$



) resultant force :

o $|\underline{R}|^2 = 6^2 + 7^2 - 2(6)(7)\cos 60$

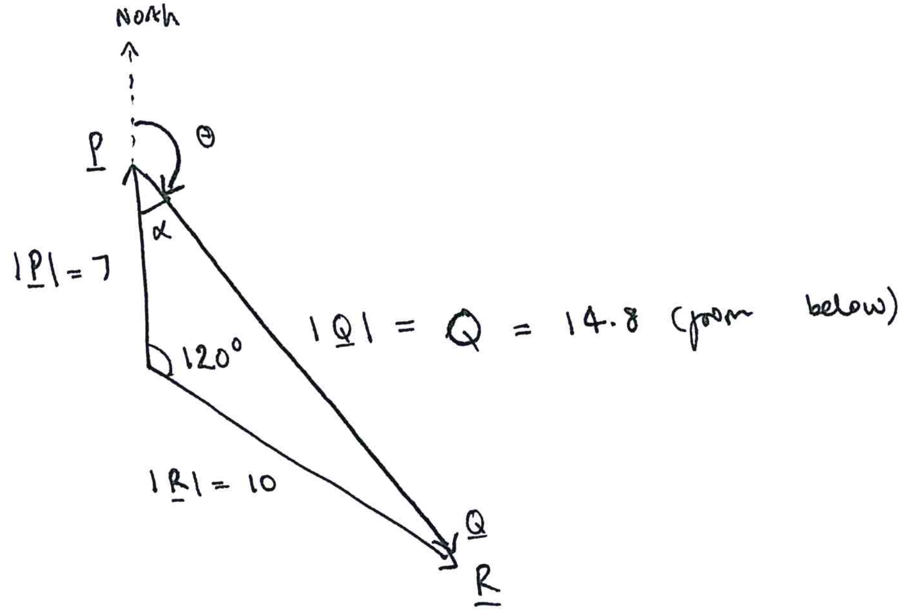
$|\underline{R}| = 6.6N.$ (2sf)

) $\dfrac{\sin\theta}{7} = \dfrac{\sin 60}{R}$

$\theta = \sin^{-1}\left(\dfrac{7\sin 60}{R}\right) = 68°$

Jan 2006 Q4

Draw vector diagram



so $\quad Q^2 = 10^2 + 7^2 - 2(10)(7)\cos 120$

$$Q = \sqrt{10^2 + 7^2 - 2(10)(7)\cos 120}$$

$$= 14.8\text{N} \quad (3sf)$$

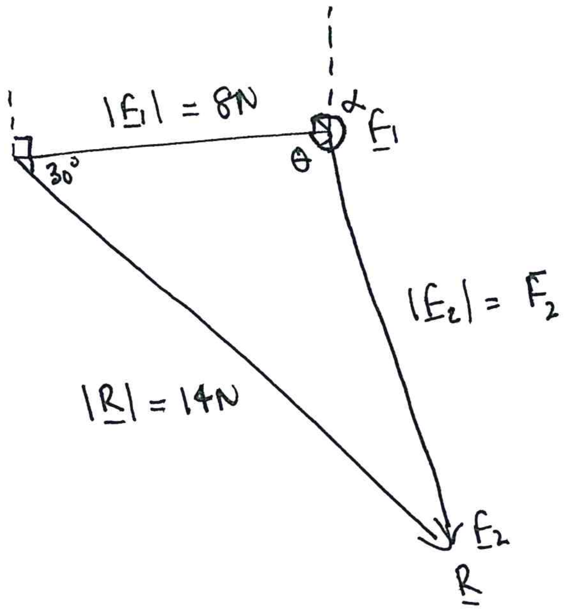
Hence

$$\frac{14.8}{\sin 120} = \frac{10}{\sin\alpha}$$

$$\sin\alpha = \frac{10\sin 120}{14.8} \Rightarrow \alpha = 35.8°$$

Hence bearing is $180 - 35.8 = 144°$ (3sf)

M1 Oct 2017 IAL Q4



i) $|\underline{F}_2| = F_2$; use cosine rule to find F_2.

$\Rightarrow \quad (F_2^2) = (8)^2 + (14)^2 - 2(8)(14)\cos 30$

$F_2 = \sqrt{8^2 + 14^2 - (16)(14)\frac{\sqrt{3}}{2}}$

$= 8.1N$ (2sf)

i) Bearing $= \alpha$: $\alpha = 360 - 90 - \theta$

Find θ : $\frac{\sin\theta}{14} = \frac{\sin 30}{8.124..}$

$\theta = \sin^{-1}\left(14 \times \frac{\sin 30}{8.124..}\right)$

$= 120.51°$

So bearing : $\alpha = 360 - 90 - 120.51..$

$= 149°$ (nearest degrees)

The following problems involve adding vectors together without using the vector addition / subtraction rules.

Usually, if the ~~fo~~ vector eg force vector is given with constants in, you are not required to draw the vectors out and form a vector triangle. (You will still have to draw out a vector if you want the angle between that vector and say $\hat{\imath}$ or $\hat{\jmath}$).

You must know that if a ~~vector~~ ~~say~~ ~~eg~~ ~~$x\hat{\imath}+y\hat{\jmath}$~~ ~~is~~ ~~parallel~~ force $\underline{F}_1$ for instance, is parallel to a vector $x\hat{\imath}+y\hat{\jmath}$ let's say, this means

$\underline{F}_1 \propto x\hat{\imath}+y\hat{\jmath}$

$\underline{F}_1 = a(x\hat{\imath}+y\hat{\jmath})$

can think of this as a proportionality constant.

In these questions it is vital you set up your equations using $\Sigma \underline{F}$ = resultant force, where $\Sigma \underline{F} = 0$ if there is equilibrium. Then collect your $\hat{\imath}$ and $\hat{\jmath}$ on the LHS and compare with $\hat{\imath}$, $\hat{\jmath}$ coefficients on RHS.

Note: I write $\hat{\imath}$, $\hat{\jmath}$, instead of $\underline{i}$, $\underline{j}$ (to signify that these are <u>unit vectors</u>).

M1 June 2017 Q1

In equilibrium so $\Sigma \underline{F} = 0 \; (= 0\hat{\underline{i}} + 0\hat{\underline{j}})$

So $(15\hat{\underline{i}} + \hat{\underline{j}}) + (5q\hat{\underline{i}} - p\hat{\underline{j}}) + (-3p\hat{\underline{i}} - q\hat{\underline{j}}) = 0\hat{\underline{i}} + 0\hat{\underline{j}}$

collect $\hat{\underline{i}}$, $\hat{\underline{j}}$; $\hat{\underline{i}}(15 + 5q - 3p) + \hat{\underline{j}}(1 - p - q) = 0\hat{\underline{i}} + 0\hat{\underline{j}}$

LHS = RHS therefore

$15 + 5q - 3p = 0$ ①

$1 - p - q = 0$ ②

So $p = 1 - q$ from ①; $15 + 5q - 3(1 - q) = 0$

$q = -1.5$

$p = 2.5$.

M1 June 2015 (IAL) Q1

$\underline{F}_1 = (2\hat{\underline{i}} + 3a\hat{\underline{j}})$ $\quad \underline{F}_2 = (2a\hat{\underline{i}} + b\hat{\underline{j}})$ $\quad \underline{F}_3 = (b\hat{\underline{i}} + 4\hat{\underline{j}})$

n equilibrium so $\Sigma \underline{F} = 0$ (no acceleration so $a = 0$)

o $\Sigma \underline{F} = 2\hat{\underline{i}} + 3a\hat{\underline{j}} + 2a\hat{\underline{i}} + b\hat{\underline{j}} + b\hat{\underline{i}} + 4\hat{\underline{j}} = 0\hat{\underline{i}} + 0\hat{\underline{j}}$

ollect $\hat{\underline{i}}, \hat{\underline{j}}$; $\quad \hat{\underline{i}}(2 + 2a + b) + \hat{\underline{j}}(3a + b + 4) = 0\hat{\underline{i}} + 0\hat{\underline{j}}$

HS = RHS so

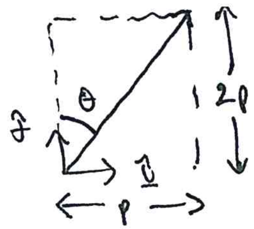
$$2 + 2a + b = 0 \quad (1)$$

$$3a + b + 4 = 0 \quad (2)$$

(2) − (1) gives $a = -2$ so $b = 2$.

M 1 June 2009 Q2

$\underline{F}_2 = p\hat{\underline{i}} + 2p\hat{\underline{j}} \Rightarrow$



o $\tan\theta = \frac{p}{2p} = \frac{1}{2}$

$\theta = \tan^{-1}\frac{1}{2} = 26.6^\circ$

o $\underline{F}_1 + \underline{F}_2 = \underline{R}$ where $\underline{R} = a\hat{\underline{i}} \; (= a(\hat{\underline{i}} + 0\hat{\underline{j}}))$

$\hat{\underline{i}} - 3\hat{\underline{j}} + p\hat{\underline{i}} + 2p\hat{\underline{j}} = a\hat{\underline{i}}$

$\hat{\underline{i}}(1+p) + \hat{\underline{j}}(2p-3) = a\hat{\underline{i}} + 0\hat{\underline{j}}$

.HS = RHS so :

$1 + p = a$ (1)

$2p - 3 = 0$ (2) $\Rightarrow \quad p = \frac{3}{2}$

M1 Jan 2012 Q3

$\underline{F}_1 = (7\hat{i} - 9\hat{j})$ N

$\underline{F}_2 = (5\hat{i} + 6\hat{j})$ N

$\underline{F}_3 = (p\hat{i} + q\hat{j})$ N

$\Sigma \underline{F} = 0$ since the particle is in equilibrium.

$\underline{F}_1 + \underline{F}_2 + \underline{F}_3 = 0 \Rightarrow (7\hat{i} - 9\hat{j}) + (5\hat{i} + 6\hat{j}) + (p\hat{i} + q\hat{j}) = 0\hat{i} + 0\hat{j}$

collect $\hat{i}, \hat{j}$; $\hat{i}(7+5+p) + \hat{j}(-9+6+q) = 0\hat{i} + 0\hat{j}$

LHS = RHS so

$7+5+p = 0$

$-9+6+q = 0$

$p = -12$

$q = 3$

)

$\underline{F}_1 + \underline{F}_2 = \underline{R}$

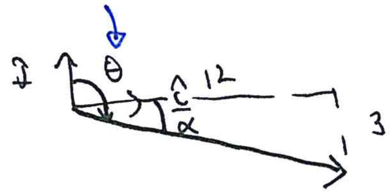
$7\hat{i} - 9\hat{j} + 5\hat{i} + 6\hat{j} = \underline{R}$

$12\hat{i} - 3\hat{j} = \underline{R}$

$\sqrt{(12)^2 + (-3)^2} = |\underline{R}| = 12.4$N (3sf)

c)

The angle we want.



so $\tan\alpha = \frac{3}{12}$

$\alpha = 14.03°$ so angle with $\hat{j}$ is $90 + 14.03$

so $\theta = 104°$ (3sf)

M1 June 2016 Q5 (IAL)

$\underline{F}_1 = 2\hat{\underline{i}} - 3\hat{\underline{j}}$ $\quad$ $\underline{F}_2 = p\hat{\underline{i}} + q\hat{\underline{j}}$ $\quad$ $\underline{F}_1 + \underline{F}_2 = \underline{R} = a(\hat{\underline{i}} + 2\hat{\underline{j}})$ ①

o sub into eq ① : $2\hat{\underline{i}} - 3\hat{\underline{j}} + p\hat{\underline{i}} + q\hat{\underline{j}} = a\hat{\underline{i}} + 2a\hat{\underline{j}}$

$\hat{\underline{i}}(2+p) + \hat{\underline{j}}(q-3) = a\hat{\underline{i}} + 2a\hat{\underline{j}}$

$2 + p = a$ ①

$q - 3 = 2a$ ② eliminate a.

so $q - 3 = 2(2+p)$ since $a = p + 2$ from ①

$q - 3 = 4 + 2p$

$0 = 2p - q + 7$ as required.

o $q = 11$: from eq ② $8 = 2a$

$a = 4$ hence $\underline{R} = 4\hat{\underline{i}} + 8\hat{\underline{j}}$ (= resultant force).

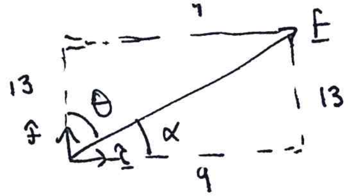
o $\Sigma \underline{F} = 4\hat{\underline{i}} + 8\hat{\underline{j}} = 2(\underline{a})$

$\underline{a} = 2\hat{\underline{i}} + 4\hat{\underline{j}}$

$|\underline{a}| = \sqrt{(2)^2 + (4)^2}$

$= \sqrt{20} = 4.5 \text{ ms}^{-2}$ (2sf)

M1 Jan 2014 (IAL) Q7



) $\underline{F} = (9\hat{i} + 13\hat{j})\,N \Rightarrow$

$\tan\theta = \dfrac{9}{13}$

$\theta = \tan^{-1}\dfrac{9}{13} = 34.7^\circ$ Alt: $\tan\alpha = \dfrac{13}{9} \Rightarrow \alpha = \tan^{-1}\dfrac{13}{9}$ so $\theta = (90 - \alpha) = 34.7^\circ$

) So $\underline{P} + \underline{Q} = \underline{F}$; $\underline{P} = a(2\hat{i} - \hat{j})$ $\underline{Q} = b(\hat{i} + 3\hat{j})$

$a(2\hat{i} - \hat{j}) + b(\hat{i} + 3\hat{j}) = 9\hat{i} + 13\hat{j}$

$2a\hat{i} - a\hat{j} + b\hat{i} + 3b\hat{j} = 9\hat{i} + 13\hat{j}$

$\hat{i}(2a + b) + \hat{j}(-a + 3b) = 9\hat{i} + 13\hat{j}$

LHS = RHS so $2a + b = 9$ (1)

$-a + 3b = 13$ (2)

Sim. equations so from (1) $b = 9 - 2a$. $\therefore -a + 3(9 - 2a) = 13$

$\Rightarrow a = 2$

so $b = 5$ $(9 - 2(2))$

Note: $\underline{P}$ is parallel to $(2\hat{i} - \hat{j})$ therefore must be a multiple of that vector.

ie $\underline{P} = a(2\hat{i} - \hat{j})$.

$\underline{Q}$ is parallel to $(\hat{i} + 3\hat{j})$

ie $\underline{Q} = b(\hat{i} + 3\hat{j})$

M1 Jan 2009 Q6

o $\underline{F}_1 = (4\hat{i} - 5\hat{j})N$, $\underline{F}_2 = (p\hat{i} + q\hat{j})N$ mass = m kg

e are told $\underline{F}_1 + \underline{F}_2 = \underline{R}$ where $\underline{R} = a(\hat{i} - 2\hat{j})$

)

$\tan\alpha = \frac{2a}{a}$

$\alpha = 63.4°$ $\Rightarrow$ $\theta = 90 + \alpha = 153°$ (3sf)

) $\underline{F}_1 + \underline{F}_2 = \underline{R}$

$4\hat{i} - 5\hat{j} + p\hat{i} + q\hat{j} = a\hat{i} - 2a\hat{j}$

lect $\hat{i},\hat{j}$ $\hat{i}(4+p) + \hat{j}(q-5) = a\hat{i} - 2a\hat{j}$

S = RHS so

$4 + p = a$ ①

$q - 5 = -2a$ ②

from eq ① : $a = 4 + p$, so sub into eq ②

$q - 5 = -2(4+p)$ yields $2p + q + 3 = 0$

) $q = 1$ $|\underline{a}| = 8\sqrt{5}$ m/s^2

$\Sigma \underline{F} = m\underline{a}$ we know $\Sigma\underline{F} = \underline{F}_1 + \underline{F}_2 = \underline{R} = (a\hat{i} - 2a\hat{j})N$.

nce $q = 1$, from eq ② $a = 2$ so $\underline{R} = 2\hat{i} - 4\hat{j}$

$\Rightarrow$ $2\hat{i} - 4\hat{j} = m\underline{a}$ $\Rightarrow$ $\underline{a} = \frac{2}{m}\hat{i} - \frac{4}{m}\hat{j}$

$|\underline{a}| = \sqrt{\left(\frac{2}{m}\right)^2 + \left(-\frac{4}{m}\right)^2} = \frac{\sqrt{20}}{m} = 8\sqrt{5}$ so $m = \frac{\sqrt{20}}{8\sqrt{5}} = \frac{1}{4}$

c) alt: find q. then find $|\underline{R}| = \sqrt{}$

hence $|\Sigma\underline{F}| = |m\underline{a}|$

$|\Sigma\underline{F}| = m|\underline{a}|$

$\sqrt{20} = m\,8\sqrt{5}$

$m = \frac{\sqrt{20}}{8\sqrt{5}} = \frac{1}{4}$

M1 Jan 2007 Q1

Note : underlined letters are equivalent to bold letters (eg in exam papers / textbooks)
→ They ~~repre~~ signify that the quantity is a vector.

MPH

When dealing with forces, we may deal with them in either vector or scalar form.

→ In vector form the force will be in bold eg $\underline{F}_1$

→ In scalar form the force will not be bold eg F

There is an intrinsic relationship between the vector and scalar form :

The magnitude of $\underline{F}_1$ is F, ie. $|\underline{F}| = F$

↳ Note that we could write that $|\underline{F}| = X$ or $|\underline{F}| = Y$, but it makes sense to say that the magnitude of the vector eg $\underline{F}_1$ ~~has~~ is F

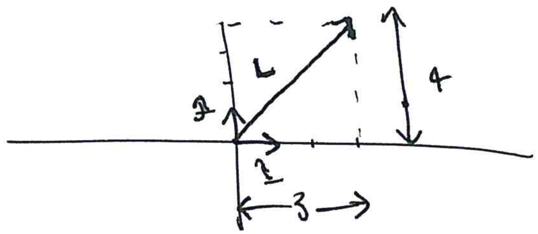
↳ Some questions will say a force $\underline{Q}$ has magnitude X N, which means $|\underline{Q}| = X$

What is the magnitude of a vector? The length of that vector!

eg the vector $3\underline{\hat{i}} + 4\underline{\hat{j}}$ has magnitude 5.

How do we calculate the magnitude / length of a vector?

We use Pythagoras (and a sketch) eg $3\underline{\hat{i}} + 4\underline{\hat{j}}$ ⇒



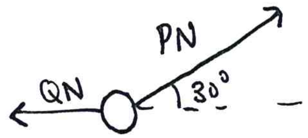
$L = \sqrt{4^2+3^2}$
$= 5.$

We could have labelled the vector $3\underline{\hat{i}} + 4\underline{\hat{j}}$ as $\underline{F}_1$, ie

$\underline{F}_1 = 3\underline{\hat{i}} + 4\underline{\hat{j}}$, so : the length of $\underline{F}_1 = |\underline{F}_1| = \sqrt{3^2+4^2} = 5.$

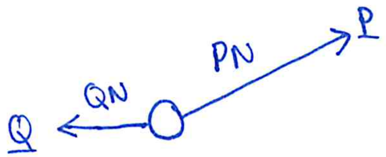
If $\underline{F}_1$ is a force then the length will have units of Newtons.

So in our example (M1 Jan 2007 Q1)



we have been given magnitudes as in the exam paper these letters are not bold.

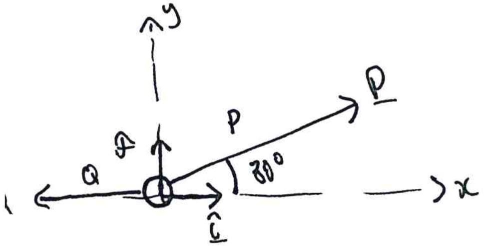
we could redraw this to show the vectors that these magnitudes came from.



ie $|\underline{P}| = P$ (N)

$|\underline{Q}| = Q$ (N)

we can actually recover the vectors $\underline{P}$ and $\underline{Q}$ from the diagram (if we place a coordinate system on the particle.



So $\underline{P} = P\cos 30\,\hat{\imath} + P\sin 30\,\hat{\jmath}$

$\underline{Q} = -Q\,\hat{\imath} + 0\,\hat{\jmath}$

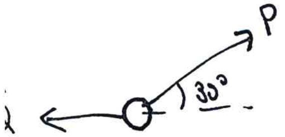
} we can actually recover the magnitudes P and Q by applying pythagoras to these vectors.

$$\Rightarrow \quad |\underline{P}| = \sqrt{(P\cos 30)^2 + (P\sin 30)^2} = \sqrt{P^2(\sin^2 30 + \cos^2 30)} = P \quad \text{since } \sin^2\theta + \cos^2\theta \equiv 1$$

$$\Rightarrow \quad |\underline{Q}| = \sqrt{(-Q)^2 + 0^2} = Q$$

Note: we could have called the vectors $\underline{X}$ and $\underline{Y}$ instead of $\underline{P}$ and $\underline{Q}$, but it makes sense to call them P and Q after their magnitudes P and Q

So in this example the particle is in equilibrium. If we were to apply Newton's II Law (N2L) to this particle in the horizontal ($\hat{i}$) and vertical ($\hat{j}$) direction, we would get these equations:



N2L for particle $R(\rightarrow)$ $\Sigma F = ma$ (not $\Sigma \underline{F} = m\underline{a}$ because we are dealing with scalars)

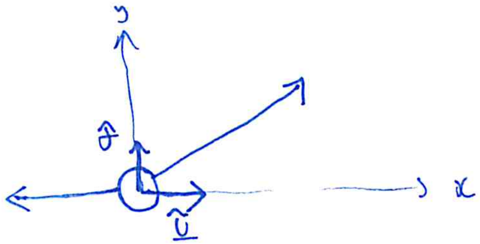
$$P\cos 30 - Q = 0 \quad (1) \quad (a=0)$$

$R(\uparrow)$ $\Sigma F = ma$

$$P\sin 30 - 24 = 0 \quad (2)$$

Now we could recover these equations by dealing with P and Q in their vector form ($\underline{P}$ and $\underline{Q}$)

we would still use N2L, and we would write:



$$\Sigma \underline{F} = m\underline{a}$$

$\underline{a} = 0\hat{i} + 0\hat{j}$ in vector form

where $\underline{F}_1 = \underline{P} = P\cos 30\,\hat{i} + P\sin 30\,\hat{j}$

$\underline{F}_2 = \underline{Q} = -Q\,\hat{i} + 0\,\hat{j}$

$\underline{F}_3 = \underline{W} = 0\,\hat{i} - 24\,\hat{j}$

The weight

so $P\cos 30\,\hat{i} + P\sin 30\,\hat{j} - Q\,\hat{i} - 24\hat{j} = 0\hat{i} + 0\hat{j}$ ($\underline{a} = \underline{0}$)

collect $\hat{i}$, $\hat{j}$;

$$\hat{i}(P\cos 30 - Q) + \hat{j}(P\sin 30 - 24) = 0\hat{i} + 0\hat{j}$$

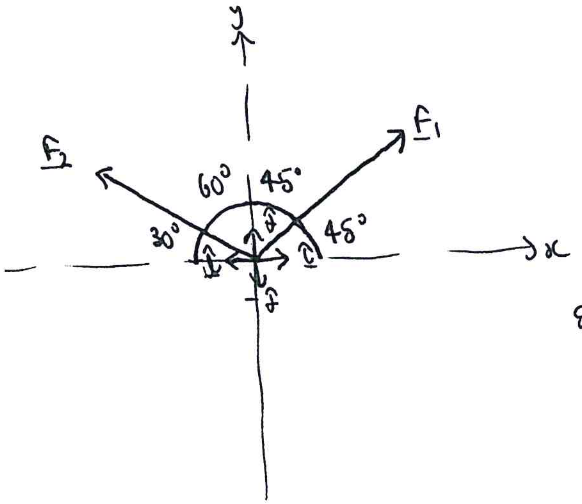
LHS = RHS so

$$\left.\begin{aligned} P\cos 30 - Q &= 0 \quad (1) \\ P\sin 30 - 24 &= 0 \quad (2) \end{aligned}\right\} \text{ same equations as above!}$$

So, resolving as we normally do avoids dealing with vectors, but the vectors are hidden in there regardless.

Challenge p96 A2

MPH



we are told resultant force is $3\hat{i} + 5\hat{j}$

ie $\underline{F_1} + \underline{F_2} = \underline{R}$

$\underline{F_1} + \underline{F_2} = 3\hat{i} + 5\hat{j}$ (a) (If we want to solve using vectors we must put $\underline{F_1}$ and $\underline{F_2}$ into vector form, using $\hat{i}$ and $\hat{j}$ unit vectors)

so $\underline{F_1} = |\underline{F_1}| \cos 45\, \hat{i} + |\underline{F_1}| \sin 45\, \hat{j}$

let's ~~label~~ ~~call~~ write $|\underline{F_1}|$ as simply F_1 because we know $|\underline{F_1}|$ is a scalar and ~~fu~~ F_1 represents a scalar and follows same naming convention 'F_1'

so $\underline{F_1} = F_1 \cos 45\, \hat{i} + F_1 \sin 45\, \hat{j}$ (1)

$\underline{F_2} = -F_2 \cos 30\, \hat{i} + F_2 \sin 30\, \hat{j}$ (2)

sub (1), (2) into eq (a)

$$F_1 \cos 45\, \hat{i} + F_1 \sin 45\, \hat{j} - F_2 \cos 30\, \hat{i} + F_2 \sin 30\, \hat{j} = 3\hat{i} + 5\hat{j}$$

collect $\hat{i}, \hat{j}$;

$$\hat{i}(F_1 \cos 45 - F_2 \cos 30) + \hat{j}(F_1 \sin 45 + F_2 \sin 30) = 3\hat{i} + 5\hat{j}$$

e resolve

RHS = LHS so

$F_1 \cos 45 - F_2 \cos 30 = 3$ (3)

$F_1 \sin 45 + F_2 \sin 30 = 5$ (4)

} Then we would solve for F_1, F

Method ② for Challenge p96 A2

$\Sigma F_{\rightarrow} = 3$

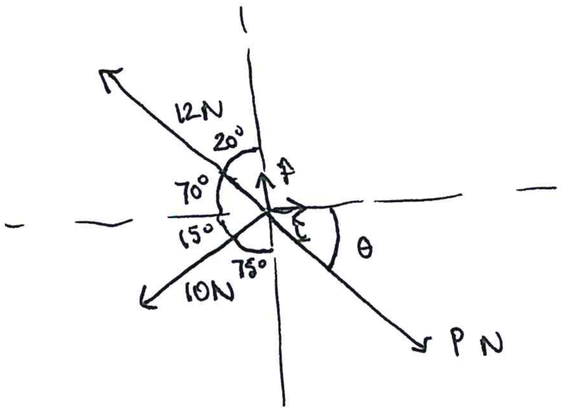
$R(\rightarrow)$ $F_1 \cos 45 - F_2 \cos 30 = 3$

$\Sigma F_{\uparrow} = 5$

$R(\uparrow)$ $F_1 \sin 45 + F_2 \sin 30 = 5$

This does seem a lot easier, and it is, but it is good to know where these equations come from.

Because we are told that the resultant force is $3\underline{i} + 5\underline{j}$, we know that in the $\underline{i}$ direction $(\rightarrow)$ the resultant force is 3; So we can ~~set~~ resolve in the horizontal direction ($\underline{i}$ direction) and set this equal to 3. The same applies for the $\underline{j}$ (vertical) direction.



system is in equilibrium:

$$\Sigma \underline{F} = m\underline{a} = 0$$

$R(\uparrow)$ $\Sigma F_{\uparrow} = 0$

$12\cos 20 - 10\cos 75 - P\sin\theta = 0$ ①

$R(\rightarrow)$ $\Sigma F_{\rightarrow} = 0$

$12\cos 70 + 10\cos 15 = P\cos\theta$ ②

To find θ: eq ① can be written as $P\sin\theta = 12\cos 20 - 10\cos 75$ ①

eq ② $P\cos\theta = 12\cos 70 + 10\cos 15.$ ②

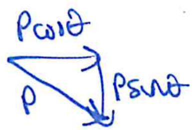
So if we do $\frac{①}{②}$ we eliminate P: $\tan\theta = \dfrac{12\cos 20 - 10\cos 75}{12\cos 70 + 10\cos 15}$

$= 0.631..$

$\theta = 32.2°$ (3sf)

So we sub in θ to eq ① or ② to find P.

alternatively: recognise that we have found the components of P; $P\sin\theta$ and $P\cos\theta$. ie



so $P^2 = (P\cos\theta)^2 + (P\sin\theta)^2 = (12\cos 70 + 10\cos 15)^2 + (12\cos 20 - 10\cos 75)^2$