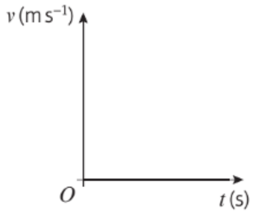
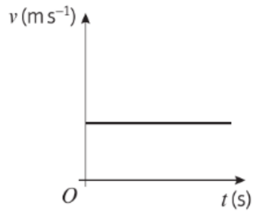
Velocity-Time Graphs

You can represent the motion of an object on a velocity–time graph. Velocity is always plotted on the vertical axis and time on the horizontal axis.

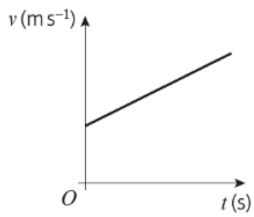
In these graphs v represents the velocity of an object in metres per second and t represents the time taken in seconds.



The velocity is zero and the object is stationary.



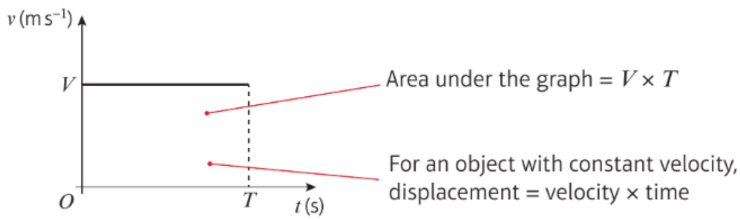
The velocity is unchanging and the object is moving with constant velocity.



The velocity is increasing at a constant rate and the object is moving with constant acceleration.

- **Acceleration is the rate of change of velocity.**
  - **In a velocity–time graph the gradient represents the acceleration.**
  - **If the velocity–time graph is a straight line, then the acceleration is constant.**

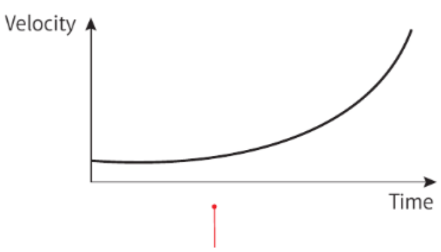
This velocity–time graph represents the motion of an object travelling in a straight line at constant velocity V m s^{-1} for time T seconds.



- **The area between a velocity–time graph and the horizontal axis represents the distance travelled.**
  - **For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point *t* represents the displacement at time *t*.**

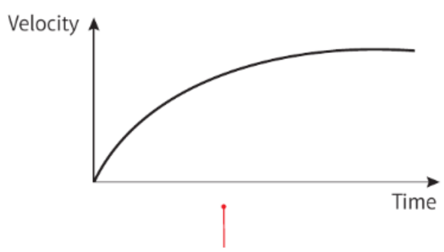
These velocity–time graphs represent the motion of a particle travelling in a straight line. They show examples of increasing and decreasing acceleration.

Increasing acceleration



The rate of increase of velocity is increasing with time and the gradient of the curve is increasing.

Decreasing acceleration



The rate of increase of velocity is decreasing with time and the gradient of the curve is decreasing.

2. Velocity-time graphs.

Any point on such a graph will have coordinates (t, v), in which v is the velocity after a time t.

Worked Example 2.

Figure 2 shows the velocity-time graph for the motion of the tennis ball described in example 1.

It was thrown into the air with a velocity of 7 m s^{-1}. It has zero velocity at 0.71 seconds and returns to the ground after 1.62 seconds with a velocity of -8.8 m s^{-1}.

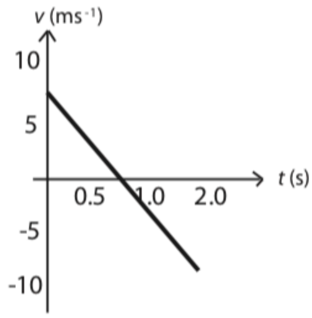


Figure 2

The gradient of a velocity-time graph is acceleration

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Example Question:

3. Figure 3 shows the velocity-time graph for a moped, which travelled between two sets of traffic lights on a straight road.
   (a) What is the moped's acceleration in each of the time intervals OX, XY and YZ?

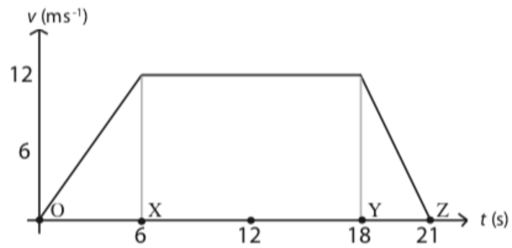

Figure 3

   (b) What was the total distance between the two sets of traffic lights?

3. (a) OX: a = 2.0 m s^{-2}, XY: a = 0.0 m s^{-2}, YZ: a = −4.0 m s^{-2}

3. (b) Distance = $(0.5 \times 6 \times 12) + (12 \times 12) + (0.5 \times 3 \times 12)$ = 198 m = 200 m (2 s.f.).

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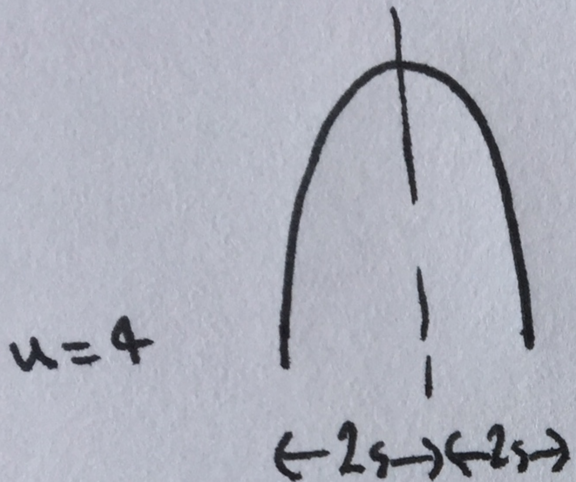
M1 Jan 2009 Q2a

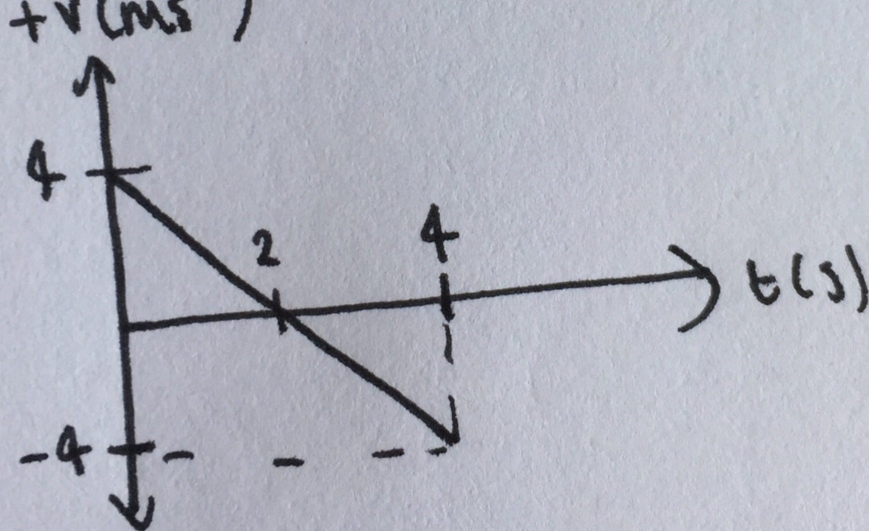
A small ball is projected vertically upwards from ground level with speed u m s^{-1}. The ball takes 4 s to return to ground level.

(a) Draw, in the space below, a velocity-time graph to represent the motion of the ball during the first 4 s.

M1 Jan 2009 Q2a



a)



M1 Jan 2013 Q5

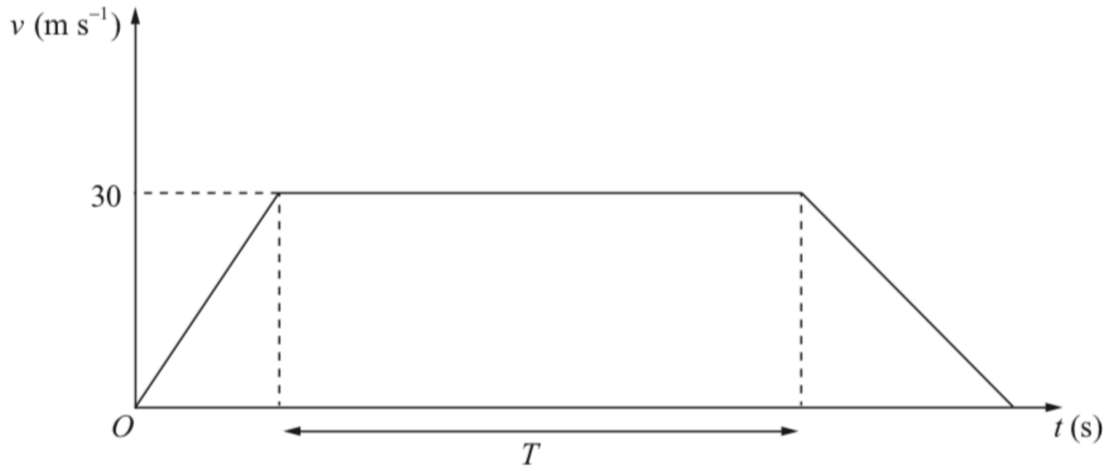


Figure 4

The velocity-time graph in Figure 4 represents the journey of a train P travelling along a straight horizontal track between two stations which are 1.5 km apart. The train P leaves the first station, accelerating uniformly from rest for 300 m until it reaches a speed of 30 m s^{-1}. The train then maintains this speed for T seconds before decelerating uniformly at 1.25 m s^{-2}, coming to rest at the next station.

(a) Find the acceleration of P during the first 300 m of its journey.

(2)

(b) Find the value of T.

(5)

A second train Q completes the same journey in the same total time. The train leaves the first station, accelerating uniformly from rest until it reaches a speed of V m s^{-1} and then immediately decelerates uniformly until it comes to rest at the next station.

(c) Sketch on the diagram above, a velocity-time graph which represents the journey of train Q.

(2)

(d) Find the value of V.

(6)

a) $S = 300$
$u = 0$
$v = 30$
$a =$
$t =$

$v^2 = u^2 + 2as$

$\frac{v^2 - u^2}{2s} = a$

$\frac{30^2 - 0^2}{2(300)} = a = 1.5 m/s^2$

b) Area under line = total displacement = 1500m.

Area of first triangle = 300m

Area of rectangle = $(30 \times T)$

Area of third triangle ⇒ need to work it out : $S =$, $u = 30$, $v = 0$, $a = -1.25$, $t =$

$\frac{v^2 - u^2}{2a} = s$

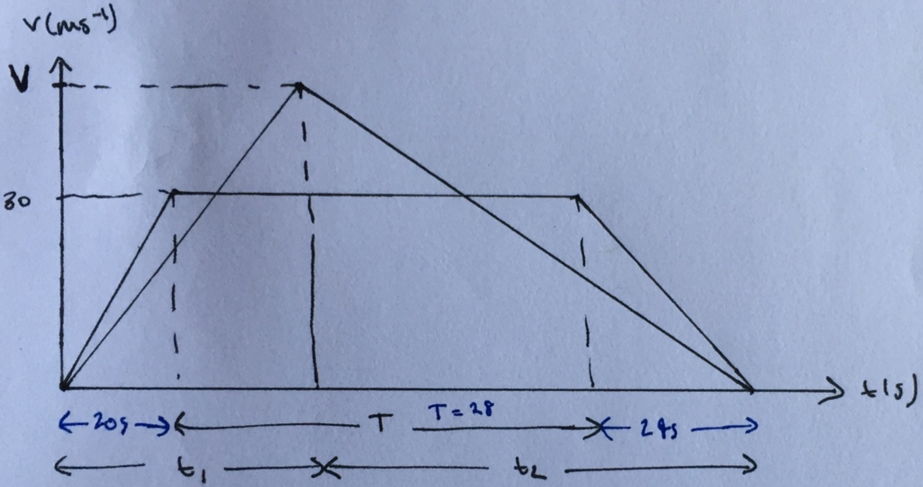
$\frac{0^2 - 30^2}{2 \times -1.25} = s = 360m$

= 360m

Therefore $300 + 30T + 360 = 1500$

$T = 28$

c)



Train Q travels same displacement as P and same time taken.

d) Method 1: $\frac{1}{2} V t_1 + \frac{1}{2} V t_2 = 1500$ (area under triangle = 1500m (total displacement))

$\frac{1}{2} V (t_1 + t_2) = 1500$ (1)

$t_1 + t_2$ is total journey time which we can get from train P.

$t_1 + t_2 = 72s$ (using SUVAT) so $V = 41.7 m/s$

$S =$, $u = 0$, $v = 30$, $a = 1.5$, $t =$ } $30 = 0 + 1.5t$, $t = 20s$

$S =$, $u = 30$, $v = 0$, $a = -1.25$, $t =$ } $0 = 30 - 1.25t$, $t = 24s$

Total time is $20 + 24 + 28 = 72s$

so $t_1 + t_2 = 72s$ then sub that into eq (1)

M1 June 2014 Q3

3. A ball of mass 0.3 kg is released from rest at a point which is 2 m above horizontal ground. The ball moves freely under gravity. After striking the ground, the ball rebounds vertically and rises to a maximum height of 1.5 m above the ground, before falling to the ground again. The ball is modelled as a particle.

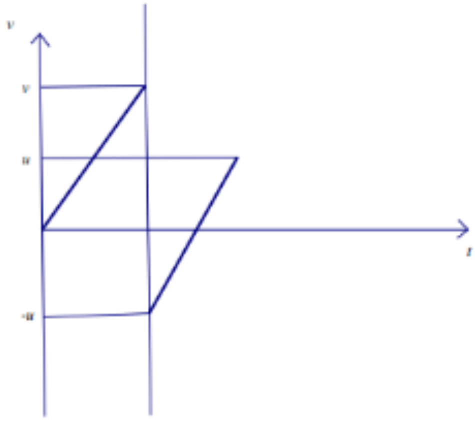
(a) Find the speed of the ball at the instant before it strikes the ground for the first time.
(2)

(b) Find the speed of the ball at the instant after it rebounds from the ground for the first time.
(2)

(c) Find the magnitude of the impulse on the ball in the first impact with the ground.
(2)

(d) Sketch, in the space provided, a velocity-time graph for the motion of the ball from the instant when it is released until the instant when it strikes the ground for the second time.
(3)

(e) Find the time between the instant when the ball is released and the instant when it strikes the ground for the second time.
(4)

Question Number	Scheme	Marks
3a	Using $v^2 = u^2 + 2as$: $v^2 = 4g$, $v = \sqrt{4g}$ or 6.3 or 6.26 (m s^{-1})	M1,A1 (2)
b	Rebounds to 1.5 m, $0 = u^2 - 3g$, $u = \sqrt{3g}$, 5.4 or 5.42 (m s^{-1})	M1A1 (2)
c	Impulse = $0.3(6.3 + 5.4) = 3.5$ (Ns)	M1A1 (2)
d	If speed downwards is taken to be positive:  First line Second line $-u, u,$	B1 B1 B1 (3)
e.	Use of suvat to find t_1 or t_2, $\sqrt{4g} = gt_1$ $t_1 = \sqrt{\frac{4}{g}} = 0.64$ s $\sqrt{3g} = gt_2$ $t_2 = \sqrt{\frac{3}{g}} = 0.55$ s Total time = $t_1 + 2t_2 = 1.7$ s or 1.75 s	M1A1 (t_1 or t_2) **DM**1A1 (4) [13]

M1 Jan 2016 IAL Q4

4. A small stone is projected vertically upwards from the point O and moves freely under gravity. The point A is 3.6 m vertically above O. When the stone first reaches A, the stone is moving upwards with speed 11.2 m s^{-1}. The stone is modelled as a particle.

(a) Find the maximum height above O reached by the stone.

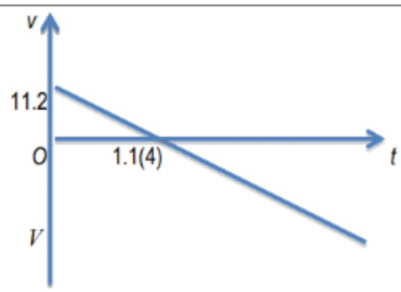
(4)

(b) Find the total time between the instant when the stone was projected from O and the instant when it returns to O.

(5)

(c) Sketch a velocity-time graph to represent the motion of the stone from the instant when it passes through A moving upwards to the instant when it returns to O. Show, on the axes, the coordinates of the points where your graph meets the axes.

(4)

4(a)	$0^2 = 11.2^2 - 2gd$ $d = 6.4$ max ht. $= 3.6 + 6.4 = 10$ m	M1 A1 A1 A1 (4)
ALT	$11.2^2 = u^2 - 2g \times 3.6$ $u = 14$ $0^2 = 14^2 - 2gh$ $h = 10$ m	M1 A1 A1 A1 (4)
(b)	$10 = \frac{1}{2}gt^2$ $t = \frac{10}{7}$ Total $= 2\text{x}\frac{10}{7} = 2.9$ or 2.86	M1 A1 A1 **dM1** A1 (5)
(c)		B1 single line **dB1** $V <$ -11.2 B1 11.2 B1 1.1(4) (4) **13**

M1 Jan 2017 IAL Q7

7.

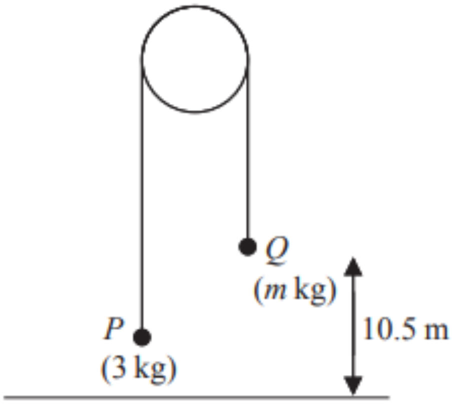


Figure 5

Two particles P and Q have masses 3 kg and m kg respectively ($m > 3$). The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut and the hanging parts of the string vertical. The particle Q is at a height of 10.5 m above the horizontal ground, as shown in Figure 5. The system is released from rest and Q moves downwards. In the subsequent motion P does not reach the pulley. After the system is released, the tension in the string is 33.6 N.

(a) Show that the magnitude of the acceleration of P is 1.4 m s^{-2}.

(3)

(b) Find the value of m.

(3)

The system is released from rest at time $t = 0$. At time T_1 seconds after release, Q strikes the ground and does not rebound. The string goes slack and P continues to move upwards.

(c) Find the value of T_1

(3)

At time T_2 seconds after release, P comes to instantaneous rest.

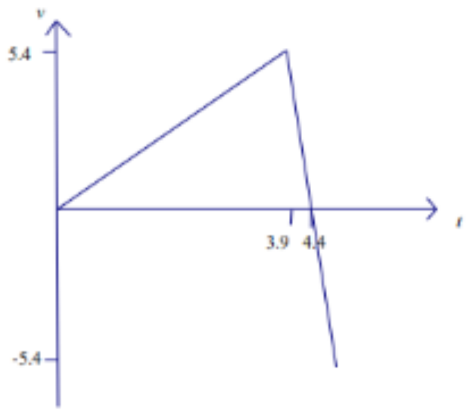
(d) Find the value of T_2

(3)

At time T_3 seconds after release ($T_3 > T_1$) the string becomes taut again.

(e) Sketch a velocity-time graph for the motion of P in the interval $0 \leqslant t \leqslant T_3$

(2)

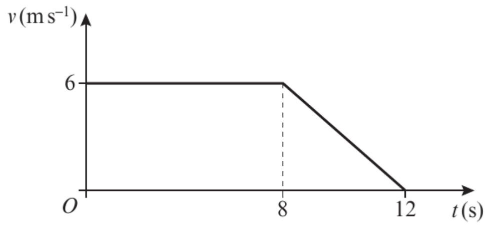
7a	Motion of P: $T - 3g = 3a$	M1
	$33.6 - 3g = 3a$	A1
	$a = 1.4\ (\text{m s}^{-2})$ ***Given Answer***	A1
		(3)
7b	Motion of Q: $mg - T = ma$	M1
	$mg - 33.6 = 1.4m$	A1
	$m = 4$	A1
		(3)
7c	Use of $s = (ut+)\frac{1}{2}at^2$: $10.5 = \frac{1}{2} \times 1.4 \times t^2$	M1A1
	$T_1 = \sqrt{15} = 3.9$ or better	A1
		(3)
7d	Use $v^2 = (u^2+)2as$ to find speed of particles when Q hits ground: $v = \sqrt{2 \times 1.4 \times 10.5}\ (= \sqrt{29.4})$	M1
	Use $v = u + at$ to find additional time for P to come to rest: $0 = \sqrt{29.4} - gt$	**DM1**
	Total time : $T_2 = \sqrt{15} + \frac{\sqrt{29.4}}{9.8} = 4.4$ or 4.43	A1
		(3)
7e		B1 Shape **DB1 ft** their values for 5.4, -5.4, 3.9, 4.4 (or T_1 T_2) (2)

Example 2 p134 AS Textbook

The figure shows a velocity–time graph illustrating the motion of a cyclist moving along a straight road for a period of 12 seconds. For the first 8 seconds, she moves at a constant speed of $6\,\text{m s}^{-1}$. She then decelerates at a constant rate, stopping after a further 4 seconds.

a Find the displacement from the starting point of the cyclist after this 12 second period.

b Work out the rate at which the cyclist decelerates.



a The displacement s after 12 s is given by the area under the graph.

$$s = \frac{1}{2}(a + b)h$$
$$= \frac{1}{2}(8 + 12) \times 6$$
$$= 10 \times 6 = 60$$

The displacement of the cyclist after 12 s is 60 m.

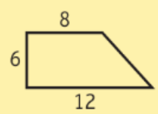
b The acceleration is the gradient of the slope.

$$a = \frac{-6}{4} = -1.5$$

The deceleration is $1.5\,\text{m s}^{-2}$.

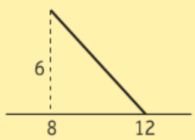
Model the cyclist as a particle moving in a straight line.

The displacement is represented by the area of the trapezium with these sides.



You can use the formula for the area of a trapezium to calculate this area.

The gradient is given by $\dfrac{\text{difference in the } v\text{-coordinates}}{\text{difference in the } t\text{-coordinates}}$



Example 3 p134-5 AS Textbook

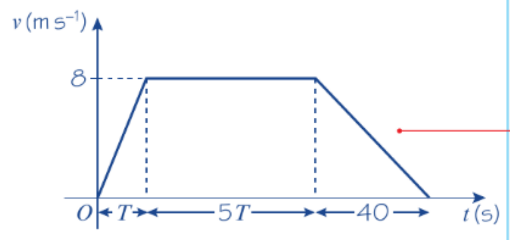
A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 m s^{-1} in T seconds. The particle then travels at a constant velocity of 8 m s^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s.

a Sketch a velocity–time graph to illustrate the motion of the particle.

Given that the total displacement of the particle is 600 m:

b find the value of T.

a



If the particle accelerates from rest and decelerates to rest this means the initial and final velocities are zero.

b The area of the trapezium is:

$s = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(5T + 6T + 40) \times 8$

$= 4(11T + 40)$

The displacement is 600 m.

$4(11T + 40) = 600$

$44T + 160 = 600$

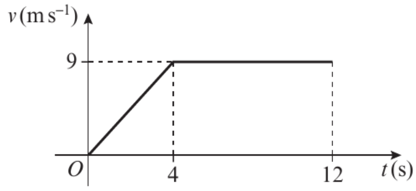
$T = \frac{600 - 160}{44} = 10$

The length of the shorter of the two parallel sides is $5T$. The length of the longer side is $T + 5T + 40 = 6T + 40$.

Exercise 9B p135-136 AS Textbook

Q1

1 The diagram shows the velocity–time graph of the motion of an athlete running along a straight track. For the first 4 s, he accelerates uniformly from rest to a velocity of 9 m s^{-1}.
This velocity is then maintained for a further 8 s. Find:



a the rate at which the athlete accelerates

b the displacement from the starting point of the athlete after 12 s.

1 a $a = \frac{9}{4} = 2.25$

The athlete accelerates at a rate of 2.25 m s^{-2}.

b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(8+12)\times 9 = 90$

The displacement of the athlete after 12 s is 90 m.

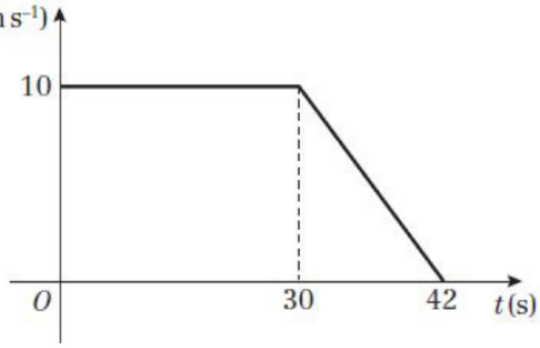
Q2

2 A car is moving along a straight road. When $t = 0$ s, the car passes a point A with velocity $10\ \text{m s}^{-1}$ and this velocity is maintained until $t = 30$ s. The driver then applies the brakes and the car decelerates uniformly, coming to rest at the point B when $t = 42$ s.

a Sketch a velocity–time graph to illustrate the motion of the car.

b Find the distance from A to B.

2 a



b $s = \frac{1}{2}(a+b)h$

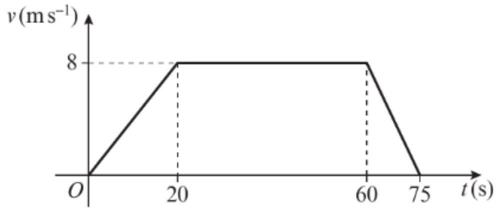
$= \frac{1}{2}(30+42) \times 10 = 360$

The distance from A to B is 360 m.

Q3

3 The diagram shows the velocity–time graph of the motion of a cyclist riding along a straight road. She accelerates uniformly from rest to 8 m s^{-1} in 20 s. She then travels at a constant velocity of 8 m s^{-1} for 40 s. She then decelerates uniformly to rest in 15 s.

Find:



a the acceleration of the cyclist in the first 20 s of motion **(2 marks)**

b the deceleration of the cyclist in the last 15 s of motion **(2 marks)**

c the displacement from the starting point of the cyclist after 75 s. **(2 marks)**

3 a $a = \frac{8}{20} = 0.4$

The acceleration of the cyclist is 0.4 m s^{-2}.

b $a = -\frac{8}{15} = -0.533$ (to 3 s.f.)

The deceleration of the cyclist is 0.533 m s^{-2}.

c $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(40+75)\times 8 = 460$

After 75 s, the displacement of the cyclist is 460 m.

Q4

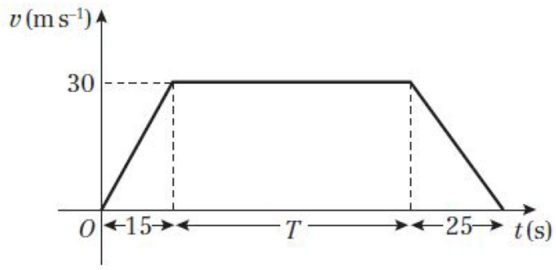
4 A motorcyclist starts from rest at a point S on a straight race track. He moves with constant acceleration for 15 s, reaching a velocity of $30\ \text{m s}^{-1}$. He then travels at a constant velocity of $30\ \text{m s}^{-1}$ for T seconds. Finally he decelerates at a constant rate coming to rest at a point F, 25 s after he begins to decelerate.

a Sketch a velocity–time graph to illustrate the motion. **(3 marks)**

Given that the distance between S and F is 2.4 km:

b calculate the time the motorcyclist takes to travel from S to F. **(3 marks)**

4 a



4 b $s = \frac{1}{2}(a+b)h$

$$2400 = \tfrac{1}{2}(T + (15 + T + 25)) \times 30$$
$$= 15(2T + 40)$$
$$2T + 40 = \tfrac{2400}{15} = 160$$

$$T = \frac{160 - 40}{2} = 60$$

The time taken to travel from S to F is $(15 + T + 25) = 100$ s.

Q5

5 A train starts from a station X and moves with constant acceleration of $0.6\,\text{m}\,\text{s}^{-2}$ for 20 s. The velocity it has reached after 20 s is then maintained for T seconds. The train then decelerates from this velocity to rest in a further 40 s, stopping at a station Y.

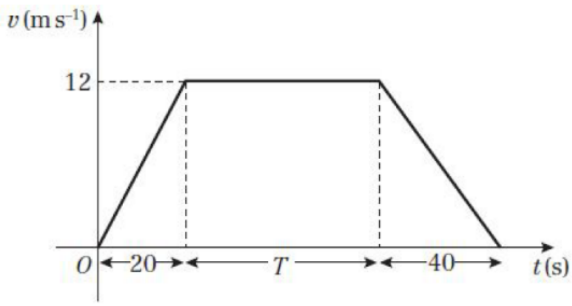
a Sketch a velocity–time graph to illustrate the motion of the train. **(3 marks)**

Given that the distance between the stations is 4.2 km, find:

b the value of T **(3 marks)**

c the distance travelled by the train while it is moving with constant velocity. **(2 marks)**

5 a The velocity after 20 s is given by



velocity = acceleration x time $= 0.6 \times 20 = 12$

b $s = \frac{1}{2}(a+b)h$

$$4200 = \tfrac{1}{2}(T + (20 + T + 40)) \times 12$$
$$= 6(2T + 60)$$
$$2T + 60 = \frac{4200}{6} = 700$$
$$T = \frac{700 - 60}{2} = 320$$

c While at constant velocity: v = 12 m s^{-1}, t = 320 s

distance travelled = 12 × 320 = 3840 m

Q6

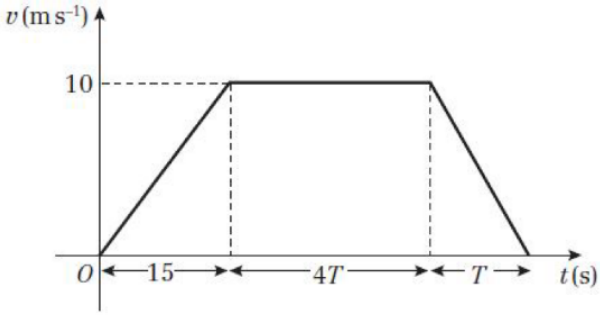
6 A particle moves along a straight line. The particle accelerates from rest to a velocity of $10\,\text{m s}^{-1}$ in 15 s. The particle then moves at a constant velocity of $10\,\text{m s}^{-1}$ for a period of time. The particle then decelerates uniformly to rest. The period of time for which the particle is travelling at a constant velocity is 4 times the period of time for which it is decelerating.

a Sketch a velocity–time graph to illustrate the motion of the particle. **(3 marks)**

Given that the displacement from the starting point of the particle after it comes to rest is 480 m

b find the total time for which the particle is moving. **(3 marks)**

6 a



6 b $s = \frac{1}{2}(a+b)h$

$$480 = \tfrac{1}{2}(4T + (15 + 4T + T))10$$
$$= 5 \times (15 + 9T)$$

$$9T + 15 = \frac{480}{5} = 96$$

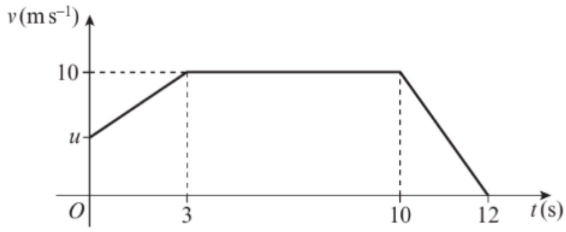
$$T = \frac{96 - 15}{9} = 9$$

Total time travelling = 15 + 5T = 15 + (5 × 9) = 60

The particle travels for a total of 60 s.

Q7

7 A particle moves 100 m in a straight line. The diagram is a sketch of a velocity–time graph of the motion of the particle. The particle starts with velocity u m s^{-1} and accelerates to a velocity of 10 m s^{-1} in 3 s. The velocity of 10 m s^{-1} is maintained for 7 s and then the particle decelerates to rest in a further 2 s. Find:



a the value of u **(3 marks)**

b the acceleration of the particle in the first 3 s of motion. **(3 marks)**

7 a Area = trapezium + rectangle + triangle

$$100 = \tfrac{1}{2}(u+10)\times 3 + 7\times 10 + \tfrac{1}{2}\times 2\times 10$$
$$= \tfrac{3}{2}(u+10) + 70 + 10$$
$$\tfrac{3}{2}(u+10) = 100 - 70 - 10 = 20$$
$$u = 20\times\tfrac{2}{3} - 10$$
$$= \tfrac{10}{3}$$

b $a = \dfrac{10 - \frac{10}{3}}{3} = \frac{20}{9} = 2.22$ (to 3 s.f.)

The acceleration of the particle is 2.22 m s^{-2}.

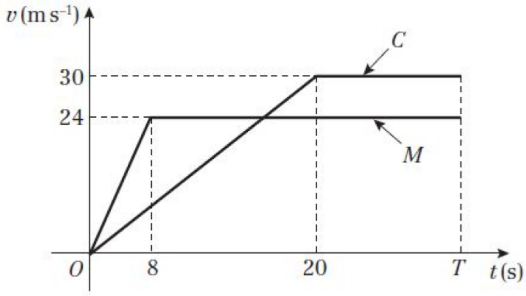
Q8

8 A motorcyclist M leaves a road junction at time $t = 0$ s. She accelerates from rest at a rate of 3 m s^{-2} for 8 s and then maintains the velocity she has reached. A car C leaves the same road junction as M at time $t = 0$ s. The car accelerates from rest to 30 m s^{-1} in 20 s and then maintains the velocity of 30 m s^{-1}. C passes M as they both pass a pedestrian.

a On the same diagram, sketch velocity–time graphs to illustrate the motion of M and C. **(3 marks)**

b Find the distance of the pedestrian from the road junction. **(3 marks)**

8 a For M, velocity = acceleration x time = $3 \times 8 = 24$



b Let C overtake M at time T seconds.

The distance travelled by M is given by

$$s = \frac{1}{2}(8 \times 24) + 24 \times (T - 8)$$
$$= 24(T - 4)$$

8 b The distance travelled by C is given by

$$s = \tfrac{1}{2}(a + b)h = \tfrac{1}{2}(T - 20 + T) \times 30$$
$$= 15\,(2T - 20)$$

At the point of overtaking the distances are equal.

$$24(T - 4) = 15(2T - 20)$$
$$24T - 96 = 30T - 300$$
$$6T = 204$$
$$T = \tfrac{204}{6} = 34$$

$$s = 24(T - 4)$$
$$= 24(34 - 4) = 720$$

Challenge

Challenge

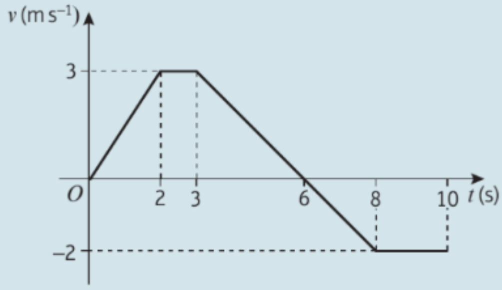
The graph shows the velocity of an object travelling in a straight line during a 10-second time interval.

a After how long did the object change direction?

b Work out the total distance travelled by the object.

c Work out the displacement from the starting point of the object after:

i 6 seconds **ii** 10 seconds.



a The object changed direction after 6 s, as this is when the velocity changed from positive to negative.

b While travelling at positive velocity:

$s_p = \frac{1}{2}(1+6)\times 3 = \frac{1}{2}\times 21 = 10.5$

While travelling at negative velocity:

$s_n = \frac{1}{2}(4+2)\times 2 = \frac{1}{2}\times 12 = 6$

The total distance travelled by the object = $s_p + s_n = 10.5 + 6 = 16.5$ m

c **i** Using the value calculated in **b**, after 6 s the displacement of the object is $s_p = 10.5$ m.

ii In the first 6 seconds, displacement is positive.
In the last 4 seconds, displacement is negative.

Hence, using the values calculated in **b**, total displacement = $s_p + (-s_n) = 10.5 + (-6) = 4.5$ m.

Ex 9C AS Textbook p141

Q12

12 A cyclist travels with constant acceleration $x\,\text{m s}^{-2}$, in a straight line, from rest to $5\,\text{m s}^{-1}$ in 20 s. She then decelerates from $5\,\text{m s}^{-1}$ to rest with constant deceleration $\frac{1}{2}x\,\text{m s}^{-2}$. Find:

a the value of x **(2 marks)**

b the total distance she travelled. **(4 marks)**

You could draw a velocity-time graph to work out the solutions. Otherwise, you could use SUVAT.

12 a $u = 0,\ v = 5,\ t = 20,\ a = x$

$v = u + at$

$5 = 0 + 20x$

$x = \frac{5}{20} = 0.25$

b While accelerating, $u = 0,\ v = 5,\ t = 20,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{0+5}{2}\right) \times 20 = 2.5 \times 20 = 50$$

12 b While decelerating, $u = 5,\ v = 0,\ a = -\frac{1}{2}, x = -0.125,\ t = ?$

$v = u + at$

$0 = 5 - 0.125t$

$$t = \frac{5}{0.125} = 40$$

Now, $u = 5,\ v = 0,\ t = 40,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{5+0}{2}\right) \times 40 = 2.5 \times 40 = 100$$

The total distance travelled is the distance travelled while accelerating added to the distance travelled while decelerating = (50 + 100) = 150 m.

Q13

13 A particle is moving with constant acceleration in a straight line. It passes through three points, A, B and C, with velocities $20\,\text{m s}^{-1}$, $30\,\text{m s}^{-1}$ and $45\,\text{m s}^{-1}$ respectively. The time taken to move from A to B is t_1 seconds and the time taken to move from B to C is t_2 seconds.

a Show that $\frac{t_1}{t_2} = \frac{2}{3}$. **(3 marks)**

Given also that the total time taken for the particle to move from A to C is 50 s:

b find the distance between A and B. **(5 marks)**

You could draw a velocity-time graph to work out the solutions. Alternatively you can use SUVAT.

13 a From A to B

$$v = u + at$$
$$30 = 20 + at_1$$
$$at_1 = 10 \qquad \textbf{(1)}$$

From B to C

$$v = u + at$$
$$45 = 30 + at_2$$
$$at_2 = 15 \qquad \textbf{(2)}$$

Dividing **(1)** by **(2)**,

$$\frac{at_1}{at_2} = \frac{10}{15}$$
$$\frac{t_1}{t_2} = \frac{2}{3} \quad \text{as required.}$$

b From the result in part **a**

$$t_2 = \tfrac{3}{2}t_1$$
$$t_1 + t_2 = t_1 + \tfrac{3}{2}t_1 = \tfrac{5}{2}t_1 = 50$$
$$t_1 = \tfrac{2}{5} \times 50 = 20$$

From A to B, $u = 20,\ v = 30,\ t = 20,\ s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{20+30}{2}\right) \times 20 = 25 \times 20 = 500$$

The distance from A to B is 500 m.

Challenge

Challenge

A particle moves in a straight line from A to B with constant acceleration. The particle moves from A with velocity 3 m s^{-1}. It reaches point B with velocity 5 m s^{-1} t seconds later.

One second after the first particle leaves point A, a second particle also starts to move in a straight line from A to B with constant acceleration. Its velocity at point A is 4 m s^{-1} and it reaches point B with velocity 8 m s^{-1} at the same time as the first particle.

Find:

a the value of t

b the distance between A and B.

You could draw a velocity-time graph to work out the solutions. Otherwise, you could use SUVAT.

a Distance s is the same for both particles: AB. For the first particle: $u = 3$, $v = 5$, time taken is t seconds $s = \left(\frac{u+v}{2}\right)t = \left(\frac{3+5}{2}\right)t = 4t$ **(1)** For the second particle: $u = 4$, $v = 8$, time taken is $(t - 1)$ seconds, because the particle starts 1 second later than the first and arrives at the same time) $s = \left(\frac{u+v}{2}\right)(t-1) = \left(\frac{4+8}{2}\right)(t-1) = 6(t-1) = 6t-6$ **(2)** $4t = 6t - 6$ **(1)** and **(2)** $t = 3$ The time for the first particle to get from A to B is 3 s.
b Substituting this value of t into equation (**1**): $s = 4t = 4 \times 3 = 12$ The distance between A and B is 12 m. [Check by substituting into equation (**2**): $s = 6t - 6 = 6 \times 3 - 6 = 12$]

Mixed Exercise 9 p152-155

Q1

1 A car accelerates in a straight line at a constant rate, starting from rest at a point A and reaching a velocity of $45\,\text{km h}^{-1}$ in 20 s. This velocity is then maintained and the car passes a point B 3 minutes after leaving A.

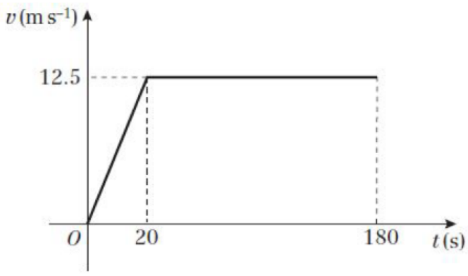
a Sketch a velocity–time graph to illustrate the motion of the car.

b Find the displacement of the car from its starting point after 3 minutes.

1 a $45\text{ km h}^{-1} = \frac{45 \times 1000}{3600}\text{ m s}^{-1}$

$= 12.5\text{ m s}^{-1}$

$3\text{ min} = 180\text{ s}$



b $s = \frac{1}{2}(a+b)h$

$= \frac{1}{2}(160+180) \times 12.5 = 2125$

The distance from A to B is 2125 m.

Q2

2 A particle is moving on an axis Ox. From time $t = 0$ s to time $t = 32$ s, the particle is travelling with constant velocity $15\,\text{m}\,\text{s}^{-1}$. The particle then decelerates from $15\,\text{m}\,\text{s}^{-1}$ to rest in T seconds.

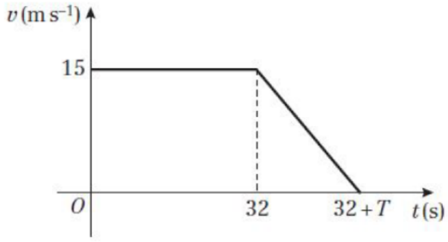
a Sketch a velocity–time graph to illustrate the motion of the particle.

The total distance travelled by the particle is 570 m.

b Find the value of T.

c Sketch a displacement–time graph illustrating the motion of the particle.

2 a



b $s = \frac{1}{2}(a+b)h$

$$570 = \tfrac{1}{2}(32+32+T)\times 15$$

$$\tfrac{15}{2}(T+64) = 570$$

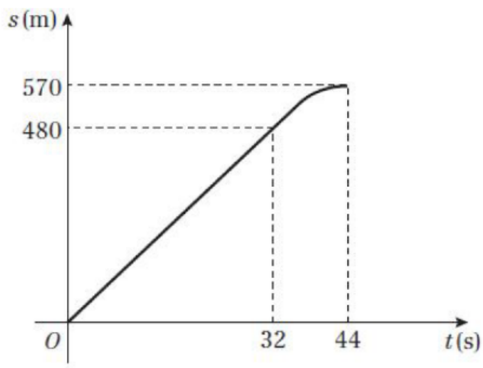
$$T+64 = \frac{570\times 2}{15} = 76$$

$$T = 76-64 = 12$$

c At $t = 32$, $s = 32\times 15 = 480$

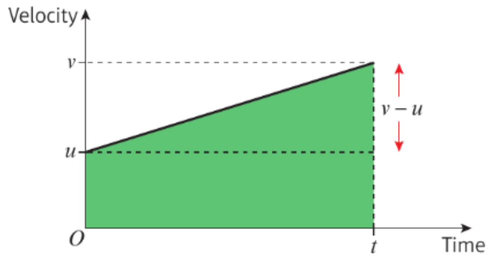
At $t = 44$, $s = 480 + \text{area of the triangle}$

$$= 480 + \tfrac{1}{2}\times 12\times 15 = 570$$



Q3

3 The velocity–time graph represents the motion of a particle moving in a straight line accelerating from velocity u at time 0 to velocity v at time t.



a Use the graph to show that:

i $v = u + at$ **ii** $s = \left(\frac{u + v}{2}\right)t$

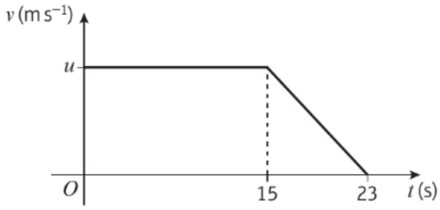
b Hence show that:

i $v^2 = u^2 + 2as$ **ii** $s = ut + \frac{1}{2}at^2$ **iii** $s = vt - \frac{1}{2}at^2$

3 a i Gradient of line = $\frac{v-u}{t}$ $a = \frac{v-u}{t}$ Rearranging: $v = u + at$ **ii** Shaded area is a trapezium area = $\left(\frac{u+v}{2}\right)t$ $s = \left(\frac{u+v}{2}\right)t$
b i Rearrange $v = u + at$ $t = \frac{v-u}{a}$ Substitute into $s = \left(\frac{u+v}{2}\right)t$ $s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$ $2as = v^2 - u^2$ $v^2 = u^2 + 2as$
ii Substitute $v = u + at$ into $s = \left(\frac{u+v}{2}\right)t$ $s = \left(\frac{u+u+at}{2}\right)t$ $s = \left(\frac{2u}{2}+\frac{at}{2}\right)t$ $s = ut + \frac{1}{2}at^2$
3 b iii Substitute $u = v - at$ into $s = ut + \frac{1}{2}at^2$ $s = (v-at)t + \frac{1}{2}at^2$ $s = vt - \frac{1}{2}at^2$

Q4

4 The diagram is a velocity–time graph representing the motion of a cyclist along a straight road.
At time $t = 0$ s, the cyclist is moving with velocity u m s^{-1}. The velocity is maintained until time $t = 15$ s, when she slows down with constant deceleration, coming to rest when $t = 23$ s.
The total distance she travels in 23 s is 152 m.
Find the value of u.



4 $s = \frac{1}{2}(a+b)h$

$$152 = \tfrac{1}{2}(15+23)u = 19u$$

$$u = \frac{152}{19} = 8$$

Q7

Draw a velocity-time graph for this question instead.

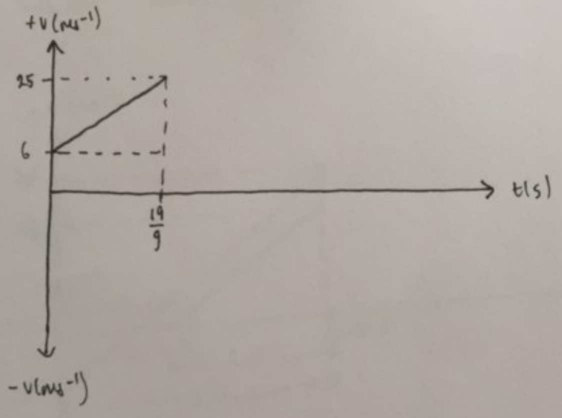
7 A ball is thrown vertically downward from the top of a tower with speed $6\,m\,s^{-1}$. The ball strikes the ground with speed $25\,m\,s^{-1}$. Find the time the ball takes to move from the top of the tower to the ground.

$s =$
$u = 6$
$v = 25$
$a = g$
$t =$

$\left.\right\} v = u + at$ so $25 = 6 + gt$

$\frac{19}{g} = t$

Take downwards direction as +v



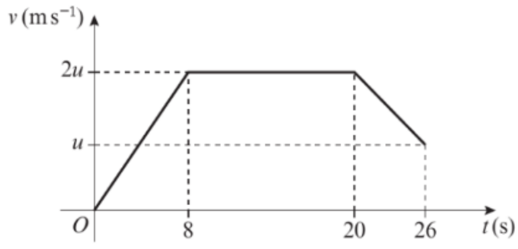
We know the acceleration (gradient) should be 'g'.

Let us calculate the gradient to check this is true.

$$\Rightarrow \frac{25-6}{\frac{19}{g}} = \frac{19 \times g}{19} = g \checkmark$$

Q9

9 A particle moves 451 m in a straight line. The diagram shows a speed–time graph illustrating the motion of the particle. The particle starts at rest and accelerates at a constant rate for 8 s reaching a speed of $2u\,\text{m s}^{-1}$. The particle then travels at a constant speed for 12 seconds before decelerating uniformly, reaching a speed of $u\,\text{m s}^{-1}$ at time $t = 26$ s. Find:



a the value of u

b the distance moved by the particle while its speed is less than $u\,\text{m s}^{-1}$.

9 a distance = area of triangle + area of rectangle + area of trapezium

$$451 = \tfrac{1}{2}\times 8\times 2u + 12\times 2u + \tfrac{1}{2}\times(u+2u)\times 6$$
$$= 8u + 24u + 9u = 41u$$
$$u = \tfrac{451}{41} = 11$$

b The particle is moving with speed less than $u\,\text{m s}^{-1}$ for the first 4 s

$$s = \tfrac{1}{2}\times 4\times 11 = 22$$

The distance moved with speed less than u m s^{-1} is 22 m.

Q12*

12 A pebble is projected vertically upwards with speed 21 m s^{-1} from a point 32 m above the ground. Find:

a the speed with which the pebble strikes the ground **(3 marks)**

b the total time for which the pebble is more than 40 m above the ground. **(4 marks)**

c Sketch a velocity–time graph for the motion of the pebble from the instant it is projected to the instant it hits the ground, showing the values of t at any points where the graph intercepts the horizontal axis. **(4 marks)**

12 a Take upwards as the positive direction.

$u = 21,\ s = -32,\ a = -9.8,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 21^2 + 2 \times (-9.8) \times (-32) = 441 + 627.2 = 1068.2$$
$$v = \sqrt{1068.2} = \pm 33 \text{ (to 2 s.f.)}$$

The velocity with which the pebble strikes the ground is -33 m s^{-1}.
The speed is 33 m s^{-1}.

b 40 m above the ground is 8 m above the point of projection.

$u = 21,\ s = 8,\ a = -9.8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$8 = 21t - 4.9t^2$$

$0 = 4.9t^2 - 21t + 8$, so using the quadratic formula,

$$t = \frac{21 \pm \sqrt{21^2 - 4 \times 4.9 \times 8}}{9.8} = \frac{21 \pm \sqrt{284.2}}{9.8} = 3.86, 0.423 \text{ (to 3 s.f.)}$$

The pebble is above 40 m between these times: $3.863... - 0.423... = 3.44$ (to 3 s.f.)
The pebble is more than 40 m above the ground for 3.44 s.

c Take upwards as the positive direction.

$u = 21, a = -9.8$

$$v = u + at = 21 - 9.8t \Rightarrow t = \frac{21 - v}{9.8}$$

From part **a**, the pebble hits the ground when $v = -33$.

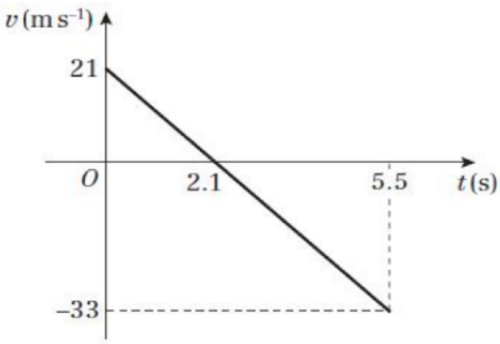
$$t = \frac{21 - v}{9.8} = \frac{21 - (-33)}{9.8} = \frac{54}{9.8} = 5.5 \text{ (to 2 s.f.)}$$

This is shown on the graph at point $(5.5, -33)$

The graph crosses the t-axis when $v = 0$.

$$t = \frac{21 - v}{9.8} = \frac{21 - 0}{9.8} = \frac{21}{9.8} = 2.1 \text{ (to 2 s.f.)}$$

So the graph passes through point $(2.1,\ 0)$



Q16

16 A car is being driven on a straight stretch of motorway at a constant velocity of $34\,\text{m s}^{-1}$, when it passes a velocity restriction sign *S* warning of road works ahead and requiring speeds to be reduced to $22\,\text{m s}^{-1}$. The driver continues at her velocity for 2 s after passing *S*. She then reduces her velocity to $22\,\text{m s}^{-1}$ with constant deceleration of $3\,\text{m s}^{-2}$, and continues at the lower velocity.

a Draw a velocity–time graph to illustrate the motion of the car after it passes *S*. **(2 marks)**

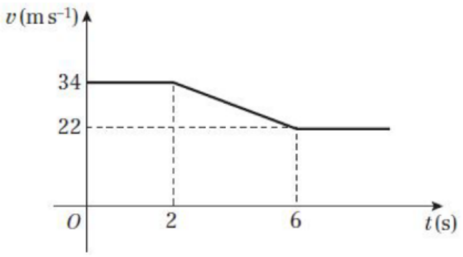
b Find the shortest distance before the road works that *S* should be placed on the road to ensure that a car driven in this way has had its velocity reduced to $22\,\text{m s}^{-1}$ by the time it reaches the start of the road works. **(4 marks)**

16 a To find time decelerating:

$u = 34,\ v = 22,\ a = -3,\ t = ?$

$v = u + at$

$22 = 34 - 3t \Rightarrow t = \dfrac{34 - 22}{3} = 4$



b distance = rectangle + trapezium

$s = 34 \times 2 + \frac{1}{2}(22 + 34) \times 4$
$= 68 + 112 = 180$

Distance required is 180 m.

Q17

17 A train starts from rest at station A and accelerates uniformly at $3x\,\text{m s}^{-2}$ until it reaches a velocity of $30\,\text{m s}^{-1}$. For the next T seconds the train maintains this constant velocity. The train then decelerates uniformly at $x\,\text{m s}^{-2}$ until it comes to rest at a station B. The distance between the stations is 6 km and the time taken from A to B is 5 minutes.

a Sketch a velocity–time graph to illustrate this journey. **(2 marks)**

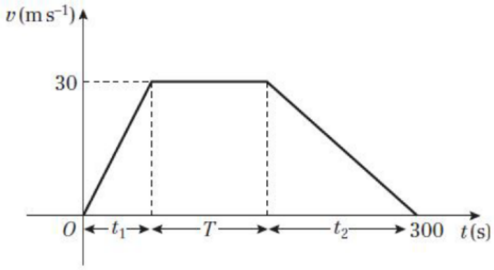
b Show that $\frac{40}{x} + T = 300$. **(4 marks)**

c Find the value of T and the value of x. **(2 marks)**

d Calculate the distance the train travels at constant velocity. **(2 marks)**

e Calculate the time taken from leaving A until reaching the point halfway between the stations. **(3 marks)**

17 a



b Acceleration is the gradient of a line.

For the first part of the journey, $3x = \frac{30}{t_1} \Rightarrow t_1 = \frac{30}{3x} = \frac{10}{x}$

For the last part of the journey, $-x = -\frac{30}{t_2} \Rightarrow t_2 = \frac{30}{x}$

$t_1 + T + t_2 = 300$

$\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300$, as required

c $s = \frac{1}{2}(a+b)h$

$6000 = \frac{1}{2}(T+300) \times 30 = 15T + 4500$

17 c $T = \frac{6000-4500}{15} = 100$

Substitute into the result in part **b**:

$\frac{40}{x} + 100 = 300 \Rightarrow \frac{40}{x} = 200$

$x = \frac{40}{200} = 0.2$

d From part **c**, $T = 100$

At constant velocity, distance = velocity × time $= 30 \times 100 = 3000$ (m)

The distance travelled at a constant speed is 3 km.

e From part **b**, $t_1 = \frac{10}{x} = \frac{10}{0.2} = 50$

Total distance travelled = 6 km (given) so halfway = 3 km = 3000 m

While accelerating, distance travelled is $\left(\frac{1}{2} \times 50 \times 30\right)$ m = 750 m.

At constant velocity, the train must travel a further 2250 m.

At constant velocity, time $= \frac{\text{distance}}{\text{velocity}} = \frac{2250}{30}$ s = 75 s

Time for train to reach halfway is (50 + 75) s = 125 s

Challenge

Complete the question then draw a velocity-time graph to illustrate the motion of both balls.

Challenge

A ball is projected vertically upwards with speed 10 m s^{-1} from a point X, which is 50 m above the ground. T seconds after the first ball is projected upwards, a second ball is dropped from X. Initially the second ball is at rest. The balls collide 25 m above the ground. Find the value of T.

Challenge

Find the time taken by the first ball to reach 25 m below its point of projection (25 m above the ground). Take upwards as the positive direction.

$u = 10,\ \ s = -25,\ \ a = -9.8,\ \ t = ?$

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ -25 &= 10t - 4.9t^2 \\ 0 &= 4.9t^2 - 10t - 25 \\ t &= 10 \pm \frac{\sqrt{102 + 4 \times 4.9 \times 25}}{9.8} \\ &= 3.5 \text{ (to 2 s.f.)} \end{aligned}$$

As we discard the negative solution. Find the time taken by the second ball to reach 25 m below its point of projection. Take downwards as the positive direction.

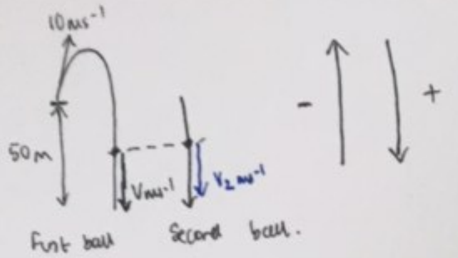
$u = 0,\ \ s = 25,\ \ a = 9.8,\ \ t = ?$

$$\begin{aligned} s &= ut + \tfrac{1}{2}at^2 \\ 25 &= 4.9t^2 \\ t &= 2.3 \text{ (to 2 s.f.)} \end{aligned}$$

Combining the two results:

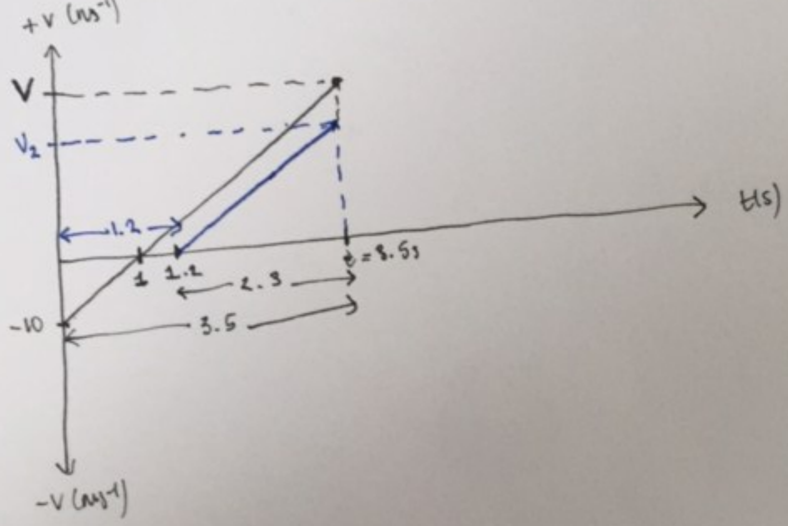
$T = 3.4989\ldots - 2.2587\ldots = 1.2$ (to 2 s.f. using exact figures)

Rest of solution overleaf.



V: speed ball A travels at at moment of collision

V_2 : speed ball B(2) travels at at moment of collision



First ball takes 3.5s to reach 25m below projection point

Second ball takes 2.3s to fall 25m downwards.

So 2nd ball is projected 1.2s after first ball

for first ball: s, $u = -10$, $v = 0$, $a = g$, t } $\frac{v^2 - (-10)^2}{2 \times g} = s$

$v = u + at$

$0 = -10 + gt$

$t = \frac{10}{g} \approx 1s.$

Ex 11C AS Textbook

Q3

3 At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is $v\,\text{m}\,\text{s}^{-1}$, where:

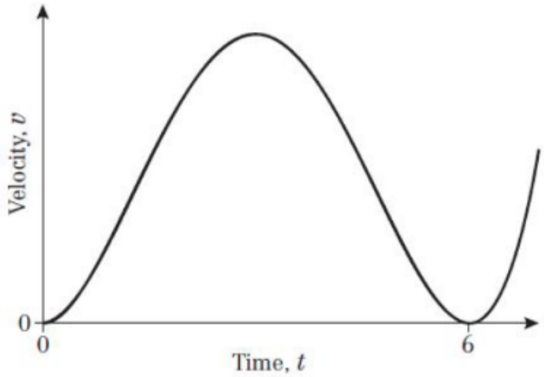
$$v = t^2(6 - t)^2,\ t \geqslant 0$$

a Sketch a velocity–time graph for the motion of P.

b Find the maximum value of v and the time at which it occurs.

3 a $v = t^2(6 - t)^2$

Velocity is zero when $t = 0$ and $t = 6$.
The graph touches the time axis at $t = 0$ and $t = 6$.
Graph only shown for $t \geq 0$, as this is the range over which equation is valid.



b $v = t^2(6 - t)^2$
$= t^2(36 - 12t + t^2)$
$= 36t^2 - 12t^3 + t^4$

$$\frac{dv}{dt} = 72t - 36t^2 + 4t^3$$

$\frac{dv}{dt} = 0$ when
$72t - 36t^2 + 4t^3 = 0$
$4t(18 - 9t + t^2) = 0$
$4t(3 - t)(6 - t) = 0$

The turning points are at $t = 0$, $t = 3$ and $t = 6$.
$v = 0$ when $t = 0$ and $t = 6$, therefore the maximum velocity occurs when $t = 3$.

When $t = 3$,
$v = 3^2(6 - 3)^2 = 9 \times 9 = 81$

The maximum velocity is 81 m s^{-1} and the body reaches this 3 s after leaving O.

Mixed Exercise 11 AS Textbook

Q5

5 A particle P passes through a point O and moves in a straight line. The displacement, s metres, of P from O, t seconds after passing through O is given by:

$$s = -t^3 + 11t^2 - 24t$$

a Find an expression for the velocity, $v \text{ m s}^{-1}$, of P at time t seconds. **(2 marks)**

b Calculate the values of t at which P is instantaneously at rest. **(3 marks)**

c Find the value of t at which the acceleration is zero. **(2 marks)**

d Sketch a velocity–time graph to illustrate the motion of P in the interval $0 \leqslant t \leqslant 6$, showing on your sketch the coordinates of the points at which the graph crosses the axes. **(3 marks)**

e Calculate the values of t in the interval $0 \leqslant t \leqslant 6$ between which the speed of P is greater than 16 m s^{-1}. **(6 marks)**

5 **a** $s = -t^3 + 11t^2 - 24t$

$v = \dfrac{ds}{dt}$

$= -3t^2 + 22t - 24 \text{ m s}^{-1}$

b P is at rest when $v = 0$.

$-3t^2 + 22t - 24 = 0$

$3t^2 - 22t + 24 = 0$

$(3t - 4)(t - 6) = 0$

$t = \frac{4}{3}$ or $t = 6$

P is at rest when $t = \frac{4}{3}$ and $t = 6$.

c $a = \dfrac{dv}{dt}$

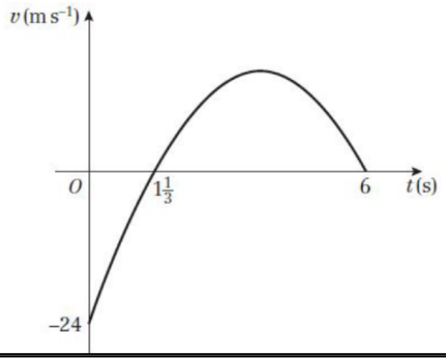
$= -6t + 22$

$a = 0$ when $0 = -6t + 22$

$t = \frac{11}{3}$

The acceleration is zero when $t = \frac{11}{3}$.

d



e The speed of P is 16 when $v = 16$ and $v = -16$.

When $v = 16$, $-3t^2 + 22t - 24 = 16$

$3t^2 - 22t + 40 = 0$

$(3t - 10)(t - 4) = 0$

$t = \frac{10}{3}$ or $t = 4$

When $v = -16$, $-3t^2 + 22t - 24 = -16$

$3t^2 - 22t + 8 = 0$

$$t = \frac{22 \pm \sqrt{22^2 - 4 \times 3 \times 8}}{2 \times 3}$$

$$= \frac{22 \pm \sqrt{388}}{6}$$

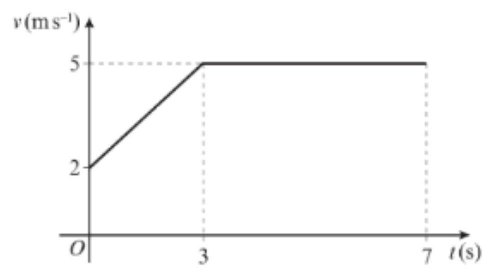
$= 0.384$ or 6.95

From the graph in part **d**, the required values are $0 \le t < 0.384$, $\frac{10}{3} < t < 4$.

Review Exercise 2 AS Textbook

Q1

1



The figure shows the velocity–time graph of a cyclist moving on a straight road over a 7 s period. The sections of the graph from $t = 0$ to $t = 3$, and from $t = 3$ to $t = 7$, are straight lines, The section from $t = 3$ to $t = 7$ is parallel to the t-axis.

State what can be deduced about the motion of the cyclist from the fact that:

a the graph from $t = 0$ to $t = 3$ is a straight line **(1)**

b the graph from $t = 3$ to $t = 7$ is parallel to the t-axis. **(1)**

c Find the distance travelled by the cyclist during this 7 s period. **(4)**

1 **a** For the first 3 s the cyclist is moving with constant acceleration.

b For the remaining 4 s the cyclist is moving with constant speed.

c area = trapezium + rectangle

$s = \frac{1}{2}(2+5)\times 3 + 5\times 4$

$= 10.5 + 20 = 30.5$

The distance travelled by the cyclist is 30.5 m

Q2

2 A train stops at two stations 7.5 km apart. Between the stations it takes 75 s to accelerate uniformly to a speed 24 m s^{-1}, then travels at this speed for a time *T* seconds before decelerating uniformly for the final 0.6 km.

a Draw a velocity–time graph to illustrate this journey. **(3)**

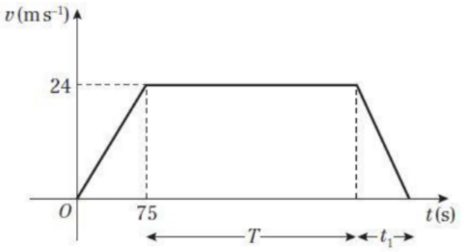
Hence, or otherwise, find:

b the deceleration of the train during the final 0.6 km **(3)**

c the value of *T* **(5)**

d the total time for the journey. **(4)**

2 a



b Let time for which the train decelerates be t_1 s.

While decelerating

area $= \frac{1}{2}\text{base} \times \text{height}$

$600 = \frac{1}{2} t_1 \times 24$

$t_1 = \frac{1200}{24} = 50$

Acceleration is represented by the gradient.

$a = -\frac{24}{t_1} = -\frac{24}{50} = -0.48$

The deceleration is $0.48\,\text{m s}^{-2}$

Q9

9 A ball is projected vertically upwards with a speed u m s^{-1} from a point A, which is 1.5 m above the ground. The ball moves freely under gravity until it reaches the ground. The greatest height attained by the ball is 25.6 m above A.

a Show that $u = 22.4$. **(3)**

The ball reaches the ground T seconds after it has been projected from A.

b Find, to three significant figures, the value of T. **(3)**

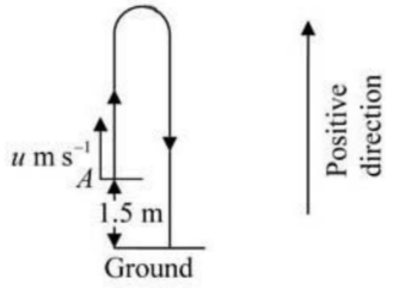
The ground is soft and the ball sinks 2.5 cm into the ground before coming to rest. The mass of the ball is 0.6 kg. The ground is assumed to exert a constant resistive force of magnitude F newtons.

c Find, to three significant figures, the value of F. **(4)**

d Sketch a velocity–time graph for the entire motion of the ball, showing the values of t at any points where the graph intercepts the horizontal axis. **(4)**

e State one physical factor which could be taken into account to make the model used in this question more realistic. **(1)**

9



9 a From A to the greatest height, taking upwards as positive.

$v = 0,\ a = -9.8,\ s = 25.6,\ u = ?$

$v^2 = u^2 + 2as$

$0^2 = u^2 + 2 \times (-9.8) \times 25.6$

$u^2 = 2 \times 9.8 \times 25.6 = 501.76$

$u = \sqrt{501.76} = 22.4$, as required.

b $u = 22.4,\ s = -1.5,\ a = -9.8,\ t = T$

$s = ut + \frac{1}{2}at^2$

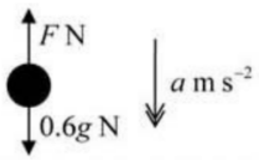
$-1.5 = 22.4T + \frac{1}{2}(-9.8)T^2$

$4.9T^2 - 22.4T - 1.5 = 0$

$$T = \frac{22.4 + \sqrt{(-22.4)^2 - 4 \times 4.9 \times -1.5}}{2 \times 4.9}$$

$= 4.637... = 4.64$ (3 s.f.)

c



To find the speed of the ball as it reaches the ground.

$u = 22.4,\ s = -1.5,\ a = -9.8,\ v = ?$

$v^2 = u^2 + 2as = 22.4^2 + 2 \times (-9.8) \times (-1.5) = 531.16$

To find the deceleration as the ball sinks into the ground.

$u^2 = 531.16,\ v = 0,\ s = 0.025,\ a = ?$

$v^2 = u^2 + 2as \Rightarrow 0^2 = 531.16 + 2 \times a \times 0.025$

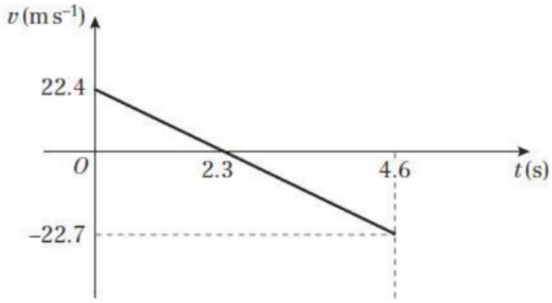
$a = -\frac{531.16}{0.05} = -10\,623.2$

$F = ma$

$0.6g - F = 0.6 \times (-10\,623.2)$

$F = 0.6g + 0.6 \times 10\,623.2 = 6380$ (3 s.f.)

d



9 e Consider air resistance during motion under gravity.

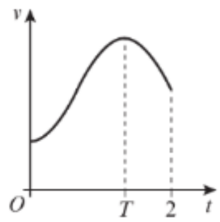
Exam Style Practice AS Level: Statistics and Mechanics

Q10

10 A particle, P, moves in a straight line through a fixed point O. The velocity of the particle, $v\,\text{m s}^{-1}$ at a time t seconds after passing through O is given by

$v = 3 + 9t^2 - 4t^3$, $0 \leqslant t \leqslant 2$.

The diagram shows a velocity–time graph of the motion of P.



Find the distance of P from O at time T seconds, when the particle is moving with maximum velocity. **(7)**

10 $v = 3 + 9t^2 - 4t^3$

When the particle is moving at maximum velocity, $a = \dfrac{dv}{dt} = 0$

$$0 = \frac{d(3 + 9t^2 - 4t^3)}{dt}$$
$$= 18t - 12t^2$$
$$= 6t(3 - 2t)$$

10 At $t = 0$, the particle moves at minimum velocity (see graph).
The particle has maximum velocity at $t = \frac{3}{2}$ seconds.

$$s = \int v\,dt = \int_0^{\frac{3}{2}} (3 + 9t^2 - 4t^3)\,dt$$

$$= \left[3t + \frac{9t^3}{3} - \frac{4t^4}{4}\right]_0^{\frac{3}{2}} = \left[3t + 3t^3 - t^4\right]_0^{\frac{3}{2}}$$

For $t = 0$, all terms are zero, so this becomes:

$$s = 3 \times \left(\frac{3}{2}\right) + 3 \times \left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^4$$

$$= \frac{9}{2} + \frac{81}{8} - \frac{81}{16} = \frac{153}{16}$$

The particle is moving at maximum velocity when it is $\frac{153}{16}$ m from O.

References:

Pearsonactivelearn AS Statistics and Mechanics textbook p1

www. mathscentre.ac.uk p2

Edexcel Mathematics IAL and 2o08 normal specification papers p3

Edexcel Mathematics IAL and 2o08 normal specification papers and mark schemes p5-12

Pearsonactivelearn AS Statistics and Mechanics textbook p13-42

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